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CONFIDENTIAL

Notations.

IV) All numerical values of nuclear cross- sections given in this paper are in units of 10^{-24} cm.^{2}

1.) $\sigma_c(C)$ is the capture cross section of carbon for thermal neutrons.

2.) $\sigma_{sc}(C)$ is the scattering cross- section of carbon for thermal neutrons.

3.) ε is the ratio of the number of thermal neutrons and the number of resonance neutrons absorbed by a sinlge uranium sphere ~~and~~ under the following circumstances: A single uranium sphere is embedded in an infinite space filled with carbon. Neutrons are generated in the carbon and the numbers of thermal neutrons and resonance neutrons produced per cc and sec are equal and have the same value trhoughout the whole infinite mass of carbon.

4.) E_0 is the energy at which the resonance absorption line of uranium has its maximum.

5.) v is the velocity of thermal neutrons .

6.) E_1 is the lower end of the resonance region of uranium below which we consider the ratio of the absorption coefficients for thermal neutrons and for resonance neutrons as constant.

7.) E_2 is the upper end of the resonance region of uranium above which we neglect the resonance absorption in uranium.

8.) k^{th} is the average number of collisions which a neutron ~~moving~~ "diffusing" in carbon survives within the energy region

9.) k^{res} is the average number of collision which a thermal neutron diffusing in carbon survives before being captured by carbon.

10.) $\lambda(C)$ is the mean free path for scattering of thermal neutrons in carbon .

11.) ρ is the density of thermal neutrons .

12.) Q is the number of thermal neutrons produced per cc and sec. in carbon.

13.) R is the radius of a sphere of uranium or uranium oxide which is embedded in carbon.

14.) A is the range of thermal neutrons in carbon defined by

$$A = \frac{\lambda(C)}{\sqrt{3}} \sqrt{\frac{\sigma_{sc}(C)}{\sigma_c(C)}}$$

15.) J_0^{th} is the number of thermal neutrons absorbed by a single uranium sphere embedde d in carbon provided that the uranium sphere absorbs each thermal neutron which reaches its surface.

16.) B is the range of the resonance neutrons in carbon defined by

$$B = \frac{\lambda^{res}(C)}{\sqrt{3}} \sqrt{k^{res}}$$

17.) J^{res} is the number of resonance neutrons absorbed by a single uranium sphere embedded in carbon.

18.) $\lambda^{res}(C)$ is the mean free path for scattering for neutrons of energy E_0 in carbon.

19.) $\mathcal{E}_0$ is the value of $\mathcal{E}$ for a uranium sphere which absorbs each thermal neutron which reaches its surface.

20.) J^{th} is the number of thermal neutrons absorbed by a single uranium sphere embedded in carbon.

21.) φ is defined by $J^{th} = \varphi J_0^{th}$ or $\mathcal{E} = \varphi \mathcal{E}_0$

22.) $\lambda(U)$ is the mean free path for scattering in the substance of the uranium sphere.

23.) N_U is the number of uranium atoms per c.c. in the substance of the uranium sphere.

24.) $\sigma_a(U)$ is the absorbing cross-section of uranium for thermal neutrons which inclusdes both the cross-section for fission and radiative capture but which does not include the cross-section for scattering.

25.) $\sigma_{sc}(U)$ is the cross-section of uranium for scattering of thermal neutrons.

26.) U is the range of thermal neutrons in the substance of the uranium sphere defined by

$$U = \frac{1}{\sqrt{3}} \sqrt{\frac{\lambda(U)}{N_U \sigma_a(U)}}$$

and in the particular case of pure uranium metal

$$U = \lambda(U) \sqrt{\frac{\sigma_{sc}(U)}{3\sigma_a(U)}}$$

Notations 3

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27.) α is the fraction of the thermal neutrons absorbed by carbon, (in an infinite mass of carbon) which contains a lattice of uranium spheres.

28.) J^{th} is the number of thermal neutrons absorbed per sec by a uranium sphere within a lattice of uranium spheres embedded in carbon.

29.) J^{res} is the number of resonance neutrons absorbed persec by a uranium sphere within a lattice of uranium spheres embedded in carbon.

30.) q is the fraction of the resonance neutrons produced in carbon which is absorbed as a thermal neutron by the lattice of uranium spheres if the number of thermal neutrons and ~~the~~ resonance neutrons produced per cc and sec. in the carbon are equal and have the same value throughout the whole infinite mass of carbon.

31.) α_m is the value of α for which q becomes maximum.

32.) q_m is the maximum value of q

33.) $q_{corr.}$ is the correct value of the fraction of the resonance neutrons produced in carbon which is absorbed as a thermal neutron by the lattice of uranium spheres ~~if~~ obtained by taking into account that the number of thermal neutrons produced per cc and sec. in the carbon near the uranium spheres is reduced due to the absorption of resonance neutrons by the uranium.

34.) V Is the volume of carbon per uranium sphere.

35.) L is the distance between neighbouring uranium spheres in a cubic or hexagonal close packed lattice.

36.) n is the number of H atoms per uranium atom in a mixture of uranium oxide and water.

37.) τ_0 and τ_{int} is the mean lifetime of a thermal neutron in water or a mixture of uranium oxide and water,

38.) $\sigma_c(H)$ is the capture cross section of hydrogen for thermal neutrons.

39.) p is the fraction of the resonance neutrons generated in a homogenous mixture of uranium oxide and water which is captured by uranium at resonance.

40.) s is the density of water in gm per cc in a homogeneous mixture of uranium oxide and water.

41.) i is the first factor in

$$\varepsilon = \left\{\frac{A^2}{B^2}\right\} \times \left\{\frac{1+R/A}{1+R/B}\,\varphi\right\}$$

42.) j is the second factor in the same expression.

43.) l is the critical radius of a graphite sphere which contains a lattice of uranium spheres giving the value of the radius for which the chain reaction becomes divergent.

In order to calculate we take into account that inside the uranium sphere the thermal neutron density obeys the equation

(15)

having as its solution

(16)

where

(17)

and for pure uranium metal we have

(18)

From equations 3, 4, and 16, we find that the number of thermal neutrons diffusing into the sphere per second is given by

where

(19)

For uranium in its pure state we have from No. 14, 18, and 19

(20a)

Where G stands for

For we can write , the difference being about 3.5% for

The first factor in expression No. 20a increases proportionately with the reciprocal value of the capture cross-section of carbon. The second factor

If the distance L of the neighboring uranium spheres in the lattice is large compared to the radius of the spheres the effect of the thermal neutron absorption of <u>one</u> uranium sphere on the thermal neutron absorption of its neighbors will be negligible. Nevertheless, the average thermal neutron density may be greatly reduced in the carbon by the presence of the uranium spheres, in particular if the range A of the thermal neutrons in carbon is lafge compared to the distance L. Under such conditions the average thermal neutron density determines with good approximation the number of thermal neutrons absorbed by one sphere in the lattice and we can write

(20)

In reality, the thermal neutron absorption will be somewhat higher so that No. 20 represents a conservative value, but the correction is small if the volume of the uranium spheres is small compared to the volume of the carbon. This can be seen, for instance, from Equation No. 5 which shows that for large values of r/R the thermal neutron density is close to , even for a uranium sphere which is black for thermal neutrons.

Further, since the distance L between neighboring uranium spheres within the lattice will be large compared to B, the range of the resonance neutrons in carbon, and we have for , the number of resonance neutrons absorbed by a uranium sphere within the lattice

(21)

From this it follows that q the fraction of all the neutrons which are absorbed by the uranium sphere in the thermal region alone is given by

(22)

or

(23)

This expression has its maximum value for

(24)

and for the maximum value for q we have

(25)

(26)

or

(27)

and from this we find

(31)

and if q has its maximum value q we have

and we have

(32)

This gives for the ratio of the volumes of carbon and uranium

(33)

or using (14) and (27) we obtain

(33a)

For L the distance between neighboring uranium spheres in a hexagonal or cubic closepacked lattice we have

(34)

Conditions for a chain reaction

If q denotes the fraction of fast neutrons emitted by uranium which are slowed down to the thermal region and are absorbed as thermal neutrons by uranium and if μ denotes the number of fast neutrons produced on the average by uranium for one thermal neutron absorbed by uranium then obviously

(49) $$\mu q > 1$$

is the condition for the possibility of a chain reaction. If this condition is fulfilled then a divergent chain reaction can be maintained in a sufficiently large system from which only a small fraction of the neutrons emitted by the uranium within can escape across the boundary of the system without being absorbed within.

In order to be on the conservative side we shall consider as a sufficient condition for a chain reaction

(50) $$\mu q\, 0.9 > 1$$

From this we find using equation No. 27 for ε

(51) $$\varepsilon > 11.3$$

as a sufficient condition.

In order to see now whether a chain reaction is possible we have to calculate from our formulae the numerical value of ε. We shall do that in the following under the assumption that the energy liberated in the chain reaction will maintain the carbon at a temperature of about 900C and in order to be on the conservative side we shall assume that the temperature of the uranium spheres in which most of the energy is liberated is, in spite of efficient cooling, about the same.

Since we have at room temperature $\sigma_c < 0.01$ we shall have at 900 C. a capture cross-section of carbon half of this value. The scattering cross-section of uranium for thermal neutrons we take to be $\sigma_{sc}(U) \sim 9$. Finally, at room temperature we take $\frac{\sigma_a(U)}{\sigma_{sc}(U)} = \frac{1}{2}$ and correspondingly we take at 900 C. $\frac{\sigma_a(U)}{\sigma_{sc}(U)} = \frac{1}{4}$. For a density of graphite of 1.7 and a density of uranium of 15 we then obtain from No. 14 for $R = 8$ cm

$$\varepsilon = 14$$

This being larger than the value required by No. 51 we conclude that in the circumstances we can expect a divergent chain reaction to take place in the system which we have investigated.

In reality the capture cross-section of carbon is perhaps much smaller than