CRITICAL DIMENSIONS

For a large sphere of graphite which contains a large number of small spheres of uranium l, the critical value for the radius of the graphite sphere for which the chain reaction becomes divergent may be calculated for various distributions of uranium within the graphite sphere. The optimum distribution of uranium is not uniform within the graphite sphere and will either decrease or increase with r according to whether we want to have a minimum amount of uranium or a minimum value for l. The treatment of this question may as well be postponed until the value of the carbon capture cross-section is known. It will then be possible to find the optimum distribution of uranium as a function of the distance from the center of the graphite sphere and give a value for l. In the meantime, a very rough approximation may give an idea of the order of magnitudes which are involved. In graphite of 1.7 density the average distance $\sqrt{\overline{r^2}}$ to which a fast neutron emitted by uranium diffuses away from its point of origin until it becomes a thermal neutron and reacts with uranium or carbon is about 50 cm. For $\mu q - 1 \cong \frac{1}{8}$ we find for the critical radius l from

$$l \sim \sqrt{\frac{3\overline{r^2}}{\mu q - 1}} \quad \text{or} \quad l \sim \sqrt{\frac{7500}{\mu q - 1}}$$

$l \sim 250$ cm. This corresponds to about 100 metric tons of graphite. If the carbon capture cross-section is lower, then l will be smaller ~~and the amount of graphite required might perhaps be as low as 20 tons~~.

The amount of uranium required can be calculated from equation No. 33a

$$\frac{4\pi}{3} R^3 / v = \frac{1 - q_m}{6} \frac{R^2}{B^2} \frac{1}{1 + R/B} .$$

It may be ~~[illegible]~~ kept down by choosing a smaller value for R than the value corresponding to the maximum value of ε. For R = 5cm. and $q_m = 0.6$ 33a gives $\frac{4\pi}{3} R^3 / v = 0.0336$ corresponding to about 30 tons of uranium for 100 tons of graphite. *29 (*) (29)

~~For larger values of q_m we find a smaller ratio of uranium to carbon.~~

~~For q we would have as the ratio of weights .~~

0.0222 $\frac{0.4}{6}$ $\frac{25}{42}$ $\frac{1}{1.877}$ $\frac{15}{1.7} = \frac{150}{700}$

21.3 tons

$\frac{10}{450}$

R = 5 cm ~~isoto~~ is at room temp close to the value which makes the expression for ε given in 20a 21 a maximum. repl.

CRITICAL DIMENSIONS

For a large sphere of graphite which contains a large number of small spheres of uranium l, the critical value for the radius of the graphite sphere for which the chain reaction becomes divergent may be calculated for various distributions of uranium within the graphite sphere. The optimum distribution of uranium is not uniform within the graphite sphere and will either decrease or increase with r according to whether we want to have a minimum amount of uranium or a minimum value for l. The treatment of this question may as well be postponed until the value of the carbon capture cross-section is known. It will then be possible to find the optimum distribution of uranium as a function of the distance from the center of the graphite sphere and give a value for l. In the meantime, a very rough approximation may give an idea of the order of magnitudes which are involved. In graphite of 1.7 density the average distance $\sqrt{\overline{\tau^2}}$ to which a fast neutron emitted by uranium diffuses away from its point of origin until it becomes a thermal neutron and reacts with uranium or carbon is about 50 cm. For $(\mu q - 1) \cong \frac{1}{\rho}$ we find for the critical radius l from

$$l \sim \sqrt{\frac{3\overline{\tau^2}}{\mu q - 1}}$$

$l \sim 250$ cm. This corresponds to about 100 metric tons of graphite. If the carbon capture cross-section is lower, then l will be smaller and the amount of graphite required might perhaps be as low as 20 tons.

The amount of uranium required can be calculated from equation No. 33a

$$\frac{4\pi}{3}R^3/V = \frac{1-q_m}{6}\,\frac{R^2}{B^2}\,\frac{1}{1+R/B}$$

It may be kept down by choosing a smaller value for R than the value corresponding to the maximum value of ε. For $R = 5$cm. and $q_m = 0.6$ 33a gives $\frac{4\pi}{3}R^3/V = 0.0336$ corresponding to 30 tons of uranium for 100 tons of graphite. For larger values of q_m we find a smaller ratio of uranium to carbon.

~~For q we would have as the ratio of weights .~~

CRITICAL DIMENSIONS

If a large sphere of graphite is used ~~and a neutron source placed in the center~~ 1, the critical value for the radius of the graphite sphere for which the chain reaction becomes divergent may be calculated ~~approximately~~ under various assumptions. The optimum distribution of uranium is not uniform within the sphere and will either decrease or increase with r according to whether we want to have a minimum amount of uranium or a minimum value for 1. ~~the critical radius of the graphite sphere~~. The treatment of this question is perhaps best postponed until the value of the carbon capture cross-section is known. It will then be possible to find the optimum distribution of uranium as a function of the distance from the center of the graphite sphere and give a value for 1. In the meantime, a very rough approximation may be presented only for the purpose of giving some idea of the order of magnitudes which are involved. If $\sqrt{\overline{r^2}}$ denotes the average distance to which a fast neutron emitted by uranium diffuses away from its point of origin in graphite until it becomes a thermal neutron and reacts with uranium or carbon, then the critical radius 1 of the graphite sphere is of the order of magnitude of

$$l \sim 3\sqrt{\frac{\overline{r^2}}{\mu q - 1}}$$

Taking as a reasonable value in graphite of density 1.7, $\sqrt{\overline{r^2}} = 50$ cm and $(\mu q - 1) \sim \frac{1}{3}$ we would then have $l \cong 250$ cm corresponding to about 100 metric tons of graphite. The corresponding amount of uranium can be taken from equation No. 33. For $A \cong 76$ cm ; $q_m \cong 0.7$; $\varphi = 0.4$; and R = 5 cm we have

$$\frac{V}{\frac{4\pi}{3}R^3} \sim 100$$

With the densities of 15 for uranium metal and 1.7 for graphite we find for the ratio of the weights about 1 to 10 or about 10 tons of uranium for 100 tons of graphite. The amount of uranium required may be reduced by choosing a smaller value for R than the value corresponding to the maximum value of ε.

CRITICAL DIMENSIONS

For a large sphere of graphite which contains a large number of small uranium spheres ~~is~~ the critical value l, for the radius of the graphite sphere for which the chain reaction becomes divergent may be calculated if the value of the nuclear constants involved is known. The treatment of this question may ~~as well~~ perhaps be postponed until the value of the carbon capture cross-section has been measured. In the meantime, a very rough approximation may give an idea of the order of magnitudes which are involved. In our system "the mean distance" $\tilde{r}$ to which a fast neutron emitted by uranium diffuses away from its point of origin until it becomes a thermal neutron and reacts with uranium or carbon is about 50 cm. For $(\mu q - 1) = \frac{1}{8}$ we find for the critical radius l from

$$l \cong \sqrt{\frac{3(\tilde{r})^2}{\mu q - 1}}$$

l = 250 cm. This corresponds to about 100 metric tons of graphite.

The amount of uranium required can be calculated from equation No. 33 a b

~~This amount~~ The uranium amount may be kept low by choosing a smaller value for R than the value corresponding to the maximum value of ε. For R = 5 cm. and $q_m = 0.6$ equation 33a would give $\frac{4\pi R^3}{3}/v = 0.0336$ corresponding to 30 tons of ufanium for 100 tons of graphite. For larger values of q we find a smaller ratio of uranium to carbon.

NOTE FOR MEMORANDUM OF July, 1940

This formula corresponds to a treatment which was first applied for th the determination of the critical radius in 1934. Appendix No. . is a copy of pages contained in an American Patent application which was filed in 1935 and subsequently withdrawn to prevent its publication in conformity with the pledge given to the British Admiralty. in 193

The equation derived on these pages leads to a critical radius of where stands for and stands for .