

Thank you, Dr. Liebermann.

You have omitted the main reason why I want to talk to young people. The main reason is that young people are much more clever than old people, and, according to my experience, the older you get, the stupider you get. And my main purpose tonight is strictly to try and hide this inexorable process for at least one hour, if I possibly can. Now, I said I would talk about science proper. I found out today that two of my very good friends, George Gamow and Marvin Stearn, have just published a book by the title of "Mathematical Puzzlers." It is an extremely amusing book, and I hope in looking into it, you will find nothing that I will tell you tonight, because that book is just very, very pure science. What I will try to tell you is a little bit more serious, and is something of a slightly different kind. I am not going to talk to you about things that you will understand. I certainly will not talk to you about things that you completely understand. Even when you think you have understood it, you will not have understood all of it, and this is one point in science, it is interesting from the very first, but you must never believe, even about the simplest subjects in science, that you have completely understood it. There is always a little bit more to learn about it. What I wish you would do in listening to what I say, is to pay some attention, to try to remember a little about it, and then puzzle about it later. And when we finish one topic, I hope you will not continue to puzzle about it right away, because there will be something amusing in the next one.

Now I would like to start with a magic trick, an act of mind-reading, you know. For that purpose I have to have an expert mathematician of the younger generation at the blackboard, and Dr. Liebermann has promised to catch such a mathematician for me, by hook or crook.

(Little girl comes up)

Now the first thing I will do before I give you a dangerous instrument like a piece of chalk will be to break this piece of chalk in two, because if I don't do this, the piece of chalk will squeak, and why it does it, I will not tell you, that is a puzzle. The second thing I will do is to give you half of the piece of chalk. And now I will be very honest, in fact, so honest as to go behind the blackboard, so that I cannot possibly see you. Excuse me, what's your name? Monica? Monica, will you please put a number on the blackboard, and make a big one, five or six digits, and make the numbers big, too, otherwise nobody can read it. Fine, now I want you to write on the top of it another number, and the last digit of that new number shall be 4. Now ahead of that, you put all the digits which you have put down before, but scramble them, put them down in any order you like. All done? Well then, evidently the second number you put down is bigger than the first number, so subtract the first from the second. And don't make any mistakes. Finished? Good, now will you add up the digits in the difference which you got. All done? Now I don't know, but I guess that this probably is a two-digit number. Is it? Alright, add up the two digits. The result is 4. Right? Well, I didn't look. Thank you very much.

The question is, how did I do it? Well, of course, as a magician, I shouldn't let you in on my tricks, and I won't, but I will try to explain to you, knowing that you won't understand it anyway, so everything will be alright. At least, I will give you a hint. You see this number, 62,341, and this number here, 7,432,614; instead of subtracting these two numbers from each other in the way in which they are usually subtracted, I will subtract them in a rather different way. Let's start with this 4, and from this 4, there is subtracted a 4 here. That is $400,904$, alright? This is evidently as much as $4 \times 100,000 - 1$, right? So $100,000 - 1$ would be $99,999 \times 4$. Alright, this is $400,000 - 40$, so this is $4 \times$ how much? $100,000 - 10$, alright? And $4 \times 100,000 - 10$

is $4 \times 99,990$. Now this is evidently divisible by 9. So when I have the difference between this and that, I get something that's divisible by 9, correct? If I now take the difference between this and that, that is $30,000 - 300$. You again will find that it is divisible by 9, namely it will be $3 \times 40,009 - 300$, that is $3 \times 10,000 - 100$, and that will be $9,900$, evidently divisible by 9. Then I take 205. $20,000 - 20,000$, that is zero, which is again divisible by 9. Now here, I have another difference, and in this difference, I have to subtract from $660,000$; well, that's a negative number, but positive or negative, it's divisible by 9, because it is $6 \times 10,000 - 100$, and $10,000 - 100$ is divisible by 9. And you can go on in this way — this difference is divisible by 9, and this difference is, and this difference is, and $10 - 1$ is also divisible by 9, right? And in this way, if I took all of this part of the top number, and all of this part of the bottom number, you can very easily show that the difference is divisible by 9. But the top number has an extra figure, the 4, right? The result is that the difference that you have written down here is the number which, when divided by 9, should leave as the residue, 4. Now, what does that have to do with adding up the digits here? It's all very simple. You have replaced here, when you added up the digits, $300,000 \times 3$. If I put 3 instead of $300,000$, I have diminished the number with which I have operated, by $3 \times 100,000 - 1$. And this is divisible by 9. Then, when I wrote 7 instead of $70,000$, then I have again taken too little by how much? By $7 \times 10,000 - 1$, and that is again divisible by 9. When you go all through it, you will find that the sum of these numbers when divided by 9, must leave the same residue as this number, and when you divide 22 by 9, indeed, the residue is 4, right? But I didn't stop here, I said let's add up these two figures, that is, I replaced 20 by 2; I can do so because the difference is divisible by 9, and in this way, when I have worked my way down to a single digit, it must be divisible by 9, and except for the residual 4, and since the single digit is all that could have remained,

it was that 4. Now, you didn't understand...you surely didn't understand, but you can perform the trick anytime you like, and if I would have put 3 at the end, then the end result would have been 3, no matter what the original number was that I put down. And if you think about it day after tomorrow morning at 8 o'clock, and think about it hard, maybe you'll understand it. But after you have understood it, don't believe you have understood it, anyway, because when you think more about it, you will find that with the same trick, the same general idea, you can perform lots more tricks, and quite different ones from those that we have here on the blackboard. Now, in order that you should not be annoyed for not having understood the trick, I will erase it. And instead, I will tell you a story, which will be simpler. This story you may have heard, but it is one of my favorite stories. It is about a tyrant. He was a Greek tyrant, but he did not live in Greece. His name was Hero, and he lived in Syracuse, that's in Sicily, just south of Italy, and since he was a tyrant, and was not a legal king, he had no crown, which annoyed him. And therefore, he asked a goldsmith, a very famous goldsmith, whose name has been forgotten in the intervening 2,000 years, to make him a golden crown, using the gold which he gave that goldsmith. Well, the goldsmith made a beautiful crown and brought it back and Hero was satisfied. But a little later, a rumor went around that the crown was not made of pure gold. It was a very beautiful crown, and I don't know why this should have annoyed Hero so much, except that since he was not a real king, it annoyed him that he did not have a crown of real gold. But he wanted to find out, so he called the most clever person who lived in those parts, namely Archimedes, and he said, "Find out whether this crown is entirely of gold, or whether it is, as I've heard, just gold on the outside, with silver inside. But don't scratch it."

So, Archimedes went home and on his way, he was thinking very hard. He thought, "Well, I can weigh this crown, and I can find out whether there is the same weight to it as the gold given the goldsmith by the king. Silver is not as dense as gold; a cubic inch of silver weighs a little less than $\frac{2}{3}$ as much as a cubic inch of gold, so if he has substituted silver for gold, this crown will have to be bigger. But it has such a complicated shape. How do I find out how much space this crown takes?" And he was very perplexed. On arriving home, he did what he always did when he had a difficult problem, and what he sometimes did when he didn't have a difficult problem. I can draw you a picture of what he did. He had a bathtub like this, and first of all he did something which Mrs. Archimedes did not like, and many ladies don't like either. He filled up his tub like this, an the very top so that it was quite full, and then because he had a very difficult problem, he sat in it like this. You can easily imagine what happened; the water ran out and made a puddle. And this Archimedes, as usual saw, with pleasure. And then he had a splendid idea. It was simply that the volume of water that made the puddle must have been precisely the same as the volume which he displaced. So all he had to do was to fix a full vessel, put the golden crown into it, and find out how much water ran out of the vessel. Thus, he could see how much room the golden crown took, and then he would see how heavy the golden crown was, and from these two facts, he could figure out whether it was all gold or whether it was light with silver. Because a golden crown, for the given weight, takes less room than a silver crown, or a partly silver crown of the same weight. And he was so excited about this that he jumped out of the tub as he was, and ran down the street saying this: you will try to pronounce it -- you pronounced it quite correctly, and because he was such a clever man, nobody was surprised at his behavior.

Now you think you have understood this story, but you didn't, because

Archimedes was so pleased about his discovery that he continued to think about it in all kinds of ways, and made many other discoveries connected with bathtubs, full or otherwise, and these discoveries, even though 2,000 years have passed, have been much better remembered than this goldsmith who may, or may not, have lost his head, but I don't happen to remember whether the crown was solid gold or not.

Now I want to tell you a different story about another great gentleman who spent most of his time in southern Italy, and this gentleman was called Pythagoras, and Mr. Pythagoras had a theory about which he was so happy that he sacrificed a hundred oxen and had a big party and the hundred oxen were eaten. And this was the theory: that if you draw a triangle in such a way that one of its angles shall be a right angle, and then on top of each of these sides, you erect a square, then the theorem of Mr. Pythagoras says that the area of this square is equal to the sum of these two areas. And I have no question connected with that, except is it true? And if it is true, how do we prove it? Now let me first write it down mathematically. Let us say that this side here is called A, and this side here is called B. There may be any number of inches, and A gives the number of inches of this side and B gives the number of inches of that side, and C gives the number of inches for that side. Then this area is $A \times A$ square inches, which we write, because we are so highbrow, as A squared. And this area here gives the times B square inches, and Mr. Pythagoras says that if you add these two areas, you'll get the area of this square, which is $C \times C$ square inches. Now how shall we prove it? It's very easy, and when you learn it at school, it's usually difficult, but in reality it is simple, and once you have seen it, you will have to work very hard to forget it. It goes like this. I'm going to make here a square, and now I'll make here another square. These two squares are supposed to be equal. And now, into each of these two squares, I shall

put 4 equal triangles -- 8 triangles, all equal. I shall draw them like this, see? These are 4 triangles, and if I have drawn this and this side equally, then all triangles turn out to be equal. And into this square, I also will put the same 4 triangles, this will be one, but the others, I will put in a different shape, like this, in a different position, and if you draw it correctly, you will find that these 4 are also the same. And now consider this area here, this area, which I have shaded, is obviously the big square less the 4 triangles. And the shaded area here is again the same big square less the same 4 triangles. Therefore, the shaded area here must add up to the shaded area here. You will notice that this shaded area, if I call this A here, is A squared, this shaded area is B squared, and this is again a square, and it is nothing else but C squared. And that is all there is to the theorem of Pythagoras, and there is just one thing wrong with it, and you won't guess what that is -- the only thing that's wrong with it, is that the theorem of Pythagoras had been known for more than a thousand years to the Babylonians before Pythagoras ever discovered it. But that doesn't really matter.

Now, so far, I have told you some stories, and none of them was connected with any other of them, but still they were science stories and science puzzles, because there was about each one something more to be learned which I have not mentioned. Now I will do the kind of thing that scientists really like to do, and should do. They are not, and shouldn't be, satisfied with one puzzle, but it is their business to heap the puzzle upon another puzzle, and upon that they heap still a puzzle, until there is a whole complicated system, and the whole thing becomes more interesting, and much more beautiful. So now I will tell you a little bit more about things which are somehow connected with the theorem of Pythagoras. And I will take away the process, and only remember the result, except that I will write it a little differently. I will make a triangle here, where I will call this X and this, Y.

And the third side I will call R , for reasons of my own. And then, I will write here, instead of A square and B square equals C square, X square plus Y square equals R square. And now I will tell you something about a strange mathematical object, which in mathematics, we like to call a vector. And if you want to remember a vector, you best remember it by drawing an arrow.

A vector really is an arrow like this, and what it means in its simplest possible form is going from this place to that place, or at any rate, making a displacement, a movement of this magnitude and this direction...that's a vector.

Now one thing you can do with vectors is that you can add them. You first make one displacement, and then on top of it you make another displacement, like this. And then the sum of the two displacements is this, so the sum of two vectors will be a new vector, a third vector, and really, it does not make any difference, as you can easily see, whether you make this displacement first and this second, or the other way around. I could have made this displacement first and this second, or the other way around. I could have made this displacement first and then this one, and that would have given the same sum. And this is known as the theorem of the parallelogram, obviously because I have drawn a parallelogram. Now there is something much more amusing about it than most people know. I have talked about vectors as displacements. I would like to talk to you for a moment about vectors as forces. A force is also something which has a magnitude, and has a direction, and I want to explain the fact that if you add two forces together, or more than two forces, they may balance or reinforce each other, and be replaced then by a resultant force, and I will try to show you that the forces have to be added, at least in some simple cases, precisely like displacements. Let me do it this way -- I would like to know how to add two equal forces which are at right angles to each other. Look, if I have two equal forces at right angles to each other, like this and like that, then the resultant force will have to be somewhere

on the bisector between the two, because why should it be closer to the one than to the other in case the two original forces are equal. The thing by which these ^{two} forces can be replaced is a third force, but how shall I get it? If it would be a displacement I am talking about, I would have to draw a line like this, and then I would have a right angle triangle here, and I would know that the length of this thing is such, that its square is equal to the square of this, plus the square of that. Let's say this is just a foot long, and this is just a foot long, then the square of this will be a square foot, and this will be another square foot, and so the square of this should be two square feet, and what is the number the square of which is 2? No, the number whose square is 2 is squared with those two, and that is accurate, whatever it is. Now, this is what I won't get if this would be displacement, but they are forces, and how do I know what forces add in the same way as displacements do? I don't know; let me try again. If a problem is too hard, I will make it harder. I will add two more forces, this one here over again, and then this one -- and now these two forces here, this one and that one, add up to a force like this. And the rest of that force, I didn't draw it right on purpose, just not to give away what it should be, and if the length of this was 1, then the length of this shall be something, I don't know what. But because it's an unknown quantity, I will call it U for the unknown. Now I add these 2 together, all right? Then they will give a force in the bisector of the same length, U, unknown. Now I add everything together, namely I add this U to that U. Now this is here in the bisector, so this is half of a 90° and this is half of a 90°. And therefore, the sum of these two will be something that lies right in their bisectors, and will be longer than U in the same proportion as U was longer than 1. It will be $U \times U$, and the end result will be U squared for the length.

But I could have proceeded differently. I have here added these two to get U, and these two to get U, and added these two resultants to get U squared. Instead, I can go right ahead and add the original 4. Now this one, and this one are opposite and equal, therefore they must cancel. These two are equal and in the same direction, and therefore, they add up, and they should give 2. The resultant should be 2, and therefore, U squared must be equal to 2, and U must be squared with those two, which is just the result you expected if we had displacements. Therefore, you can see that this idea of parallelograms, at least in some cases, can be proved from simpler and more obvious ideas.

Now, I want to tell you a little bit more about these vectors. I'm beginning to pile up one puzzle on top of the other, and you won't understand it, but you will understand approximately what I talk about, not the details, but the subject matter. And then when you hear it again, or when you think about it again, you might find it more easy the next time. I want to say this, when I take any vector, I can always do this to it. Here is a vector, and if I call this vector R, then I can draw these two perpendicular lines and think of this vector as the sum of this vector, and this vector. And I will call this X and this Y, and I get R from the theorem of the parallelogram. Then I will see that R squared is equal to X squared or Y squared. With this idea of the vector, you can do all kinds of funny things, even practical things. Now I will consider the following little problem:

I draw here two lines, and these lines I will consider as two mirrors, only I will draw everything in a plane, so as to make it simpler. Now I will assume that the light beam comes in like this. Now you know what a mirror does with a light beam; it throws it back in such a way that this angle shall be equal to this angle, right? And now the light will follow this mirror, and this mirror will throw it back so that this angle shall be equal to this angle, and the light beam goes out like this, and it's very easy to

see that the outgoing light beam will go back toward the sun, will be parallel to the incoming beam, and just go in the opposite direction. You can prove it in many ways, but I want to prove it in my own way. Let us represent the velocity of this light beam by a vector, and this vector has two components, the velocity with which the light beam moves in the X direction, like that, and the velocity with which the light beam moves in the Y direction. It's moving in this direction, and at the same time in that direction, only in the Y direction, it moves more slowly, and therefore it is flatter. Now, it is very easy to see when the light beam falls on this mirror what will happen to its motion in the X direction. It will continue to move in the X direction just as fast as it has been moving, but its motion in the Y direction has been reversed. Instead of going like this, it will now be bouncing back like that. So this mirror revises the velocity in the Y direction; similarly, the mirror revises the velocity in the X direction, you see? And when the mirror is through with the light, then the light has returned its X velocity and its Y velocity. That means that these velocities are together, and its moving backward from where it is. Now then, why did I talk about it in this funny manner? I did it because we are using these mirrors every day, not just two of them, but three of them. We use very often three mirrors, three perpendicular mirrors, and one can make those very nice and very small -- you know what they are called -- they are called cats eyes, and these little mirror arrangements are used on the road sides. And when you drive at night, they shine right back at the car, which has illuminated it. And to see that in two dimensions is easy in anything, but in three dimensions, you must be real clever to see it. But if you have followed me so far, you will see at once how it's done in three dimensions. The light falls first on the wall meter, and that reverses the velocity of light that started in the X direction. The other meter reverses the velocity in the Y direction, and the third, in the V direction. Just think it out in detail and you will find

that the light every time will go back precisely toward the headlight that has sent it out. Therefore, just the one person who wants to look at it can look at it.

Now I have worked in this idea that we can have vectors in 3 dimensions, and a displacement in 3 dimensions has 3 components, the X component, the Y component, and the Z component. This needs very deep thought, and you should think about it extremely hard for 7-1/2 minutes, and maybe draw a few sketches on some piece of paper. You will find that if I have two points out in space — and now I will draw something funny. I will draw a coordinate system. This shall be the X direction, this shall be the Y direction, you can imagine here a plane which , and this is the Z direction here. Then you can think of a point out here, which I will draw on the blackboard, and this will be the vector R. The vector R has 3 components, X, Y, and Z. One can then show very easily that R^2 will not be X^2 or Y^2 , but X^2 , plus Y^2 , plus Z^2 . I won't prove it to you, it's quite easy, and you can do it yourself. And there is something very funny, because you can use instead of these three directions X, Y and Z, three new directions in space. These three new directions I can call by three new names, U, V, and W. You have three new components. You find that all three will be something new, but U^2 , plus V^2 , plus W^2 , will be the same thing, namely R^2 . You see, you can start on your course in a system around, and the sum of these squares will always remain the same. It is something mathematicians like, I don't know why.

And now in the remaining ten minutes, I will explain everything about the theory of relativity. That is really what I led up to, because if you have understood what Pythagoras' theorem is about, and what vectors

are about, then it should become a little bit simpler to understand what relativity is about. Relativity is the answer to a puzzle, which was puzzling to the most clever people, even to Einstein. The puzzle is one simple fact, and don't ask me to tell you how they found out this fact, because it would take too long and I would lose your interest. The fact is that light velocity is light velocity. It sounds simple, but it's complicated, and it's not complicated; it's strange, it's surprising. If I stand here, and a light beam goes by, it goes by at a certain velocity, let's say 170,000 miles/sec. Now I take a plane and fly after the light beam. Now, how much faster does the light beam go than my plane? You know the answer -- 170,000 miles/sec. I didn't make any headway, and if I was any kind of a very fast rocket, no matter how fast, the light would still seem to go faster than the rocket by 170,000 miles/sec. This is strange, and this people found, and they couldn't understand it. Now Einstein said something very funny, and I will tell it to you. I have said that if you have one system of axes, and you project on this system of axes X, Y and Z, then this equation is true, but if I take another system of axes, then U squared, plus V squared, plus W squared will be the same thing, namely R squared. Now Einstein said, "Well, I will talk not about points in space, but about another strange assembly of things which I will call points in the word, and a word-point will be characterized not by three letters, X, Y, and Z, which give the position in space, but by four letters, X, Y, Z, and T, the time. And the word-point is nothing but an event, something that happens at a given time in a given place. And I will now be interested not in just one word-point, but in two, just as R was really the distance between two points in space, so I will be considering two events in the word, which occur at different places and at different times. Just to make things simpler, let's forget about Y and V. I could write them in, too, but it's getting late, so I will be very brief. Let's say that the two

events take place both in the same elevation, so Z is the same, and in the same latitude, so that X and Y are the same — but longitudes would be different, you see? Tools should have different X values, so the two events are in different places from each other. The difference in the positions, I will call X , and the difference in the time, I shall call T . Now, light velocity is usually called feet, and in the time difference, light will have gone a distance $3 \times T$. The fact that light can go from one word-point to another just in the time that it arrives from one point to the other, is expressed by the statement that X is equal to $3T$. There is such a distance between the two points, that light just could go from one to the other. For instance, two word-points shall be something that happens here and now, and something that happens precisely 1 second later on the moon. Light has just time to go from here to the moon in one second. These two word events, the start of the light here, and its arrival on the moon 1 second later, are two word-points for which X is equal to $3T$. The difference in position X is equal to the light velocity times the difference in time. Or, I can say X squared is equal to $3T$ squared. Or, I can say X squared, minus $3T$ squared, which is the same, must be equal to $?$. Now look, there is something similar between this equation and that equation. You have here something about squares and here something about squares. Now Einstein says that if somebody else moving with the same velocity is looking at the same situation, he will find that X for him will be different. And we are quite accustomed to that, because if I take two word events, let us say, the word event here and now, and the word event there, starting 3 second later. Then for me, there is a difference between these two points, or whatever it is, 10 feet, all right? I'll make that statement again — a word event here and now, and a word event 3 seconds later, and I will call these two events now and then. And I say now 1, 2, 3, then. Now I walked from one word event to the other, and if I carried the coordinate system with myself, I could say both of them happened at the same

place, namely at the place where my hand was. So for these two cases, in the one case X was different, but in the other case, it was just nothing. Now if two people are looking at the same thing, and one says the word events will differ from each other by X, and the other says the word events will differ from each other by as much as T, and another person says no, that it wrong, those two word events were different from each other by a different quantity. I will call that quantity capital X, and will call the time difference capital T. And I say these are the right time and space differences. Then Einstein says, fine, all that we can say is that if for two word events, light has just time to go with velocity speed from one to the other, it must have had time with the velocity speed to go from one to the other for the other fellow, too. So I have to write here $X^2 - 3T^2$ is equal to . Now let me tell you what is funny about it, because I used some numbers, some figures on the blackboard which may confuse you. I have demonstrated to you that for the one person, and for the other person, X might mean something quite different, but we are used to the idea that the time difference between two events is the same, no matter who looks at it. We are used to the idea that the time difference between two word events is always the same. But Einstein says no, this cannot be true, because otherwise, light could not appear to have the same velocity every single time you look at it. And, we have to assume something strange, we have to assume that a time interval is different for different persons who are looking at it.

Now let me say just one word to finish up. These last things you have not understood, but you have understood that there is something exceedingly strange, namely that we have found out by some kind of experiment about light, that light is running with the same velocity, no matter who is looking at it -- and that this strange fact can be explained only if you assume that for different observers, time seems to pass at a different rate. These ideas are funny to you because you see them for the first time. They are really not more strange

than the statement that if I measure the distance between two points, they are always the same, even though their distance, their separation in this direction or in that direction, may be different in each case. For instance, if I look at an object like this from here, then its two ends appear to be separated, but if I look at it endwise, then its two end-points seem to coincide. And it is in this sense that Einstein says time and space cannot be considered separately. If you change your point of view, you will see something different. Now, please do not go away from here with the idea that relativity is something very complicated, because it isn't. Relativity is not more complicated than simple statements like this, that the earth is round. When people heard that for the first time, they felt very uncomfortable about the poor people in Australia who have to walk upside down. Yet the idea of the earth being round is not complicated, it was only unaccustomed and strange. Relativity is not more complicated than the theorem of Pythagoras, with which you can become familiar in a short time. Only, the theorem of Pythagoras does not tell you anything which would clash with ideas that you have. Whereas relativity tells you about things of which you have never heard before. And this is the final result of science study -- that if you go on and on distant in a straightforward way, each of these problems is relatively simple; in the end you get to something which is entirely unsuspected, entirely new and at the same time, not more complicated, but in fact, more simple than the kind of picture you had about the world before you started to think. Now I know that much of this sounds new to you, or strange to you, but the more you think about any part of it, the simpler it will look, and the more you will be interested in finding what the next complicated thing is which will become simple when you look at it a bit more closely.

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