Dose of reality added to soap bubble question

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On its surface, the question appears as easy as child's play.

Take a piece of wire, bend it into an arbitrarily shaped loop, and then stick it into soapy water. Now, what is the shape of the soap bubble formed within the boundary of the wire?

Though seemingly simple, the question took mathematicians nearly a century to solve.

Now, a team of three researchers have taken the problem one step further by adding, in essence, a dose of reality.

In research described in an article in the October 1 issue of the Proceedings of the National Academy of Sciences (PNAS), the three scientists add thickness and weight to the problem to find its solution.

In this manner, the researchers conclude, the problem can be moved from the surreal world of mathematics--where points and surfaces have no weight or thickness--and into the real world where objects have dimensions.

The results not only offer the potential for solving intractable problems in diverse fields such as robotics and the design of computer chips, they also might lead to a new branch of mathematics called dimensional geometry.

"Just as the subject of probability started from a gambler's question and the subject of geometry started from partitioning flooded land in the Nile River valley in Egypt, so the question of minimum surfaces with thickness and weight may lead to a new branch of mathematics," said T. C. Hu, a professor of computer science and engineering at the University of California, San Diego, who co-authored the PNAS article with Andrew Kahng of UCLA and Gabriel Robins of the University of Virginia.

It all began about 150 years ago when Joseph Antoine Ferdinand Plateau, a Belgium physicist, asked the question about soap bubble shapes within wire boundaries. The problem, dubbed both the "Plateau problem" or the "minimal surface problem," aroused the curiosity of many of the great 20th Century mathematicians.

In 1930, a general analytical solution to the problem was finally described by Jesse Douglas of Rice University and Tibor Rado of Ohio State University. For any given curve, their solution offered a mathematical description of the surface of the minimum area defined by the borders of the curve.

This work was considered a major breakthrough, and it garnered Douglas the first Fields Medal in Mathematics--the equivalent of the Nobel Prize for mathematics.

However, the Douglas-Rado solution could only give equations that describe the minimum surface--it could not actually construct the surface. And despite its elegance, the solution made sense only in an idealized world since it was based on the notion that the surface of the minimum area (the bubble enclosed within the wire) had no depth and no weight. In mathematics, a point does not have any dimensions and a line does not have any width. But in the real world, any surface or line--no matter how light or thin--has dimensions.

In 1963, R. E. Gomory and Hu, then working at the IBM Research Center, decided to find a way to bridge this reality gap. Using what they considered nontraditional methodology, Gomory and Hu conceived the idea of constructing minimum surfaces that had dimension. This idea was largely ignored until 1991 when Hu, then a UCSD faculty member, Kahng (who received his Ph.D. under Hu at UCSD) and Robins (who received his Ph.D. under Kahng at UCLA) decided to prove its validity and then use it to attack the Plateau problem.

In the PNAS article, the research team describes their method for explicitly constructing a solution to the Plateau problem for a given wire shape and a specified thickness for the surface (in this case, a curved slab defined by a wire shape which is embedded along the inside wall of a cylinder). Using a computer, they then demonstrated how their methodology worked as the surface's thickness was reduced until it approached zero (the original Plateau solution).

"We solved for the optimum thick surface (like an orange peel) and then progressively changed this dimension so the surface became thinner and thinner, and smoother and smoother," said Hu.

"The computational experience confirmed the validity of our approach," he added.

Hu said the new approach might have implications for a variety of fields, including the construction of smaller and faster computer chips or even motion planning for robots.

"Suppose you want a robot to go from point A to point B in the quickest, safest possible way," he said. "Before, we had solution methods that could only come up with the ideal paths which had zero width. Since robots have physical dimension and can stray slightly from the paths we plan for them, it is nice that we can now find the best-possible path which has a prescribed width, so that it is safer and more realistic."

As for its place in mathematics, Hu said he didn't know how well the solution might be adopted by professional mathematicians, since the approach offered an approximate, "discrete" surface rather than a precise answer.

"Mathematics is so beautiful in part because it is so pure," he said. "And pure mathematicians hate to do something that seems a little bit inexact.

"But mathematicians can benefit from looking at real problems. And perhaps they can benefit from this approach of constructing a model from a real application."

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