

Report

MEMORANDUM

On Cooling of the Power Plant

L. Szilard

May 30, 1942

The following considerations apply to a system in which the power plant is cooled by a gas like helium or a liquid like bismuth. Certain principles of constructions are illustrated by the attached figures. These principles of constructions are based on general principles which apply if there is a direct heat transfer from the uranium to the cooling gas or liquid. Other systems will be considered in separate memoranda.

The uranium is assumed to be present as metal but the use of uranium carbide and fused uranium-oxide is also envisaged. In computing the values for the helium cooling machine we shall assume that the uranium is present in the form of metal. In computing the bismuth cooled machine we shall, however, also ~~discuss~~ *consider* the possibility that the uranium is present not in the form of metal but in the form of ~~oxide~~ *pure dioxide* or carbide.

General Requirements:

The following method of operation ~~was envisaged at the~~ *would be* ~~time when these plans were drawn up.~~ *desirable* ~~The~~ *a* power plant, ~~accord-~~ *should* ~~ing to these plans,~~ producing about 10^6 kw, ~~would~~ be allowed to run for a long period of time, perhaps 6 months or longer. *Then* ~~After this~~ the chain reaction ~~will~~ *would* be shut off and the cooling ~~will~~ *would* be kept on for another period of time, perhaps one month.

may be

After this period the uranium ~~is~~ dissolved in situ by circulating ~~an acid~~ *a solvent* through the power unit. If necessary the uranium metal could be burnt to uranium oxide in situ before it is dissolved. The solution would be pumped into a tank and if possible the uranium would be precipitated together with the element 94 as peroxide. The remaining solution which would contain most of the fission products could be pumped away. The uranium oxide could be redissolved and re-precipitated, etc., until it is sufficiently inactive to permit a separation of element 94 from uranium.

Products, the principles illustrated by the apparatus
~~However, the constructions discussed further below are not~~
~~limited~~ *may permit* to this mode of operation and ~~it is quite feasible~~ *is feasible* to

remove the uranium from the power unit and dissolve the uranium outside of the power unit.

Insufficient with a different mode of op. and may permit to

uranium metal or

forms which 5

Figure 1 shows various forms in which ~~the~~ uranium carbide or fused uranium oxide could be used. ~~Forms of the types showing in A. B. and C~~ *show* have been called clusters in previous ~~memoranda~~ *discussions*. A is a cluster which

has the contours of a cube and which consists of square bars of uranium metal of perhaps .6 cm side and 8 cm long. B is a cluster which has the contour of a short cylinder of perhaps .8 cm diameter and 8 cm length.

picture

It is built from uranium metal sticks of about 1.6 cm diameter and 8 cm length. Two alternatives are ~~drawn~~ *illustrated* in figure B. *half of the* On the right ~~hand side~~ we see empty spaces between the uranium sticks, whereas *half* on the left ~~hand~~ side we see a part of the empty spaces between the uranium sticks

~~are~~ filled with narrower uranium sticks. In the case of gas cooling, the alternative which is preferred is the one shown at the right hand side of figure B, in which the empty spaces between the uranium sticks are unobstructed. *In the case of air cooling*

Figure C shows a cluster which has a contour of a short cylinder of about 8 cm diameter and 8 cm length. The cluster is formed of balls of uranium carbide or uranium oxide. More or less regularly shaped ~~granules~~ *ules* can take the place of such balls and this arrangement is considered in connection with bismuth cooling rather than in connection with helium cooling.

Figure D shows a short cylinder of uranium metal with a number of holes going axially through the cylinder. Such a uranium metal "plug" could be cast by using a beryllium oxide sticks as cores in the casting process.

Double Stream Method

Figure 2 illustrates the principle of the proposed cooling method. In figure 2, 1 is a graphite cylinder of several meters diameter and about equal height. Helium or bismuth enter into this graphite pile on top through certain ducts, for instance duct 2, and the cooling liquid leaves the pile at the bottom through other ducts, for instance, duct 3.

The cooling liquid passes from one system of ducts to the other system of ducts by streaming through uranium clusters or a uranium plug. This uranium cluster or uranium plug is placed in a cavity. One of these cavities is designated by No. 4 in figure 2.

One possible arrangement of the ducts and the uranium clusters is shown in figure 3. In this figure we see duct 2 through which the gas enters cold and from which it gradually passes through uranium plugs, for instance, the plug designated by ~~No.~~ 4 into the duct 3. The hot gas leaves the pile through the duct 3. A square graphite column which could be, if necessary, removed as one piece from the pile has a number of equally spaced holes which are fit to take the uranium plugs. ⁶ Ans. 2

May 30, 1942

L. Szilard

5

A slightly different arrangement of the ducts is shown in figure 7. The arrangement shown in figure 7 arises out of the arrangement shown in figure 3 in the following way: 1 row of cells of the type shown in figure 3 is assembled and then two input ducts of two neighboring cells are merged in one single ^{input} gap (gap 2 in figure 7) similarly two neighboring output ducts are merged into one single ^{output} gap (gap 3 in figure 7). In figure 7 the gaps are drawn slightly wider than the uranium plug 4. This however is not necessary and gaps can be made less wide. The function of these gaps in their relation to the general reaction is not negligible. By choosing fairly heavy uranium plugs and fairly wide gaps we may achieve an increase in k . In order to keep the leakage small it will however be necessary to have the gaps narrower towards the top and the bottom of the pile. For instance, in the zone of about $1/4$ of the total height both at the top and at the bottom the gaps may be narrow near the center. At about $1/2$ of the total height they can be wide and their general arrangement can be made as shown in figure .

Figure illustrates the assembly of the power unit. The unit shown in this figure is cylindrical. The graphite cylinder being about as high as its diameter. The gas inlet is on the top and the outlet is on the bottom. From a point of view of the tensile strength of the tank it might be better to use in practice, a sphere-like shape of both of the graphite pile and the tank.

Helium Cooling.

the following ^{gives the} ~~the~~ outlines ~~of~~ the cooling plant which can dissipate ~~1 million kw.~~ for a power unit which can dissipate 10^6 KW.

The exit velocity of the hot gas is chosen to be 177 meters per second at which velocity kinetic energy of the gas is 1% of the heat which is transported by the gas provided we allow the temperature to rise by 300°C. At this velocity the pressure loss in the ducts may be about 1%. The pressure loss in the uranium plug is again about 1%, and the pressure loss due to the loss of the kinetic energy at the exit is again about 1% of the transported exit.

Assuming a uranium plug of a cross section of 64 cm² and 8 cm length leaving free for the gas a cross section of 22 cm², we have about 7 kg of uranium per plug. In the standard machine (which develops 10⁶ kw per 10 tons of uranium) the heat developed per uranium plug would amount to 170 large calories.

The amount of helium flowing through 22cm² (the free cross section in the plug) with an exit velocity of 177 meters at 600°C is equal to 60 gm moles and taking for c_p the value of 5, and for the temperature rise 300°C, the heat transported away amounts to 90 large calories or slightly more than 1/2 of the standard value, i.e., 500,000 kw per 10 tons.

This value is not the limiting factor however which, if we use uranium metal sticks of .6 cm diameter is rather due to the heat transfer from the metal to the gas. Taking for the heat transfer 18 small calories per cm² which corresponds to an average temperature difference of 250°C ^{at} the velocity of 177 meters, we find for the radius of uranium cylinder at which the heat transfer would correspond to 10⁶ kw per 10 tons of uranium a radius of 0.075 cm. Accordingly for a diameter of .6 cm, or a radius of 13 cm. we obtain heat transfer of 1/4 of the standard machine or 250,000 kw per 10 tons of uranium. ~~This value is moreover optimistic inasmuch as it neglected the fact that the average velocity is only 2/3 of the velocity exit and accordingly the heat transfer is somewhat smaller.~~

Our results should be correct on this score by a factor of 0.86

Systems Res to 600 ΔT = 200 correct by factor 4/5
 Total factor

Friction Loss.

The loss due to friction is primarily due to the fact that the energy reached by the velocity of the gas at the exit is lost. Such a loss occurs twice. Once in the plugs and once at the exit of the ducts. In addition to this there is a friction loss in the ducts which goes through the pile. The friction loss in the duct is smaller than would correspond to the exit velocity since the velocity changes by a factor of 2 in the duct. Moreover the ducts can be built in such a way that their total cross section should slightly increase towards the exit. Consequently it would appear that a total loss in the ducts will almost certainly be below 2% of the transported heat. The loss in the uranium plugs will also be about 2% of the transported heat, 1% being due to friction and 1% being ^{due} to loss in exit. The best estimate of the loss which we can give at present is altogether 3% of the transported heat.

Assuming that we have about 40 tons of uranium in the power unit we would then dissipate 1 million kw and the friction loss would amount to 3%.

MEMORANDUM ON MY VISIT TO CHICAGO, JANUARY 23-24.

COPIES TO:

January 26, 1942

H. Anderson	Alvarez
E. Fermi	A. H. Compton
B. Feld	E. O. Lawrence
John Marshall, Jr.	
E. Teller	
W. H. Zinn	

I arrived in Chicago on the evening of Friday, January 23, and had a conversation with Compton, Lawrence, and Alvarez. Compton told us that he would try to bring the Princeton group, which includes Wigner, to Chicago and build up the work in Chicago while the work in New York would also be expanded. Compton intended to have an office in New York, and he proposed spending a considerable amount of his time in New York. Lawrence, Alvarez and I felt that this plan was very bad since our main task is to create a working organization as quickly as possible and this would not be achieved. Lawrence and Alvarez were dead against it. I told Compton that if we were faced with a choice between this plan and Berkeley and had no third alternative, I personally would much prefer to go to Berkeley in spite of the fact that I fully shared Fermi's objections to the West Coast. Therefore, if we had no third alternative, I would like to return to New York and make an attempt to win over Fermi to the Berkeley solution rather than split the project in this way. I emphasized that I believed that if Compton decided to move the Chicago group to Berkeley or to any other place which he considered suitable, we would all do our best to get Fermi's approval, and that I was rather confident that in such circumstances Fermi would overcome his strong preference for Columbia in the interest of concentration. I asked for a few days to accomplish this since it would obviously be wrong to decide in favor of a place as long as Fermi had a strong objection.

Compton gave us his reasons for thinking that Columbia University was not a suitable place for concentration. I told

Professor Compton that on Thursday, January 22nd, Anderson, Feld, Marshall, Zinn and I talked over this matter at great length. We came to the conclusion that immediate concentration under his exclusive direction is imperative. We realized that it is impossible to get anywhere with this question as long as each group exerts pressure for its own university. Therefore, we decided that each of us would let Compton know by telegram that our first loyalty is to the project, and therefore that if he were not satisfied that Columbia is the right place for this work, and if he wanted to move both the Columbia and Chicago groups to a third place of his choice, we felt that he ought to have the whole-hearted support of all of us. The telegrams were supposed to go out Thursday night. Unfortunately, we were unable to locate Fermi and it was felt that Fermi should have an opportunity to take a stand and send a wire to Compton if he so desired. The following morning we contacted Fermi, but instead of sending the telegrams I decided to fly to Chicago, acquaint Compton with the views expressed in those telegrams, and give him such additional information as appeared necessary.

I raised tentatively the question with Compton whether he would consider Harvard a possible place for the project, and also whether he would consider moving the Chicago group to Princeton and leaving the Columbia group for the time being at Columbia. Princeton being of commuting distance from New York, frequent meetings which might be arranged between the two groups could serve as a substitute for a complete concentration. I asked Compton for an opportunity to make a complete statement on the whole subject under discussion, and an interview was fixed for the next day, Saturday, at 10 a. m.

At this point, Lawrence, Alvarez and I left Compton and the three of us continued the conversation. Lawrence told me that in the circumstances he would no longer press for a concentration at Berkeley and asked Alvarez and me to tell Compton that, in his view, it would be best to concentrate at Columbia, Princeton, or Harvard, or

any other convenient location which appeared acceptable. Alvarez stressed that, in his opinion, Harvard would be a very good place for the project.

I saw Compton Saturday morning as arranged and learned that he had decided in favor of concentration at Chicago and had already telephoned Conant and considered the matter as definitely settled. The text of the telegrams of January 22nd written by Anderson, Feld, Marshall, and Zinn, was then submitted to Compton. I expressed concern about the decision he had reached, and also about the manner in which the decision was arrived at. A full statement of my immediate reaction to it was given, and I said that I would communicate my final reaction after I had had an opportunity to consult with Fermi and Wigner.

L. Szilard

THE UNIVERSITY OF CHICAGO

DATE February 6, 1942

To Metallurgical Group

DEPARTMENT

FROM N. Hilberry

DEPARTMENT

IN RE:

Dr. Compton has asked me to call to your attention the following memorandum just received from Irvin Stewart concerning the proper classification of our documents and communications.

"Dr. Conant and I have been discussing the proper classification of documents and communications dealing with the subject with which you are primarily interested. It was agreed that all communications which deal with the scope of the work or its progress or in any way reveal the fact that work in the field is going on at a particular place should be classified as SECRET and governed by the precautions applying to secret matter.

"On the other hand, such matters as appointments, travel or correspondence of a routine character couched in phraseology which does not disclose the nature of the work may be handled as open matter. There is no middle ground. If the communication is one which might be revealing to an enemy if its contents were in my hands, it should be SECRET. If it is not, it may be open."

KTF

February 11, 1942

Gregory Breit

A. H. Compton

Weekly Reports

May I ask you to accept the responsibility for receiving the weekly reports from the men who are actively at research. These would include the following: Mr. Fermi, Mr. Allison, Mr. Wigner, Mr. Wheeler, Mr. Szilard and Mr. Breit. Mr. Fermi and Mr. Allison will themselves, presumably receive reports from the leaders of the projects going on under their immediate supervision. Mr. Doan will report direct to me with regard to the operation of the Laboratory. May I thank you for undertaking this assignment.

cc: Mr. Fermi
Mr. Allison
Mr. Wigner
Mr. Wheeler
Mr. Szilard
Mr. Doan

March 4, 1942

MEMORANDUM

On the Question of a Chain Reaction
in a System Containing Hydrogen and
Uranium

It was shown by Fermi in June 1939 that a heterogeneous system of uranium oxide and water is preferable to a homogeneous system composed of these substances, and he attempted to calculate in detail a system composed of layers of water and uranium oxide. A lattice of uranium spheres embedded in paraffine is rather similar in behaviour, though perhaps slightly more efficient.

The efficiency of both arrangements is unfavorably influenced by the fact that the cross section for scattering increases for hydrogen by a factor of about 3 if we go over from resonance neutrons to thermal neutrons. The purpose of this memorandum is to point out that it is possible to get rid of the harmful effect of this phenomenon by using an arrangement in which spheres of uranium are surrounded by a cavity.

In such a system, the thermal neutron density in the paraffine is everywhere the same and about equal to the thermal neutron density at the surface of the uranium sphere. Surface resonance absorption of the uranium is

about the same as it would be without cavity.

Figure 1 shows an example for a system in which we have about 2 atoms of H for 1 atom of Uranium.

- R1 the radius of the uranium sphere (density 20 grs. per cc.) is 2.5 cm.
- R2 the mean radius of the spherical paraffine shell is 5 cm.

The thickness of the paraffine shell is $1/3$ cm. This gives about 11 grams of H for 1200 grams of uranium, or 2 H atoms for 1 U atom.

The average paraffine density is about $1/5$ of the real paraffine density.

It should perhaps also be mentioned that a uranium paraffine carbon system of the type illustrated in Figure 2 might be considered. There is at present no reason to expect that the multiplication factor in an infinite medium would be higher than for a uranium carbon system. However, if the multiplication factors were about the same, the point of divergence would be reached for about half as much graphite in the U-H-C system as in the U-C system.

A beneficial effect of a somewhat different nature may be derived from the use of a cavity in a carbon uranium system also. This point is being taken up with Wigner, who may have considered this question along with others by studying the properties of the U-C system.

LS:j

A beneficial effect of a somewhat different

nature may be derived from the use of a cavity in a
carbon uranium system also. This point is being taken
up with Wigner, who may have considered this question
along with others by studying the properties of the U-C
system.

Robert

12:1

MEMORANDUM

March 14, 1942

The following points need to be remedied:

(1) In the past, decisions have been taken on important issues by persons who were not working full-time on the project, and those who were working full-time did not have authority to act in connection with all relevant phases of the work. Although the situation is much better at present than in the past, some of the old trouble still remains.

(2) In the past, frequent changes in the authority of the various committees, changes in personnel of the committees, and all sorts of re-organizations, have taken place. These may have been necessary. However, if this process continues, it will be impossible to settle down and do the work which we have before us. At present, people like Wigner, Szilard, Fermi and Teller are quite uncertain as to how long they will be connected with this work, and the general atmosphere is not conducive to the long range planning which this subject of high complexity requires. The present situation does not make it possible to gather a staff which could form the backbone of an organization that

would have its hands full for at least five years even though the first practical success may be achieved at a much earlier date.

(3) At present, the decisions are being taken by a planning board of which neither Urey nor Compton nor Lawrence is a member. Some of these decisions are taken without intimate knowledge of the subject matter and cause considerable disturbance.

(4) There is a considerable shortage of physicists who are capable of carrying on an independent investigation of certain lines in the field of nuclear physics. Men like Auger, Rossi, Rossetti, or Goldhaber are not doing any defense work, but the present organization makes it impossible to use them, even though all physicists who know them may be satisfied that they could be trusted 100 per cent.

There was a certain period of time in the past when we suddenly discovered that we were not allowed to talk to Wigner. At present we are not allowed to talk to Teller. Fermi and Szilard are not supposed to know certain lines of development. The psychological effect of this situation is very serious. It would seem that all this trouble would disappear if the work would be carried out in a framework slightly different from the

present framework. The re-organization proposed would retain all persons who are connected with the present organization and would change only very slightly their inter-relationship.

It is proposed to have the work carried on with the framework of a corporation which may be called the "Corporation for Scientific and Industrial Research." This corporation would have several divisions, one division headed by Urey, one division headed by Lawrence, and one division headed by Compton. All officers of this corporation would have to serve full-time.

A Board with Bush as Chairman, and Conant, Briggs, Murphree and the heads of the divisions of the corporation as members, would make recommendations to the Government concerning the allocation of money to the corporation and would also determine the distribution of the allocated sum to the divisions of the corporation which will have separate individual budgets. Funds would be allocated on the basis of a two year plan, with recommendations for additional funds to be made during that period if and when required.

Accordingly, the picture would be as follows:

Board

Chairman	-	Bush	
Members	-	Conant	Compton
		Briggs	Lawrence
		Murphree	Urey

Full-time Officers

Managing Director - Compton or Murphree

In Charge of Division	1.	-	Urey
" "	"	"	2. - Lawrence
" "	"	"	3. - Compton

It is proposed that all persons, whether citizens or aliens, employed by the corporation be cleared in the following manner:

If the corporation employs a persons who has not been cleared by means of a standard procedure, or otherwise, the corporation will submit the name to the F B I and to a special committee which will have at least five members and which will include, ex officio, the heads of the divisions. Any information obtained by the F B I will be transmitted to this committee and the committee will decide whether or not the person can be trusted.

MEMORANDUM

concerning the use of metal in the
~~IN FAVOR OF LARGE CELLS~~
di. 71

L. Szilard

April 27, 1942

I wish to put forth arguments in favor of the view that if we use metal spheres embedded in graphite the most favorable arrangement will be represented by large cells; for instance, metal spheres of 4 cm. radius surrounded by 22,000 to 29,000 cc. of graphite corresponding to a weight ratio C/U from 7.5 (at room temperature) to about 10, *(with cooling)* temperature differentiated.

By taking into account the mass resonance absorption which was studied at Princeton, Wigner calculated for uranium oxide the most favorable lattice composition; and on the basis of this calculation the last three exponential experiments were performed with small lumps of uranium oxide of about 2 kilogram weight and a cell size of about 20 cm. cube.

may well be Though it may have well been that for uranium oxide *(of 5.5 gm/cc)* this composition is very close to *(the optimum)*, it is more likely than

not that if we go over to ^{from a cube} the uranium ~~atom~~ ^{metal of the same} uranium spheres of ^{uranium} 4 cm. radius or larger and cubic graphite cells up to, perhaps, ³⁰ ~~40~~ cm. side ^{or more} will represent the most favorable arrangement and will give a multiplication factor which is about 10% or more above the multiplication factor obtained by 2 kilogram oxide lumps, assuming the same degree of impurities.

The following ^{two} ~~three~~ factors contribute to this prognosis:

1. Fission by fission neutrons

In this respect metal offers considerable advantage due chiefly to its greater density and to less ^{or} extent to the absence of a slowing down agent like (oxygen) within the sphere. ~~On the~~ ^{premise is} ~~The estimates of this effect were based on~~ observations of Marshall and Szilard concerning fission by fission neutrons (memorandum dated) and ~~from the~~ observations of Szilard and Zinn on the slowing down of fast neutrons by inelastic collisions in uranium (memorandum dated).

~~According to these rough preliminary measurements we have~~

^{time} for the product of the cross sections caused by fission and ^{of fission for neutrons}

the number of neutrons emitted in this process ✓
and for the $\sigma_v = 1.3 \times 10^{-24} \text{ cm}^2$
 The cross section of inelastic collision in uranium was found
 to be about $2.7 \times 10^{-24} \text{ cm}^2$. (Based on these figures Feld and
 I estimated the contribution of fission neutrons to the multi-
 plication factor for uranium spheres embedded in graphite
 (memorandum to be presented shortly). For large spheres we
 calculated with a diffusion equation, ~~and~~ ~~for~~ smaller metal
 spheres and for oxide spheres we made a rough estimate by
 neglecting elastic collisions within the sphere but taking
 into account on the basis of the diffusion equation the back-
 scattering of fast neutrons from the graphite into the
 uranium sphere. In assuming $\nu = 2.6$, we find the following
 contributions due to the fission by fission neutrons:

It may be of interest

~~It is best~~ to correlate these results with Fermi's first exponential experiment in which he used uranium oxide lumps of about 50 lbs at a density of 3.1. A uranium metal sphere of equal weight would give a fast neutron contribution which is increased by a factor of over the fast neutron contribution in Fermi's experiment.

While it would be possible to improve the mathematical part of our estimate, it should be pointed out that the constants on which these estimates are based are not sufficiently well known to be able to foretell the value of the fast neutron ~~contribution~~ *contribution* diffusion. Even if the rough measurements of Marshall and Szilard were to be taken at their face value, it should be remembered that these measurements give the value for σv , whereas the value which enters into the calculation is based on

$\sigma(v-1)$

While we have assumed that 2.6 neutrons are emitted by fission, there is really no basis for this assumption since little is

known about fission in U^{238} . If the number of neutrons emitted in this process were 3.6, the contribution to the multiplication factor given in the rough table would have to be multiplied by 1.6. Other ~~neutron contributions~~ ^{nuclear cross sections} which enter in a less vital manner contribute also something to this uncertainty.

In view of the fact that the fast neutron contribution can be ^{decisive} of ~~sizable~~ ^{is important} importance, it ~~is proposed~~ to carry out certain experiments, the result of which is not affected by the uncertainties in the value of γ and the other nuclear constants involved. *Such an exp. is ~~proposed~~ in another memorandum.*

2. Differential Heating

The effect of keeping the uranium cool where the bulk of the graphite is allowed to enter was recognized from the start (see Physical Review paper of February, 1941, unpublished; ^{but available as} Section Report No. ⁵⁷ ~~in some other~~) and was put forward as ~~the~~ main reserve for improving the multiplication factor at the meeting of the Uranium Sub-Committee, August 22, 1941. The contribution

which can be derived from this effect is indeed considerable if sufficiently large cells are used. Detailed calculations of this effect were completed by Feld and myself in September, 1941, and ^{(a report} can be made available ^{later should the subject} if ~~they~~ become of immediate ^{practical} interest.

One of the examples which we calculated ~~concerned a~~ ^{was} ~~that~~ a system composed of a metal sphere of 4 cm. radius embedded in a graphite sphere of 9 cm. radius, leaving 5 cm. ^{"meat"} for cooling down the neutrons. A ^{spherical} gap of ~~two~~ 2 cm. was left free between this graphite sphere and the rest of the graphite for heating ~~insulation~~ insulation, and it was assumed that the neutrons are cooled down from 1800° Absolute, which is the temperature of the bulk of the graphite, to 600° Absolute by the time they enter the uranium sphere.

For a ratio of C/U by weight of 7.5 we found at room temperature that 80% of the neutrons are absorbed by uranium and 20% are absorbed by carbon. If the carbon is allowed to heat up to 1800° Absolute, the thermal absorption drops to _____%.

Effect of Heating

The effect of allowing the uranium and the graphite to heat up is twofold. First of all, there may be an improvement in the multiplication factor if both the uranium and the graphite are heated up to the same temperature, ~~say about 600°C.~~ ^{say 300 to}

^{for instance 300°C a further} And, further, there is an improvement in the multiplication factor which is due to differential cooling and which becomes considerable if we allow the bulk of the carbon to heat up to about 1500° C. while keeping the uranium and the surrounding

^{a fixed temp for uranium 3000} graphite ~~AMMORRUM~~ The existence of both effects was clearly recognized from the start (Phys. Rev., Feb., 1940; unpublished, but available as Section Report); and the effect of differential heating was ^{presented} explained more clearly at the first meeting

^{Theoretical} of the Uranium ^{of the Uranium CMC} Sub-Committee, August 22, 1941. ~~However, there~~

~~are a number of finer points which may become of importance and which~~

Copies to: Allison Doan
Compton Hilberry
Fermi
Wigner ←

MEMORANDUM

July 1, 1942

L. Szilard

The purpose of this memorandum is to show on hand specific examples what ~~personally think ought to be done,~~ and what in fact we could do, if we had an organization that could act with the freedom of action of an industrial corporation.

Definition of the Task which is before us:

There are 4 or 5 different ways in which a power unit dissipating about 1 million kw could be built. It appears likely that further experience will gradually eliminate most of these potential possibilities leaving only one or two that may prove satisfactory from the point of view of operation ~~and~~ safety. Under the circumstances, the right policy appears to be to try out on a 10 watt unit the nuclear properties of these different systems on the campus of the University of Chicago. This 10 watt unit should be kept in operation for the purpose of such nuclear experiments rather than transformed into a power unit of 10,000 kw. Those of the different systems which appear to be most promising should be built up as speedily as possible in ^{at least} 10,000 kw size, and the most successful one should subsequently be built up in a 300,000 kw to 1,000,000 kw size. If such a 1,000,000 kw unit works satisfactorily the pattern should be frozen and ten identical units should be built in different parts of the country.

Method of accomplishing the above mentioned task:

The various systems of cooling will require different types of equipment for producing the graphite and the uranium in the form which is required. While such equipment is not very expensive, it will invariably take 6 months or more to procure it from the time when it is decided that

it is actually needed. The right policy therefore appears to be to equip a small factory of our own with various types of equipment which, if necessary, can be put into operation at a month's notice. If not necessary, this equipment will remain idle and can be disposed of later when it becomes certain that we need fall back upon it. In many cases the necessary equipment is present and in use in the hands of firms with whom we are in contact. In such a case a flexible contract coupled with an experimental order could secure for us the right of using the equipment if and when needed. Gentlemen's agreements which were made in the past for just such a purpose gave very satisfactory results. For instance, the only nichrome furnaces which Dr. Alexander has at present available for reducing uranium oxide to uranium metal were bought under just such a gentlemen's agreement by Columbia University last Fall.

Examples of action which I would recommend for immediate consideration if we were able to act with the same freedom as an industrial corporation.

1. I would enter into binding agreement with Eldorado Gold Mines for 100 to 300 tons of uranium products and in return obtain a binding assurance concerning the installation of additional equipment at the mine. The deadline for this is the 15th of July of this year, the date of departure of the last boat.

2. I would purchase and install in a blanket factory of our own equipment necessary to sinter uranium dioxide in the form of square rods of about 1 cm x 1 cm x 5 cm. The potential output of this equipment would have to be 1 ton per day. It would include an automatic press and induction furnaces.

It has been shown by Dr. Spedding's group that such uranium dioxide sticks can be made by a method suitable for industrial production. The best values so far obtained indicate a density of 10 gm/cc and a heat conductivity

The latter has been measured by Wollan.

Such sintered dioxide might be the best substance to use for a bismuth cooled plant, and in the absence of metal, also for the Helium cooled plant. It is however possible that ~~the~~ multiplication factor will not be good enough and that the plan of using sintered dioxide will have to be dropped at a later date on this ground.

3. Equipment for making crucibles for recasting uranium metal should be built up and be held in readiness in our blanket factory.

4. I am taking up with the Brush Beryllium Company the question whether it would be possible to obtain fused uranium ingots as a direct production of electrolysing uranium tetrafluoride. If their reaction is favorable we should place an experimental order ~~and make~~ a binding promise to take a production of about 10 tons per month, or alternatively to pay a premium to them for making use ourselves, or having one of our contractors make use of, processes which they may develop.

5. We should at once make a gentlemen's agreement with Union Carbon and Carbide Company concerning the production of fused uranium carbide. Again we should offer a premium for this company to work out a method for the production of this carbide in such shapes as we need and for placing at our disposal, if need be, a certain number of arc furnaces. If the preliminary experiments are favorable, and if the arc furnaces of that company are fully occupied, we should place an order for sufficient numbers of arc furnaces to have a production capacity of a ton a day.

Fused uranium carbide has similar properties to fused uranium dioxide. It might prove superior to uranium dioxide by virtue of the absence of an $n - \alpha$ reaction which may or may not be present in uranium dioxide. It might also prove to be superior from a point of view of high temperature chemistry in the pile.

The arc furnace reduction might furthermore supply a product as way between the metal and the carbide, or possibly even fairly pure metal. This metal might have different properties from metal produced by Westinghouse or Alexander, and it is conceivable that its carbon content would permit the use of bismuth cooled machine in which the bismuth is in direct contact with the metal.

6. A contract should be entered upon with Union Carbon and Carbide Company and perhaps others who are willing to work up carnotites for uranium and we should obligate ourselves to take a production of perhaps 20 to 30 tons per month for a period of a year.

7. A contract should be entered upon with Westinghouse and Alexander for a steady production of uranium metal of a *well* defined quality and we should obligate ourselves to take a certain quantity per month for a period of two years.

8. We should guarantee Dr. Alexander the salary of 3 or four first class engineers or metallurgical chemists for a period of 3 years with the proviso that if we should discontinue to use his factory he would release his men in case we want to use their services in our own organization.

Dr. Alexander told me ~~that~~ when I complained to him about the failure of his developing proper methods for fusing the uranium, that he was unable to get good men for such development work owing to his inability to guarantee them steady employment. His contract has a cancellation clause and he would have to dismiss his crew at 4 weeks notice to safeguard himself against financial loss in case his contract is cancelled.

MEMORANDUM CONCERNING THE USE OF URANIUM METAL
IN THE CHAIN REACTION

By

L. Szilard

April 30, 1942

The purpose of this memorandum is to emphasize that we may expect a considerable improvement in the multiplication factor if we go from small uranium oxide lumps to fairly large metal spheres. Various factors which are expected to contribute to this improvement in the multiplication factor are discussed below for the purpose of drawing up a program of experiments designed to determine separately these factors.

Contribution of Fast Fission Neutrons to the Chain Reaction

In this respect metal spheres offer a considerable advantage due chiefly to their greater density and to a lesser extent to the absence of a slowing down agent within the spheres. Observations of Marshall and Szilard concerning fission by fission neutrons (memorandum dated) give the product of the cross sections of fission for fission neutrons σ

and the number of neutrons ~~emitted~~ ν

emitted in this process. The value obtained from these rough preliminary measurements is

$$\sigma \nu = 1.3 \times 10^{-24} \text{ cm}^2$$

However
 If one wants to calculate the contribution of the fission neutrons in the chain reaction, one has to know the value of the product $\sigma(V-1)$,

According to whether we assume $V = 2.6$ or $V = 3.6$

we find a more or less optimistic value for this contribution

which differs by a factor of 1.62. This is perhaps the most important uncertainty in the evaluation of this phenomenon. The figures given below are based on the less optimistic assumption of $V = 2.6$

Another value which enters into the evaluation of this contribution is the cross section of uranium for inelastic scattering. This was determined by Szilard and Zinn (see memorandum dated) and found to be about $2.7 \times 10^{-24} \text{ cm}^2$. B. Feld and I tried to estimate the contribution of the fast fission neutrons to the multiplication factor in the chain reaction, taking also into account the back-scattering of fast fission neutrons into the uranium sphere from the surrounding graphite. We found tentatively for a measured sphere of 4 cm radius a contribution of about 1.13, or about 13%. This is about 7% larger than the contribution which we estimate for uranium oxide spheres of about 50 lb weight and 3.1 density, which corresponds to Anderson's and Fermi's first exponential experiment. Inasmuch as we have assumed $\nu = 2.6$, our estimate might be considered conservative.

L. Howard

MEMORANDUM

On the Multiplication Factor

May 6, 1942

In the simplest case in which the thermal neutron density is the same everywhere in the uranium and the carbon we can write for the fraction of thermal neutrons which are absorbed by uranium

5(1-p^th) = (5.5 * 10^24 / (0.2 * 10^24 + 10^24 * C)) = 5.5 / (5.5 + 5/1000 * Nc/Nu) =

and the fraction which escapes capture by uranium at resonance (mass absorption only)

(1-p*) = e^(-6.3/3.8 * 0.6 * 1/Nc/Nu) ≈ 1 - 14.2/Nc/Nu

in order to see the most favorable ratio of carbon atoms to uranium

atoms we calculate Nc/Nu from the equation

(1-p^th) = (1-p*) or 1 - 14.2/Nc/Nu = 1 / (1 + 0.9/1000 * Nc/Nu)

which means that the thermal factor and the mass resonance absorption

factor should be about equal. From this we obtain as the most favorable

ratio Nc/Nu to Nc/Nu = 133

and find: (1-p^th) = .89; (1-p*) = 0.9

We wish now to consider uranium spheres of finite size and take into

account the fact that the average thermal density in the uranium sphere

is by factor of f/f0 smaller than the thermal neutron density at

the surface of the sphere. Values for this factor are given in the last

column of sheet 1 for uranium = 1.6 cm. (density 18) which we used to

calculate thermal factor

The thermal factor for equals 133. Column 1 on sheet 2

gives the thermal factor. By multiplying this with the fast neutron

factor given in column 4 of sheet 1 we obtain column 2 of sheet 2.

Finally we obtain the thermal factor given in column 3 of sheet 2 by

multiplying with a factor that takes care of the surface resonance

Handwritten notes on the left margin: 1 - a/x = 1 + cx; x = a/2 + sqrt(a^2/4 + c); a = 14.2; c = 0.9/1000; x = 133

Handwritten notes on the right margin: 1 / (1 + 0.9/1000 * Nc/Nu)

absorption. In accordance with the Princeton results and Wigner's formula we write for this factor

It may be seen that the most favorable size is a 5 cm. radius sphere if the thermal neutron density and the resonance neutron density in the carbon are uniform.

MEMORANDUM

On the Multiplication Factor

L. Szilard

May 6, 1942

In the simplest case in which the thermal neutron density is the same everywhere in the uranium and the carbon we can write for the fraction of thermal neutrons which are absorbed by uranium

$$1-p^{\text{th}} = \frac{\sigma_a(U)}{\sigma_a(U) + \frac{N_c}{N_u} \sigma_c(C)} =$$

and the fraction which escapes capture by uranium at resonance (mass absorption only)

in order to see the most favorable ratio of carbon atoms to uranium atoms we calculate from the equation

which means that the thermal factor and the mass resonance absorption factor should be about equal. From this we obtain as the most favorable ratio to

and find:

We wish now to consider uranium spheres of finite size and take into account the fact that the average thermal density in the uranium sphere is by factor of smaller than the thermal neutron density at the surface of the sphere. Values for this factor are given in the last column of sheet 1 for uranium = 1.6 cm. (density 18) which we used earlier to calculate thermal factor

The thermal factor for C^{13} equals 133. Column 1 on sheet 2 gives the thermal factor. By multiplying this with the fast neutron factor given in column 4 of sheet 1 we obtain column 2 of sheet 2. Finally we obtain the thermal factor given in column 3 of sheet 2 by multiplying with a factor that takes care of the surface resonance absorption. In accordance with the Princeton results and Wigner's formula we write for this factor

It may be seen that the most favorable size is a 5 cm. radius sphere if the thermal neutron density and the resonance neutron density in the carbon are uniform.

MEMORANDUM

On Available Information

L. Szilard

May 8, 1942

X
Summary:

The purpose of this memorandum is to state that information is available on a number of subjects and that details can be communicated if required. Some of this information is written up in memoranda which would have to be copied from the original manuscript before they can be distributed, some other information has not yet been written up in such a form, but is on record *in some other form*.

1. Contribution of fast neutron fission in 238 to the chain reaction.

a. Statement of the Problem.

If we have a lattice of uranium metal spheres in graphite thermal neutrons diffuses into the uranium spheres and cause emission of fast fission neutrons from 235. These fast fission neutrons which are generated in a certain depth below the surface ~~diffuse~~ out of the uranium sphere into the graphite. In their passage through uranium they cause fission ⁱⁿ ~~and~~ U_{238} , and a certain number of neutrons, V , is emitted in such a fission process. This means that every fast neutron which is generated by thermal fission ^{in 235} ~~and~~ which actually causes fission in U_{238} leads to the liberation of $(V - 1)$ neutrons.

~~A fraction of these additional neutrons, originating from U_{238} leading to the liberation of secondary neutrons, etc.~~
fast neutrons
~~will again also cause fission in the U sphere~~
~~and will cause secondary neutron emission from 238 etc. etc.~~

B. Expected Effect on the Chain Reaction.

The contribution of these neutrons originating from U_{238} to the multiplication factor may be expected to be considerable.

~~B.~~ Feld and I tried to estimate the most probable value of this contribution for metal spheres from 3 to 8 cm. radius, and for oxide spheres of certain radius and density. Our estimate gives for instance, a contribution of about 16% for uranium metal spheres of 5 cm. radius at a density of 20. (factor 1.16)

should be made of 238

In order to be in the position to make such an estimate we have to know something about the fission cross-section, ν of 238 for fission neutrons of 235, and also we have to know how rapidly the fission neutrons are slowed down in uranium below the fission threshold of 238.

Fission caused by fission neutrons was observed by Marshall and Szilard (memorandum dated _____). These rough preliminary measurements give a value of $\sigma_f \nu = 1.3 \times 10^{-24}$. The value which enters into the computation is however, not $\sigma_f \nu$ but rather $\sigma_f(\nu - 1)$. According to whether we assume $\nu = 2.6$ or $\nu = 3.6$ we obtain

$$\sigma_f(\nu - 1) = \quad \text{or} \quad \sigma_f(\nu - 1) =$$

The contribution mentioned above assumes the less optimistic value of $\nu = 2.6$. Since the value of ν is not known for 238 an uncertainty of a factor of about 1.6 is introduced into the calculations of the fast neutron contribution. It is therefore possible that the fast neutron contribution is considerably

larger than mentioned above.

The slowing down of neutrons by inelastic collisions below the fission threshold of U_{238} was studied by Szilard and Zinn (memorandum dated _____). A cross-section was obtained for bismuth, lead, and uranium for D+D neutrons and for radon beryllium neutrons. The cross-section for uranium was found to be $\sigma = 2.7 \times 10^{-24} \text{ cm}^2$.

From these data

~~18~~. Feld and I estimated the contributions to the multiplication factor in two different ways. First, we treated the problem as a diffusion problem in which the fast neutrons are "absorbed" by being slowed down below the fission threshold of 238 through inelastic collisions in uranium and elastic collisions in carbon. We found, however, that the diffusion equation gives a very bad approximation for spheres which have a radius about equal to the diffusion length of the fast neutrons in uranium or graphite, and we believe, that we obtain a more reliable value by neglecting elastic scattering in uranium and assuming an absorption cross-section for fast neutrons of $\sigma = \sigma_{in} + \sigma_f = 3.2 \times 10^{-24}$. We assumed that the neutrons are created near the surface of the sphere at a distance from the center varying from .8R to .9R as the sphere is increased from 3 cm. radius to 8 cm. radius. Treating the problem thus like a problem of absorption of light, we have to take into account separately the fact of back scattering by ~~copper~~ ^{graphite}. This was ~~found~~ ^{done} in a very ~~good~~ ^{crude} way by assuming that fast neutrons leaving the sphere return to the surface of the sphere once with a probability of 1/2, with unchanged energy.

Should have from preceding page

Carbon beryllium

May 9, 1942

Subjects

could be summarized

Information is available on the following subjects:

1. Contribution of fast neutron fission in 238 to the chain reaction.
 - a. Experiments on fission by fission neutrons.
 - b. Slowing down of fast neutrons below the fission threshold of 238 in uranium and other elements.
 - c. Computations on the contribution of fast neutron fission to the multiplication factor in a graphite uranium system.
2. Considerations showing the advantage of using uranium metal spheres 5 to 8 cm. radius or other heavy bodies of uranium in large ^(lattice) cells *over using small bodies in small lattice cells*
3. Considerations showing the advantage of using cavities, neutrons channels ^{or} and neutron slits in the core of the pile.
4. Considerations showing that the multiplication factor can be considerably improved in a uranium hydrogen system by introducing cavities or channels into the system.
5. Formulae and computations of the thermal neutron absorption of a lattice of infinitely long cylindrical rods.
6. Considerations on differential cooling.
7. Considerations on the compatibility of neutron channels and differential cooling.
8. Rough experiments on the thermal conductivity of uranium metal.

2. Res.

3.7

Carbon bodies

Summary:

SUMMARY

Using the most probable values for the nuclear cross-sections it is estimated that the contribution of fast neutron fission in large uranium metal spheres is considerable, for instance, about 16% for $r = 5$ cm. This contribution may perhaps be further increased by using "contour bodies". Under the circumstances we may expect considerable increase in the multiplication factor if we go over to uranium metal ~~using large uranium spheres or other similar uranium bodies~~ *spheres in large lattice cells* and neutron channels or neutron slits. The use of channels or slits, while it increases the multiplication factor, would not in itself lead to a reduction in the critical amount of graphite in uranium, because it increases the slowing down length for fast neutrons. *It is found however that* Neutron channels ~~can, however,~~ be used to increase the multiplication factor in the core of the pile since an increase in the slowing down length limited to the core of the pile has almost no harmful effect. *In this way* It is shown that the use of neutron channels is compatible with differential cooling.

In
In this way a considerable decrease in the critical amount of graphite in uranium may be achieved by using in the core of the pile large uranium metal spheres in large cells and neutron channels or neutron slits. *Another* ~~Another~~ *a different way of achieving this*
Wigner

May 10, 1942

MEMORANDUM

On Resonance Absorption of Uranium
to May 8, 1942

*for high energy
at high neutrons
(~100KV)*

The possibility that uranium might show a much higher resonance absorption for high energy neutrons than originally assumed was first emphasized by Wigner and led to the experiments of the Princeton Group. Marshall and I used a different line of attack for the same problem by determining the capture cross section of uranium for radium beryllium photo neutrons. A rough preliminary determination showed the value of about . . . (See memorandum dated . . .). This value is indeed much higher than expected and is consistent with the results obtained at Princeton, by direct observation on the mass resonance absorption of uranium.

MEMORANDUM

On ~~the~~ Contour Bodies

May 10, 1942

Since the contribution of fast fission neutrons which split U238 appears to be considerable, it is of interest to investigate whether this contribution can be enhanced by shifting the center of gravity of the thermal neutron fission closer to the center of gravity of the uranium body than we have it in the case of uranium spheres or cylinders. This result can be achieved by using bodies which will be called contour bodies because their contour imitates a sphere or some other configuration. The attached figures show such contour bodies.

May 10, 1942

MEMORANDUM

On Resonance Absorption of Uranium

to May 8, 1942

The possibility that uranium might show a much higher resonance absorption for high energy neutrons than originally assumed was first expressed by Wigner and let to the experiment by Marshall and I used a different line of attack for the same problem by determining the capture cross section of uranium for radium beta-rays. A rough preliminary determination showed the value of

Figure 1 is a rotated H which has the contours of a ^{short} cylinder. This figure also shows that the cavity inside of the contour body is not filled with graphite in order to have the contour surface determine the surface resonance absorption rather than the real surface.

MEMORANDUM

On the Contour Bodies

May 10, 1942

Figure 2 shows the simplest form of a contour body which is a ^{short} pseudo cylinder. It ^{is believed} can be shown that the average thermal neutron density within the uranium metal is larger for such a contour body than for a sphere and that the center of gravity for the thermal neutron absorption also shifts into a more favorable position. However, since no quantitative investigation has so far been carried out these statements are made with ^{the} appropriate reservation.

The attached figures show such contour bodies.

MEMORANDUM

On the Multiplication Factor For Uranium

Metal Spheres. ~~shown by~~

L. Szilard

May 10, 1942

Based on the information concerning resonance neutron absorption which was obtained by the Princeton Group and on our estimates of the fission by fission neutrons, it is possible to make a forecast for the multiplication factor for uranium metal spheres of different sizes. This question has been investigated ^{by Feld and Rotland} along the following lines: We ~~will~~ ^{have} considered

the case in which the thermal neutron density and the resonance neutron density in the graphite is everywhere the same. This can be ~~achieved~~ ^{approached} by means of abundant use of cavities, channels, or slits. (See below). For small spheres ~~for which~~ the average thermal neutron density $\bar{\rho}_0$ within the sphere is the same as the ~~average~~ ^{surface} thermal density ρ_0 at the surface of the sphere $(\bar{\rho}_0 = \rho_0)$ and the similar condition holds for the resonance

neutron density. The most favorable ratio of carbon atoms to uranium atoms ^{is then $N_C/N_U \approx (\delta^* = 1)$} comes out to be 133. If we increase the ratio N_C/N_U and δ^* is below

1, the most favorable ratio of carbon atoms to uranium atoms falls from 133 to 69.6 for ^{spheres of 8 cm radius} 8 cm. spheres. The fraction of utilized neutrons increases from .865 to .917 as the ^{radius} range of sphere is increased from 3 cm. to 8 cm.

But the values for δ^* were calculated from a diffusion length of 1.63 cm for thermal neutrons in uranium metal of density 18 gm per cc, and δ^* was based on the Princeton measurements of uranium oxide which is a rather pessimistic assumption. If δ^* were larger, for instance, equal to δ , this would favor large metal spheres even more than it is apparent from the

figures given above.) It is, therefore, possible to conclude that by making ~~clear that the optimum~~ ^{clear that the optimum}

~~the neutron densities approximately uniform in graphite, the optimum size metal sphere will be about ^{Scumhere between 10 and 40 kg} kg., and accordingly we shall have cubic lattice cells of ~~cm. wide.~~~~

In order to estimate the multiplication factor which can be obtained we have to compare the fraction of neutrons utilized of about .92 ^{which can be obtained under the conditions} with the fraction of neutrons utilized in the ^{which we}

^{9. for} Columbia experiment (in which about 2 kg. lumps of uranium oxide were used) ^{we} ^{corrected}
~~and for which Wigner estimated the fraction of neutrons utilized to be about .75.~~ ^{For the Columbia experiment} ~~Correcting Wigner's value for the contribution of fast neutron fission in which we estimate to be .04 percent, we can write for the multiplication factor to be expected~~ ^{This value should be corrected} ^{to $\eta = .79$}

$$\text{Factor for Col. exp} \times \frac{0.92}{0.79} =$$

^{For instance if}
 If we assume that the Columbia experiment performed with pure uranium oxide ^{more so} would give a multiplication factor of about 1 ^{then large η} ~~we arrive at the~~ ^{found density of them and} ~~conclusion that large uranium metal spheres would give a multiplication factor of 1.17~~ ^{most} ^{number} ^{of}, provided the thermal neutron density and resonance neutron density can be made sufficiently uniform in the graphite.

^{omit about fast neutron fission}
 The optimum would be around 3 cm radius and $\eta = 0.795$. The fast neutron contribution drops the optimum into the neighborhood of 9 cm spheres ^{10 to 40 kg lumps and cubic cells} and increases η ^{of} ^{an} by ^{a factor of} $\frac{0.917}{0.793} = 1.15$

MEMORANDUM

On the Total Cross-Section of Uranium

May 10, 1942

L. Szilard

The total cross-section of uranium for scattering was determined by the standard method which is sensitive to small angle scattering using a neutron howitzer. The experiments were actually performed by ~~Eyer~~ ^{Reyer} on a sample of uranium metal powder. This sample contained, according to the analysis furnished by the N.B.S., 100 parts per million, boron, and .012 percent hydrogen. The ~~uncorrected value measured~~ ^{total cross section observed} was found to be 19×10^{-24} cm², corrected for the above impurities, we obtain 18×10^{-24} cm².

MEMORANDUM

On Heat Conductivity of Uranium Metal.

A rough preliminary measurement was performed on the heat conductivity of uranium metal in 1941. An apparatus was ~~devised and built~~ ^{improvised} for this purpose by Szilard and Zinn and measurements were taken by Marshall. The value obtained was about .04 cal per cm and sec, which ~~is~~ ^{on the present plant} tentatively being used in the present calculations until a more reliable value can be obtained. (Details available in the form of a manuscript written by John Marshall, Jr.).

MEMORANDUM

On the Cooling Problem

May 14, 1942

L. Szilard

A number of methods had been considered in the past and the purpose of this memorandum is to give a short review of these methods. The reason for considering such a great variety of methods is due to the fact that we have not sufficient knowledge at the present, concerning certain metallurgical and technological questions and also we do not know which of the manufacturing problems involved can be solved in a sufficiently short time to be applied in the near future.

General Statement of the Problem.

Since we would not be justified to assume that our development work is ahead of the development work in Germany, it is not sufficient to aim at the production of a moderate quantity of copper, but we have to aim at a copper production which will exceed the potential copper production of Germany. Under the circumstances the only certain advantage we have is at present the fact that Germany will probably hardly be able to secure more than 2000 tons of uranium, whereas the United Nations have a very much larger supply. Since about 50 tons of uranium will be tied up in every unit and since it is doubtful how long we will have to wait for reconditioning this uranium after the copper extraction process, it appears advisable to aim at a production of 1 kg. of copper per day per unit. Methods of cooling can be devised which go considerably beyond this figure and we shall not be satisfied with methods which fall far below it. Enclosed is a graph which shows the different methods of cooling and their interrelationship.

Cooling by Bismuth.

It might be possible to use liquid bismuth for cooling.

(The question whether bismuth alloys with uranium metal is under investigation and might make it necessary to coat the uranium metal with a layer of carbide or a thin layer of some metal which can be electro-plated on uranium). Figure shows a double stream arrangement.

General Statement of the Problem.

Since we would not be justified to assume that our develop-
ment work is ahead of the development work in Germany, it is not our
intent to aim at the production of a moderate quantity of copper, but we
have to aim at a copper production which will exceed the potential copper
production of Germany. Under the circumstances the only certain advantage
we have is at present the fact that Germany will probably hardly be able
to secure more than 5000 tons of uranium, whereas the United Nations have
a very much larger supply. Since about 80 tons of uranium will be tied
up in every unit and since it is doubtful how long we will have to wait
for reconditioning this uranium after the copper extraction process,
it appears advisable to aim at a production of 1 kg. of copper per day
per unit. Methods of cooling can be devised which are considerably bet-
ter than this figure and we shall not be satisfied with methods which fall
far below it. Enclosed is a graph which shows the different methods
of cooling and their interrelationship.

MEMORANDUM

On The Helium Cooled Power Unit

June 9, 1942

L. Szilard

Helium at 10 atm.

Assuming cylindrical or cubical cross section of 64 cm^2 , a total of which 22 cm^2 or about $1/3$ are left free for the gas, and length about 8 cm, we have for uranium metal of density 20, a cluster of about 7 kg weight. For a standard machine developing 10^6 kw per 10 ton of uranium this cluster would develop 165 large calories. If we further assume that the helium at 10 atmosphere pressure enters the cluster with 200°C and leaves it with 500°C , and that the velocity at exit is equal to the critical velocity 177 m/sec , we find that the heat carried away by the helium corresponding to 300°C temperature rise in the helium, is about 100 large calories. This corresponds to $100/165$ ~~equal to~~ 0.6 of the standard power unit, or 600,000 kw. If the cluster is built of uranium sticks of about 3 mm. diameter, we have a heat transfer of about 500,000 kw in the cluster corresponding to the mean temperature difference of 300°C between the gas and the surface temperature of the uranium metal. Since the mean temperature of the gas is 350°C , the surface temperature of the metal would have to be 650°C . In order to see this we may write

This is calculated with a heat transfer coeff. of
 $h = 75 \cdot 10^{-2} \text{ cal/grad cm}^2 \text{ sec}$

For a one-half standard machine the mean temperature in uranium sticks of

3 mm. diameter is $^{\circ}\text{C}$ higher than the temperature of the surface.

MEMORANDUM

On The Helium Cooled Power Unit

June 9, 1943

L. S. Gilford

1-11

Assuming cylindrical or spherical cross section of 3 mm. diameter, total of which is 23 cm. or about 1.3 are left for the gas, and is right above it. we have for uranium metal of density 19.3, a cluster of about 1 kg weight. For a standard machine developing 100 kw per 10 ton of uranium this cluster would develop 100 large calories. If we further assume that the helium at 10 atmosphere pressure enters the cluster with 200°C and leaves it with 600°C , and that the velocity at exit is equal to the critical velocity 177 m/sec, we find that the heat carried away by the helium corresponding to 200°C temperature rise in the helium, is about 100 large calories. This corresponds to 100,000 watts or 0.8 of the standard power unit, or 800,000 kw. If the cluster is built of uranium atoms of about 3 mm. diameter, we have a heat transfer of about 800,000 kw in the cluster corresponding to the mean temperature difference of 300°C between the gas and the surface temperature of the uranium metal. Since the mean temperature of the gas is 380°C , the surface temperature of the metal would have to be

380°C . In order to see this we may write

Approximate Dimensions

The uranium ^{clusters} plugs may have a weight somewhere between 2 and 7 kilograms. A 7 kilogram plug would have a cross section of about 64 sq. cm. of which about one-third, perhaps 22 sq. cm., are left free from the gas in the form of gaps between the uranium pencils. The length of the uranium pencils forming the cluster or plug would accordingly be about 8 cm., ~~assuming~~ ~~a uranium density of 20.~~

If we have two kilogram uranium clusters, the length of the uranium pencils forming the clusters would be reduced to 5.2 cm. In a cubic pile of 6 x 6 x 6 meters containing 40 tons of uranium we would have 740 graphite columns 6 and 27 uranium clusters in each column. This would correspond to a cubic cell of 22 cm. side. The total cross section of the ducts 2 and 3 may be chosen to be 15% of the cross section of the cubic pile, or 5.4 meters square, and we would need about 1.2³ times as much graphite for reaching the critical dimensions as without the ducts. Some of the cross section for one pair of ducts corresponds to 73 sq. cm. and accordingly the radius of duct 3 at exit is about 5 cm.

For helium at 10 atmospheres pressure and a temperature increase of 300° C. the exit velocity of the helium would be 53 meters or 1/3.35 of the critical velocity. Assuming that the kinetic energy is lost three times in the duct and at exit the loss amounts to 0.33% of the transported heat, or about 1000 KW. To this we have to add the friction loss in the uranium plug and at exit. Taking the velocity of the uranium plugs to half of the critical velocity, 90 meters per second, and assuming that the kinetic energy is lost twice in the plugs and at exit, we have a friction loss of an additional 1/2% or 1500 KW. Even if we assume that through the roughness of the pipes we double the friction loss, we shall still remain below 5000 KW and a compressor of 10,000 KW input ought to be sufficient.

This power unit can dissipate 300,000 KW of helium at 10 atmospheres pressure. The helium is assumed to come from the heat exchanger with a temperature of 200° C., and to be heated up in the power unit by 300° C. to 500° C. The highest temperature at which the uranium would be in contact with helium is estimated to be 800° C. The friction loss of the circulating helium within the power plant is estimated to amount to less than 4000 KW.

If helium is used at one atmosphere pressure the friction loss would be less than 400 KW and the heat output 30,000KW.

... pile of 6 x 6 x 6 meters containing 40 tons of uranium we would have 740
... uranium clusters in each column. This would
... correspond to a cubic cell of 22 cm. side. The total cross section of
... the ducts 2 and 3 may be chosen to be 15% of the cross section of the
... cubic pile, or 1.4 meters square, and we would need about 1.33 times as
... much graphite for reaching the critical dimensions as without the ducts.
... Some of the cross section for one pair of ducts corresponds to 73 sq. cm.
... and accordingly the radius of duct 3 at exit is about 5 cm.
... For helium at 10 atmospheres pressure and a temperature increase of
... 300° C. the exit velocity of the helium would be 53 meters or 173 ft. or
... the critical velocity. Assuming that the kinetic energy is lost three times
... in the duct and at exit the loss amounts to 0.55% of the transported heat, or
... about 1000 KW. To this we have to add the friction loss in the uranium
... pile and at exit. Taking the velocity of the uranium pile to half of the
... critical velocity, 90 meters per second, and assuming that the kinetic
... energy is lost twice in the pile and at exit, we have a friction loss of an
... additional 1/2% or 1800 KW. Even if we assume that through the roughness
... of the pipes we double the friction loss, we shall still remain below 3000 KW
... and a compressor of 10,000 KW input ought to be sufficient.

In order to prevent the fission products from precipitating in bulk together with the uranium peroxide it may be necessary to add fairly large quantities of the stable isotopes of the fission products to the solution each time before the uranium is precipitated as peroxide.

Insertion 2 (p. 4)

The uranium plugs designated by 4 may be of the construction shown in Figure 1B. Uranium pencils of about 3 mm. in diameter are bundled together to form a short cylinder. Figure 3A shows a modification of the arrangement shown in Figure 3. In this modification the axis of the uranium plugs coincides with the axis of the graphite column 6. The helium enters cold through the duct 2 and leaves hot through the duct 3. It may pass from duct 2 to duct 3 either through one uranium plug or through several uranium plugs in series. The section shown in Figure 3A shows three uranium plugs in series. This arrangement has sufficient elasticity to meet the particular requirements of the chain reacting power unit.

As Wigner ^{repeatedly} pointed out, it is necessary to have a cooling system which is adapted to deal with the specific power production three to four times (according to whether the power unit has a spherical or cubic shape) as large in the center of the power unit as the average value within the power unit. The arrangement shown in Figure 3A makes it possible to have throughout the whole power unit the temperature of the gas entering duct 3 to be approximately the same. In order to achieve this one has only to vary ^{m,} the number of plugs which are connected in series, according to the position which the graphite column 6 occupies in the pile and according to the position of the particular uranium plugs within the particular graphite column.

In the center of the pile the gas may flow from duct 2 to duct 3 by passing through one single uranium plug and the temperature of the gas may rise during this passage by ^{the same} sudden value, for instance, 300° C. In another region of the pile where the neutron ^{density} intensity is smaller by a factor $1/k$ ~~than in the center~~ we will obtain the same rise of temperature of the gas which passes from duct 2 into duct 3 by passing the gas through n uranium plugs in series.

~~If we choose~~ ^{Since} the pressure difference ~~is~~ ^{between duct 2 and 3 has} ~~everywhere~~ ^{is the same} for ~~each~~ ^{each} ~~plug~~ ^{plug} the column ~~to~~ ^{the} does not vary along the column, the velocity of the gas ^{in the plugs} will be proportional to $\frac{1}{\sqrt{n}}$ and in order to have the same temperature of the gas at entrance into duct 3 we have to choose

$$n = k^{2/3}$$

(If we had m plugs in series in the center ~~than we had~~ ^{would} ~~to write~~ ^{to write} ~~$n = k^{2/3}$~~ $n = m k^{2/3}$)

MEMORANDUM

Sept

On a Helium Cooled Power Plant.

August 10, 1942

Addition to memorandum dated June 18, 1942.

Bernard T. Feld

Previously I ~~have~~ calculated the dimensions of a 300,000 kw machine cooled by helium at 10 atmospheres. In this machine we ~~have~~ used, at the center, a plug consisting of pencils 0.3 cm in diameter and 4 cm long. The gas was circulated with a velocity of 68 m/sec, or $\frac{1}{\frac{2.6}{3.3}}$ x critical velocity. The duct size ~~is~~ ^{for} this machine is 12 percent of the total area. We now consider a 100,000 kw machine.

, if we keep the same velocity,
 The duct size ~~in this case~~ goes from 12 percent to 4 percent, and if we take a 20 cm cell, this gives a cross sectional area of 16 cm² for each duct, and a critical length of 150 cm for this duct size. Thus, the length of our duct is about 4 critical lengths. Hence, the friction loss in the duct, including the loss upon exit, equals $\frac{4 + 1}{(\frac{2.6}{3.3})^2}$ percent of the total heat transport--or .74 percent. In the plugs themselves the production of heat is 1/3 as much as previously considered, and so, in order to get a temperature rise of 300°C, we would put 3 plugs in series, at the center of the pile. The friction loss in the plug and on exit, is $\frac{.27 + .25}{1.69} = 0.31$ percent, so that the total loss in this machine is $0.31 + 0.74 = 1.05$ percent, or 1,050 kw.

If we reduce the pressure to 7 atmospheres, ~~then~~ we would ^{connect} ~~collect~~ two plugs in series at the center of the pile and would have to increase the total duct area to 6 percent. For this size duct the critical length is 200 cm, and so we have a loss of $\frac{3 + 1}{6.8} = .59$ percent in the ducts. The total loss then for this machine = $0.31 + 0.59 = 0.90$ percent of the heat transported, or 900 kw.

~~SECRET~~

*prints + handwritten
(see O.P.)*

MEMORANDUM

L. Szilard

September 19, 1942

CLASSIFICATION CANCELLED
Date 4/21/56
For The Atomic Energy Commission
C. L. Marshall/ner
Director, Division of Classification

Introduction.

These lines are primarily addressed to those with whom I have shared for years the burden of knowing that it is within our power to construct atomic bombs. What the existence of these bombs will mean we all know. It will bring disaster upon the world if the Germans will be ready before we are. It may bring disaster upon the world even if we anticipate them and win the war, but lose the peace that will follow.

to help
The requirements of keeping our work secret makes it impossible gradually to prepare the country for the role which it will have to assume immediately after the war is over. *How can we* If it wants to have peace in a world where a lone airplane can drop *an atomic bomb* a bomb over a city like Chicago. Not one single house may be left standing and the radioactive substance scattered by the bomb may make the area uninhabitable for some time to come. Some way will have to be found to *this law* ~~prepare for~~ *(that could be taken with)* adequate action after the war, those whom the constitution has entrusted with determining the policy of this country.

Most of you may feel that of more immediate concern to us is the fact that the perhaps most promising branch of this work ~~known~~, which is pursued at Chicago, is not developing as rapidly as it ought to.

What is the trouble with our project?

Roughly speaking there are two kinds of troubles which frustrate our work. Troubles which originate at the Washington end of our organization, and troubles which originate at the Chicago end of our organization.

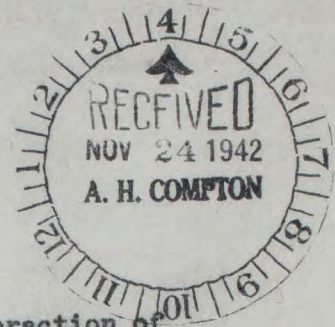
This material contains information affecting the national defense of the United States within the meaning of the espionage laws, Title 18, U.S.C., Secs. 793 and 794, the transmission or revelation of which in any manner to an unauthorized person is prohibited by law.

~~SECRET~~

*Copy of report
handed to A.H. office
Nov. 23-42*

SHORT MEMORANDUM ON BISMUTH COOLED POWER UNIT

L. Szilard
November 23, 1942



SUMMARY

If simple but thorough metallurgical tests on the interaction of liquid bismuth with steel come out satisfactorily it will be possible to build a bismuth cooled power unit that would have high operational safety. Such a power unit can be expected to work with 1000 tons of graphite and 150 tons of uranium in the form of carbide or dioxide. More than one kilogram of 94 would be produced daily by such a power unit and it is estimated that a total of about 200 kilograms of 94 might be produced by each such power unit. This estimate of the total has a much smaller degree of certainty than the calculated value of the daily production of 94.

Introduction

The proposal of using bismuth as a cooling agent was part of the original scheme of setting up a chain reaction in a uranium-graphite system. Experiments performed since that time have greatly strengthened the view that the neutron absorption of bismuth is sufficiently low to permit this application. The last experiment performed on 4 tons of bismuth at Chicago gives, according to Fermi, an about 1% loss in the multiplication factor, if the amount of bismuth in the pile is equal to the amount of uranium. This result bears out the contention that the commercial grade of bismuth, which was known to be of extraordinary purity, is sufficiently pure for our purposes. The chemical stability of a system composed of bismuth, graphite and uranium carbide, or uranium dioxide, at high temperatures, and in the presence of strong radiation, was one of the chief considerations in selecting this system of cooling. Compared to this the good heat transfer properties of liquid bismuth are of secondary importance. There is reason to believe that it is safe to have liquid bismuth in contact with steel and to pump it through steel pipes. This point has however to be further investigated before a decision of building a bismuth cooled power unit can be made.

The Use of a Bismuth Cooled Power Unit For:

a. production of 94.

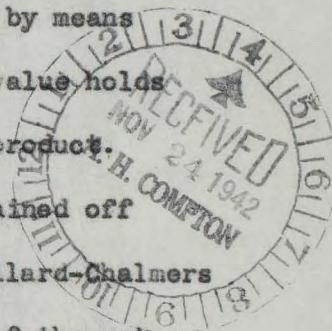
The main purpose of operating a bismuth cooled power unit during the war is the production of about 1 ton of 94. This amount might be needed in order to win the war by means of atomic bombs, though one may hope that a smaller quantity will be sufficient. Assuming that about 1/4 of the U 235 contained in 150 tons of uranium in the power unit can be transformed into 94, 270 kg. of 94 could be obtained during the operation of the power unit. In order to produce this amount in about 200 days, it is necessary to produce about 1.3 kg./day and to dissipate about 1.3 million kw. This is quite feasible in a bismuth pile without straining matters.

b. production of radioactive poisons.

The bismuth cooled power unit could produce about 40 tons of radium equivalents in radioactive poisons outside the power unit by means of fully utilizing the neutrons which escape from within. This value holds for a time of operation which is equal to half life time of the product. Therefore some such quantity of radioactive material could be drained off every few days if a suitable product were chosen. However, a Szilard-Chalmers separation may have to be performed in order to reduce the bulk of the material which has to be transported, and this may require to put each time perhaps 15 tons of material through a chemical separation.

c. production of polonium and light sources for the armed forces.

About 10 tons of radium equivalent of polonium will be produced in 140 days of operation in the 150 tons of bismuth which will be used for cooling the power unit. Polonium mixed with a luminous compound gives a light source which is practically free from harmful rays and which can be used to serve as torches, to illuminate instruments, etc. The torches would be of very small weight and would give off light "permanently", the brilliance de-



caying to its half value in 140 days. 250,000 such torches, each dissipating about one watt, could be made from the quantity of polonium produced in 140 days of operation.

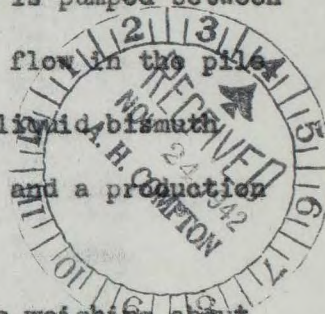
About four times this quantity of polonium could be produced outside the power unit but this would involve the exposure of many hundred additional tons of bismuth and may therefore not be expedient during the war.

In the post-war period the manufacture of polonium might become the first important industrial application of the chain reaction.

General Features of the Power Unit.

It is proposed to maintain a chain reaction in a cylindrical graphite pile 8 m. high and 9.5 m. in diameter, weighing about 1000 tons and containing about 150 tons of uranium in the form of uranium carbide. This graphite pile would be enclosed in hermetically sealed container filled with helium at normal pressure. Liquid bismuth would enter at the top of the pile having a temperature of about 300°C. and flow through grooves or bores in the graphite from top to bottom under the action of gravity leaving at the bottom at about 600°C. The pressure of the liquid bismuth would remain the same along the flow from the top to the bottom of the pile and would vary according to the volume which is pumped between 1 and 2 atmosphere gauge. The velocity of the vertical bismuth flow in the pile is estimated to be 4 m. per second. A flow of about 3.3 m³ of liquid bismuth per second corresponds to a heat dissipation of 1.3 million kw, and a production of 1.3 kg. of 94 per day.

The uranium carbide is present in the form of aggregates weighing about 2 kg. Figure 30 shows one possible shape in which the uranium carbide could be used. According to the position of these aggregates in the pile, smaller or larger number of them are in series in a parallel flow arrangement. The principle of such a parallel flow arrangement is illustrated in Figure 3, C taken from an



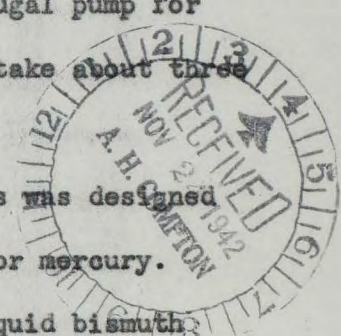
earlier memorandum. Since this figure relates to a helium cooled power unit, it does not show the dimensions which would be correct for bismuth cooling. In the case of bismuth we may have a vertical graphite column of 20 cm. x 20 cm. and the two vertical bismuth ducts have together an average cross section of 10 cm² near the axis of the pile. Near the axis of the pile the amount of bismuth is about equal to the amount of uranium, but closer to the periphery the relative amount of bismuth is less. Altogether about 100 tons of bismuth may be in the pile during operation.

Pumps.

The pumps lifting the liquid bismuth to about 10 m. height must have an output of about 3500 kw and assuming 50% efficiency they would have a power requirement of about 7000 kw.

Centrifugal pumps have been used for mercury under conditions of temperatures and pressures which are very similar to those required for the pumping of bismuth which will have a temperature of about 300°C. after passing through a heat exchanger. The General Electric Company has put at our disposal blueprints which could serve as a basis for further action. The Westinghouse Company expressed its willingness to design and build a centrifugal pump for our purpose and it was estimated that the designing work would take about three weeks and the actual building time about six months.

Another type of pump which would have certain advantages was designed by Einstein and Szilard and built and operated in 2 kw. units for mercury. This pump consists of a steel tube with an iron core and the liquid bismuth would flow through the annular gap between core and tube under the electrodynamic action of electric windings which are outside the steel tube. This pump has the advantage of having no moving parts, of requiring no lubrication or other service, and of representing an all sealed system with a minimum of



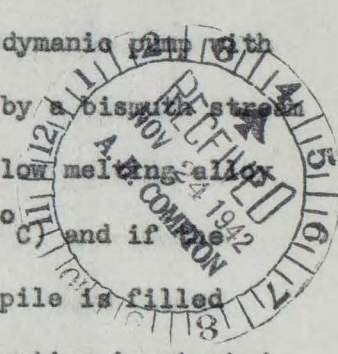
danger of leaks. It has the disadvantage that it may have to be operated on a low frequency and may therefore require a converter. Our estimate for the efficiency at $12\frac{1}{2}$ cycles is about 20% for units which have an output of about 300 kw. and which would accordingly require a power input of 1500 kw. Larger units may have a somewhat higher efficiency. The Westinghouse Company made a preliminary investigation into these questions and expressed its willingness to design such a pump if requested to do so. It was estimated that it would take about two months to complete the design.

Heat Exchanger

One may consider to have a heat exchanger at the bottom of the pile in which the heat is transferred from the bismuth to a bismuth-lead alloy having a melting point at 124°C . This alloy would then be pumped outside the pile and would transfer its heat to water.

Controls

It is proposed to control the bismuth cooled pile in a manner which does not necessitate the moving of "control rods" through stuffing boxes. Such an arrangement which contains no moving parts is illustrated in Figure 40. This shows a steel tube (of about 1 cm diameter and less than 0.5 mm. wall) going through the pile which communicates through an electrodynamic pump with a vessel outside the pile. (The tube in the pile is cooled by a bismuth stream indicated by arrows). This communicating system contains a low melting alloy containing bismuth, lead, tin, and cadmium (melting point 68°C) and if electrodynamic pump is out of action, the steel tube in the pile is filled with this liquid alloy as shown in the figure. The chain reaction is started by switching on a variable transformer (Dreh transformator) which feeds the electrodynamic pump. The liquid is then pumped out of the steel tube in the pile into the vessel outside until the pressure difference becomes equal to the pressure produced by the pump. This pressure is controlled by the trans-



former which in turn is controlled by the radiation intensity in such a manner that the voltage of the transformer increases with decreasing radiation intensity. If there is a failure in the control arrangement the pump goes out of action and the absorbing liquid fills the steel tube inside the pile and stops the chain reaction. Several steel tubes are connected to the same electrodynamic pump and two or three such systems ought to be provided for the sake of operational safety. The power consumption at each such pump amounts to a few kilowatts.

Start of Operation

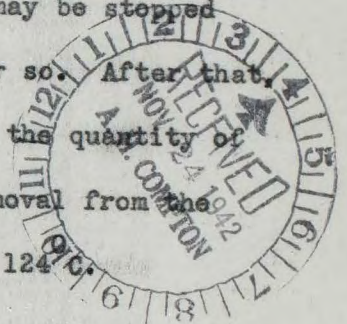
At the start of operation the graphite pile has to be heated up to a temperature above 300°C . Electric resistors can be used for this purpose and a few thousand kw. input is sufficient to heat up the pile within 24 hours. Since the bismuth pumps are not in operation during the heating up period, the electric installation required by the bismuth pumps is capable of taking care of the heating.

End of Operation

After operation of several months the chain reaction may be stopped but the bismuth circulation is maintained for another month or so. After that, by admitting liquid lead in a quantity approximately equal to the quantity of bismuth in the pile, the uranium can be cooled down before removal from the pile to about the melting point of the bismuth-lead eutectic, 124°C .

Materials

Either of the two uranium carbides or uranium dioxide could be used following designs which have been fairly well developed. Uranium dioxide has to be used in a sintered form and the technique for sintering has been worked out on a laboratory scale at Ames, Iowa. The preparing of uranium



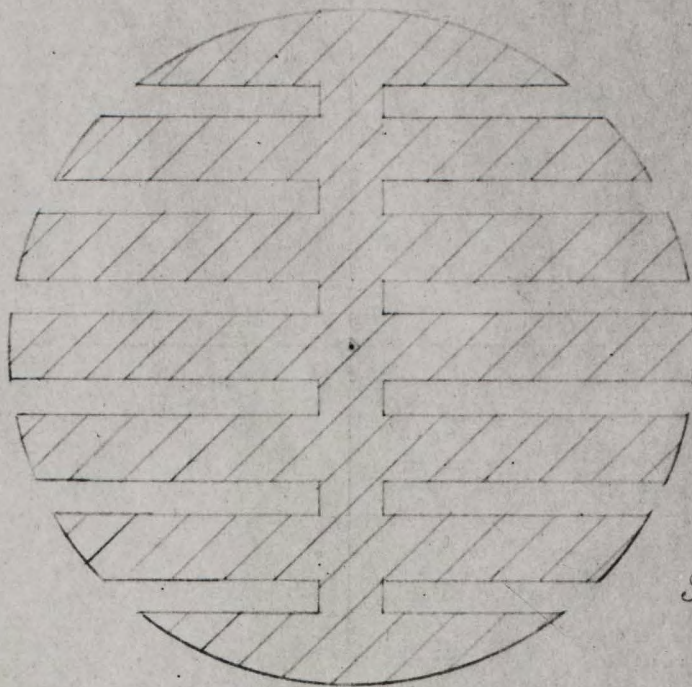
carbide and its casting into simple cylindrical shapes has also been worked out on a laboratory scale at Ames, Iowa. More complicated shapes would have certain advantages, such as the shape shown in Figure 30, have so far not been cast.

THIS DOCUMENT HAS BEEN
 TAKEN FROM THE FILES OF THE
 ARGONNE NATIONAL LABORATORY
 AND WAS REFERRED OVER TO
 DR. LEO SZILARD ON

Robert S. Johnson

Uranium metal could possibly be used, for instance, inside of graphite tubes, but the use of uranium metal in this form has so far not been sufficiently studied and it is therefore uncertain whether the use of uranium in metal form would be practicable from the point of view of operational safety.





Section A-A'

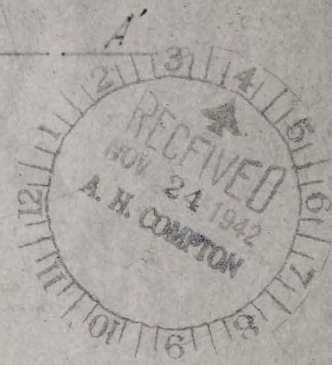
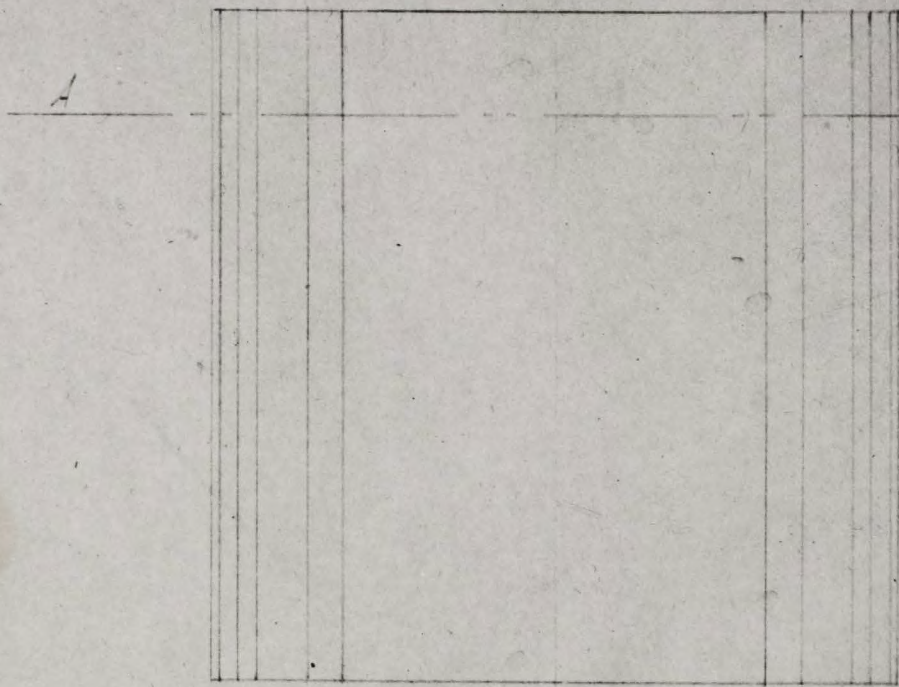


FIG. 30

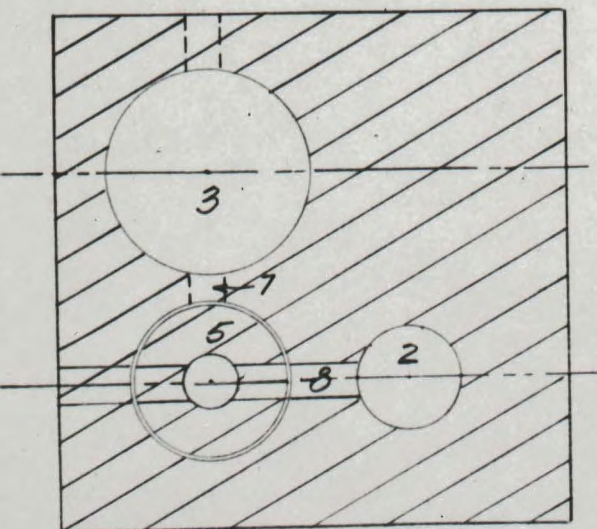
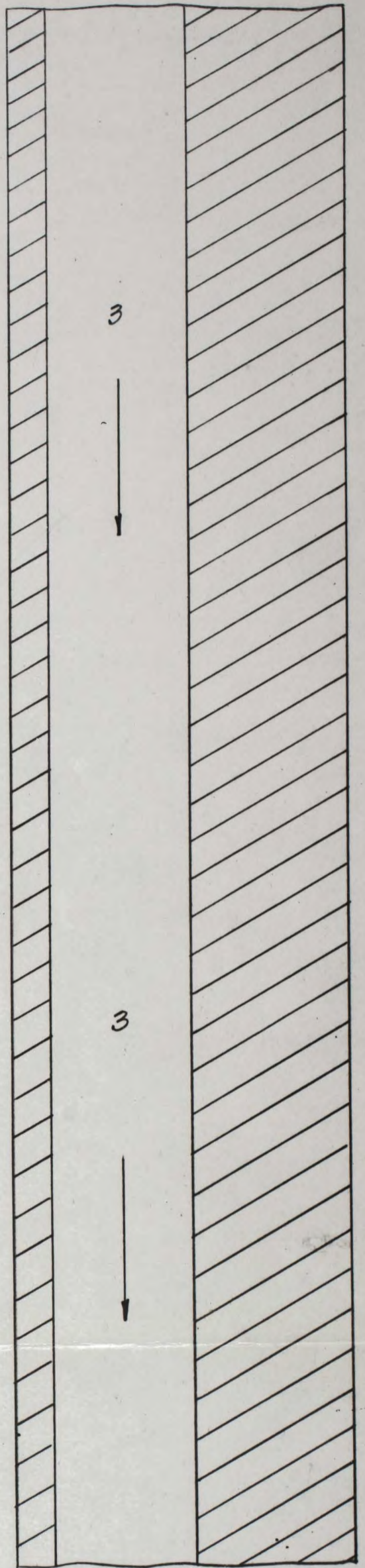
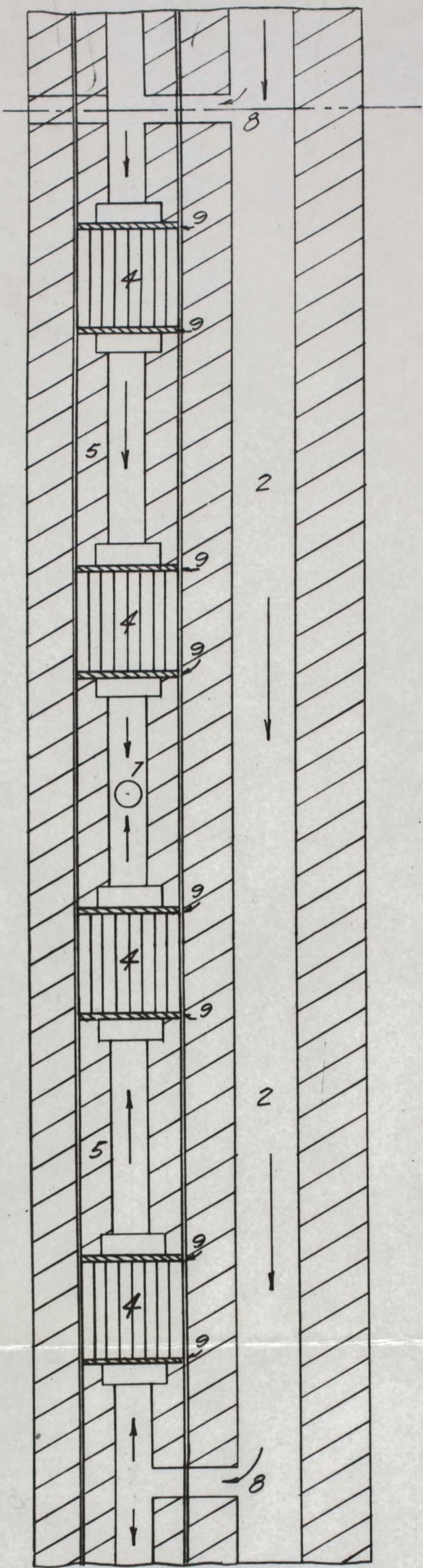


Fig. 3C

SECRET

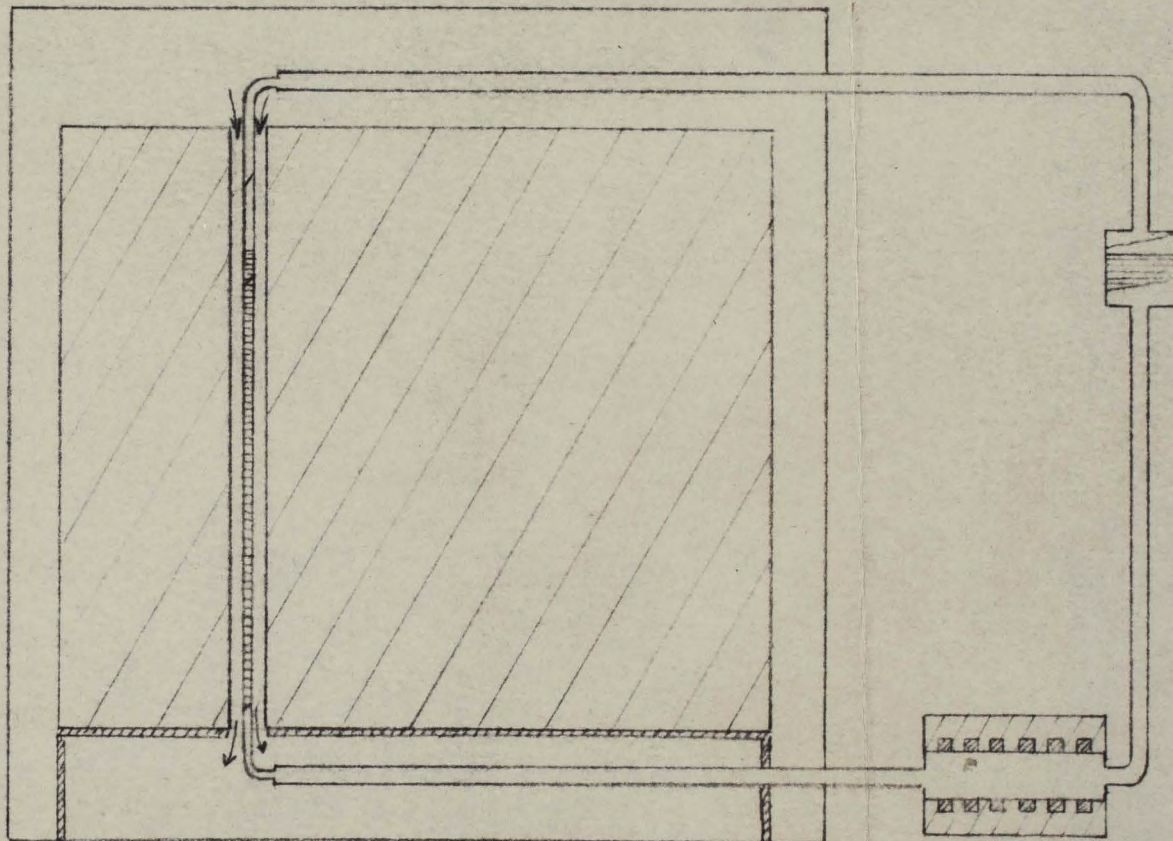


FIG 40



Depart
~~MEMORANDUM~~ *for Scotland*
L. Seilard

November 24, 1942

~~L. Seilard~~

On the controls and 94 production *we* are making the following simplifying assumptions. We assume that if an atom of 235 undergoes fission, an atom of 94 is produced, and also that if an atom of 94 undergoes fission, another atom of 94 is produced. We further assume that we have an infinite system with $k = 1$. In the circumstances, clearly the number of atoms of 235 will vanish asymptotically and an equal amount of 94 will be produced. However, since the thermal cross-section of 94 is about 1.7 times that of 235, k would, after the start of the chain reaction, rise above 1, unless we either change the amount or distribution of uranium in the system, or introduce absorbers. If we choose the former, ~~say~~ the above considerations hold. If, however, we introduce absorbers, ~~say~~ in the form of control rods, the control *rods* will have to move gradually into the chain reacting system in order to maintain k . Since there is now additional absorption, the total number of atoms of 235 plus 94 does not remain constant but diminishes with time. Since the amount of 235 approaches asymptotically zero, and since, if there is no 235 present, and if there is an absorber inside the chain reacting system, the amount of 94 will ultimately also diminish with time, it follows that after a certain time of operation, k will begin to decrease and the direction of the movement of the control rods will have to be reversed in order to maintain $k = 1$. From then on the control rods move outwards, but they will not, in a finite time, move completely out of the pile, since if they were out of the pile at a time when there is still some 235 left, the continued chain reaction would increase k beyond 1. It follows therefore,

that the total fission cross section of 235 and 94 which is present, is at all times larger than the fission cross section or the initial amount of 235, and ^{in the} a state which we are approaching ^{asymptotically the} corresponds to a number of atoms of 94 which is 1.7 times smaller than the initial number of atoms of 235.