Exhibit A attached March 9th, 1939

As a carrier of such a chain reaction it is necessary to have an element the mass of which is sufficiently high to allow its disintegration into its parts with liberation of energy, the disintegration being ordinarily inhibited (meta stable state), the inhibition being lifted in a reaction with a neutron. An example of such a meta stable element is uranium.

In order to be able to utilize a nuclear reaction in which an excess number of neutrons is liberated by neutrons for the maintenance of a chain reaction, it is not sufficient to measure the cross-section and other constants of the reaction, but it is also necessary to be aware of the laws which govern the neutron output of such reactions in function of the geometrical conditions. Once the general laws, the type of behavior is known, the exact dimensions can be easily determined in each particular case by actually measuring the neutron output. Such experimental adjustments can, however, only be made if the general type of behavior is known.

In the following we shall by way of example demonstrate certain general features of such a neutron chain reaction in a special case. In this special case the carrier of the chain reaction forms a spherical layer as illustrated in Fig. 1. In Fig. 1, 1 is a spherical layer which contains a carrier of the chain reaction, 2 is a neutron source which has fixed constant output of neutrons, r_1 is the inner radius of the spherical layer, and r_2 is the outer radius of the spherical layer. In order to be able to treat the problem as a diffusion problem, we choose r_1 very much larger than the mean free path a of the neutrons produced in the chain reaction in the substance of

the chain reaction layer. The neutrons liberated in the chain reaction layer will have to make a number of collisions with the nuclei which compose the chain reaction layer, and we designate the number of collisions necessary for causing transmutations in the carrier of the chain reaction layer which lead on the average to the liberation of one additional neutron with f. We further assume that f is a large number so that we may have conditions in which the well known equations of diffusion can be applied. With these assumptions the density s of the neutron will with good approximation be given within the chain reaction layer as a function of the radius r by the following equation:

(1) $$D\frac{d^2(rs)}{dr^2} + A(rs) = 0$$

D is a diffusion constant which is given by

$$D = \frac{aw}{3}$$

w is the mean velocity of the neutrons; a the mean free path for scattering collisions within the chain reaction layer; and f a number which says how many scattering collisions a neutron has to make in the chain reaction layer in order to produce on the average an additional neutron.

A stands for the number of neutrons produced per c.c. of chain reaction layer in a second and its value is given by $A = \frac{w}{af}$ so that $\sqrt{\frac{D}{A}} = \frac{a\sqrt{f}}{\sqrt{3}}$

For a given value of r_1 there is a certain value of r_2 and accordingly a certain value of $r_2 - r_1$ for which the number of neutrons per second diffusing out of the spherical layer into space becomes infinite for a finite neutron production of the source. This value of $(r_2 - r_1)$ we may call the critical thickness of the chain reaction layer. If the

thickness of the chain reaction layer is smaller than but very close to the critical thickness, the neutron output is very much larger than the neutron input and we may have 1000 or more times as many neutrons emerging from the chain reaction layer as the neutron input of the neutron source 2. The value of the critical thickness is a function of $\underline{r}_1$ and of the boundary conditions for $\underline{r} = \underline{r}_1$ and $\underline{r} = \underline{r}_2$. If the outer surface ($\underline{r} = \underline{r}_2$) of the spherical layer were to stand free in space, the density $\underline{s}$ would be 0 for the outer surface, and if there is no absorption of neutrons in the hollow sphere containing the neutron source, i.e. if the number of neutrons produced by the neutron source is equal to the number of neutrons diffusing outwards from the sphere $\underline{r} = \underline{r}_1$, we obtain for very large values of $\underline{r}_1$ ($\underline{r}_1 \gg \sqrt{\frac{\delta}{A}}$) for the value of the critical thickness

$$(2) \qquad l_{kr} = \frac{\pi}{2}\sqrt{\frac{D}{A}} = \frac{\pi}{2}\frac{a\sqrt{f}}{\sqrt{3}}$$

The critical thickness for these conditions we shall call further below the standard critical thickness of the substance which composes the chain reaction layer.

If the outer surface is covered by some material, for instance if the transmutation layer is covered by lead or immersed into water, the critical thickness is less than the standard critical thickness. This is due to the back scattering by lead or water, and, in the case of water, the fact that the neutrons are slowed down by water and their mean free path is thereby reduced plays an important role.

If the neutrons are allowed to escape out of the hollow sphere containing the neutron source in the interior of the spherical layer or if they are absorbed within this hollow sphere, the critical thickness is increased.

If all the neutrons were captured in the hollow sphere within $r = r_1$, the critical thickness would become independent of the value of r_1 and have a value exactly double the ~~normal~~ standard critical thickness provided that the neutron density s remains 0 at the outer surface ($r = r_2$).

Obviously the above given diffusion equation (1) holds only for stationary solutions, that is for solutions where the neutron density s is a function of r only and does not vary with time. Not for all boundary conditions will such stationary solutions exist. If we gradually increase the thickness of the ~~critical~~ chain reaction layer we reach for any given boundary conditions a thickness at which the neutron outflow becomes infinite for finite neutron production of the neutron source. This thickness is the critical thickness of the arrangement. It can be calculated for every case in the following way from given boundary conditions.

The solution of the above given diffusion equation has the form of

$$(3) \qquad s_r = C_1 \sin\left(\sqrt{\frac{D}{A}}\, r\right) + C_2 \cos\left(\sqrt{\frac{D}{A}}\, r\right)$$

For the boundary condition $s = 0$ for $r = r_2$ the solution takes the form of

$$(4) \qquad s_r = C \sin\left[(r_2 - r)\sqrt{\frac{D}{A}}\right]$$

or

$$(5) \qquad s = C\, \frac{\sin\left[(r_2 - r)\sqrt{D/A}\right]}{r}$$

If there is no absorption of neutrons in the hollow sphere (3) around the neutron source (2) and if the neutron source (2) produces a fixed number N of neutrons per second, the number of neutrons diffusing from the sphere $r = r_1$ into the chain reaction layer must be equal to N. On the other hand, the number of neutrons thus diffusing through

any sphere is given by

$$4\pi r^2 D \frac{ds}{dr}$$

and it is therefore also

$$N = 4\pi r_1^2 D \left[\frac{ds}{dr}\right]_{r=r_1}$$

For a value of $r = r_a$ for which $\frac{ds}{dr}$ becomes 0, the ratio of the number of neutrons diffusing through the outer surface $r = r_2$ to N becomes infinite. The corresponding value of $(r_a - r_1)$ is therefore the critical thickness and can thus be calculated for different values of r_1 from equation (5) for very small values of r_1 ($r_1 \ll \sqrt{\frac{D}{A}}$) it obviously becomes twice the standard critical thickness i.e. $\pi\sqrt{\frac{D}{A}}$

To take another example, if the boundary conditions are $s = 0$ both for r_1 and r_2, then a glance at equation (5) shows that $(r_2 - r_1) = \pi / \sqrt{\frac{D}{A}}$, i.e. twice the critical thickness, quite independent of the value of r_1.

Figs. 2 and 3 show such a chain reaction apparatus. A neutron radiation, the initial radiation, is generated by the high voltage canal ray tube 1 (shown in greater detail in Fig.). This tube generates fast deuterons which strike the target 28 which contains deuterium. The neutron radiation emerging from 28 acts on the matter 3 which fills the spherical transmutation space. The composition of this matter 3 will be discussed further below and is such that a chain reaction is released by the neutrons. The pumps 120, 121 and 122 pump a liquid, for instance water or mercury, through the pipe systems 107, 110, 111 thereby cooling the transmutation area 3 and driving the heated liquid through the boiler 126. The boiler supplies steam to a power plant. The neutrons emerging from the sphere 3 act on a layer 9 which is composed of an element xx that will trans-

Exh. B
dated March 9 1939

The standard critical thickness is given by

(11) $$t_{st} = \frac{\pi}{3}\,\bar{\sigma}\left(\frac{w_2 - w_0}{3(w_1 - 2w_2)}\right)^{1/2}$$

in the general case and by

(12) $$t_0 = \frac{\pi}{2}\,\bar{\sigma}\sqrt{\frac{1}{3}}$$

in the special case.

Obviously, the above diffusion equation presupposes for its validity a small value of w_2, but even for large values of w_2 it gives at least the order of magnitude for the critical thickness.

The critical thickness will in practice always be determined empirically for instance in the following way: a neutron source is surrounded by the chain reaction layer of an approximately correct thickness which is safely below the critical thickness. The radiations emitted from the chain reaction layer while exposed to this neutron source are observed by means of an ionization chamber. Then the thickness of the chain reaction layer is brought closer to the critical thickness by gradually increasing either the quantity of uranium or the quantity of hydrogen containing substances mixed with the uranium. The amount of ionizing radiation which is emitted is again observed and the thickness of the chain reaction layer is again brought closer to the critical thickness in the same way as before. In this way, by observing the increase of the emitted radiation as a function of the increasing effective thickness of the chain reaction layer the critical thickness can be extrapolated from the observed curve by plotting the intensity of the emitted neutron radiation against the effective thickness of the chain reaction layer. Instead of an ionization chamber which registers

the neutron intensity by means of recoil ions in the gas of the chamber, induced activity caused by the neutrons can be used as a measure of the radiation intensity.

Variation of Critical Thickness

If slow neutrons are used the critical thickness can be increased by having a slow neutron absorber within the hollow sphere in the center of the spherical arrangement. If the inner radius of the spherical shell of the chain reaction layer is much larger than the critical thickness (to be accurate we have said the standard critical thickness given by the above formulas), and if all slow neutrons are absorbed, for instance by a cadmium layer covering the inner surface of the spherical chain reaction layer, the critical thickness of the arrangement is increased. By removing such absorbing matter from the inside of the chain reaction layer, the critical thickness may be reduced below the actual thickness, and thus an explosion may be brought about. The explosion will be all the more violent the more quickly the absorbing substance is removed. A similar increase in the critical thickness of a spherically symmetrical chain reaction layer can be brought about by removing a section of the layer and thereby producing an opening through which the neutrons can escape. For instance a conical section corresponding to a few percent of the spherical chain reaction layer can be so arranged as to be easily moved out of its place and replaced, and thereby the critical thickness may be reduced or increased.

Explosions may be brought about by a variation of the critical thickness for the purpose of creating a large amount of radio active elements. In such a case it is

Fig 2

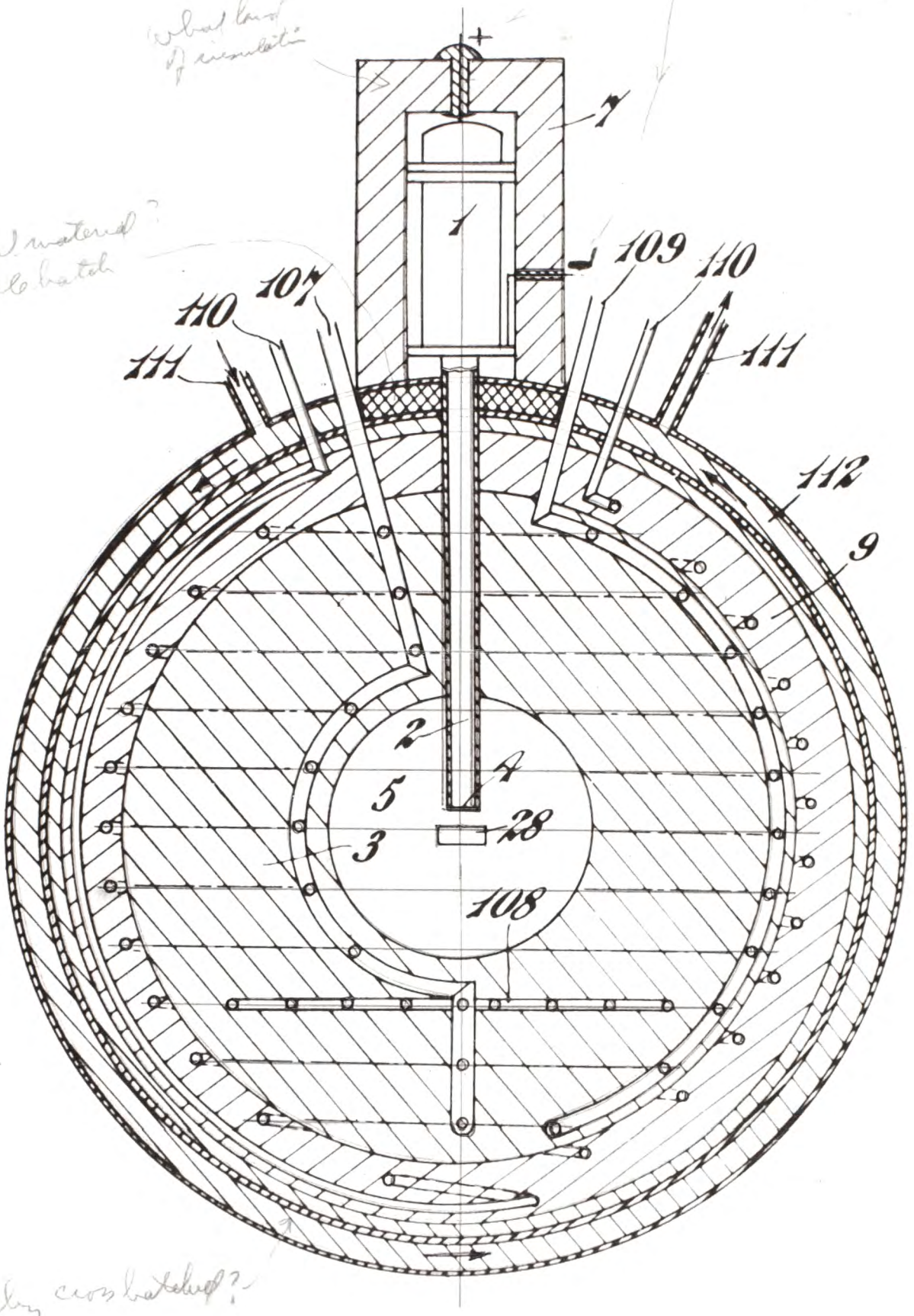

Fig 2

Leo Szilard

Print of drawing as originally filed.

Fig 3

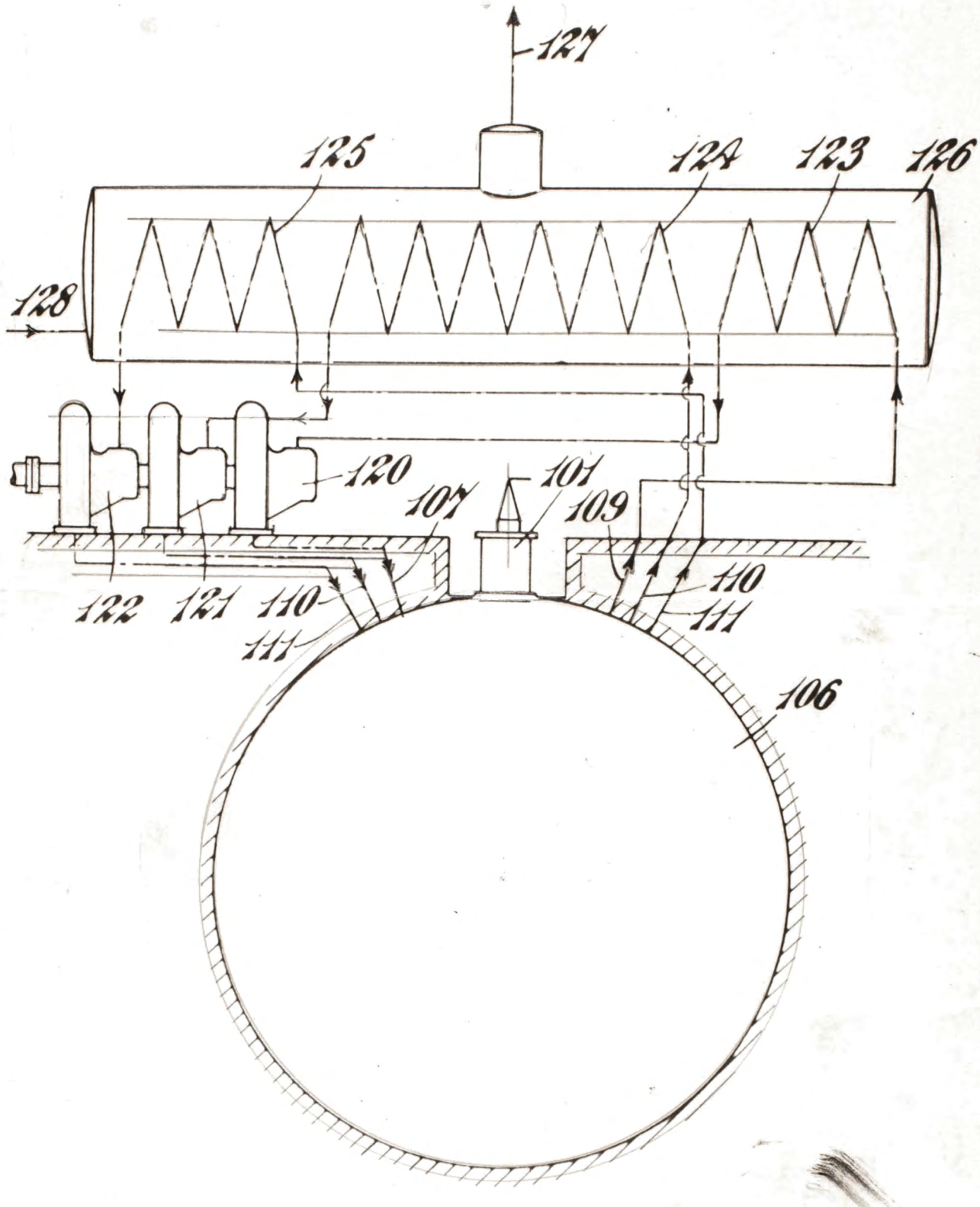

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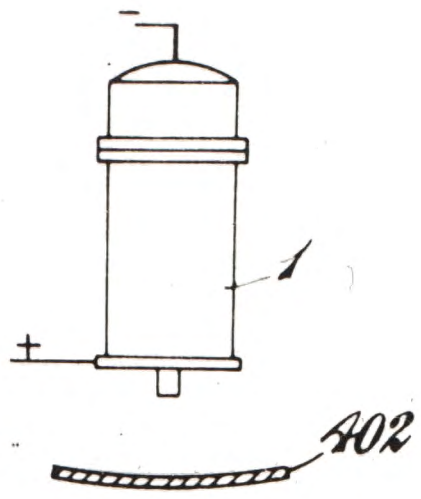
Fig 8B



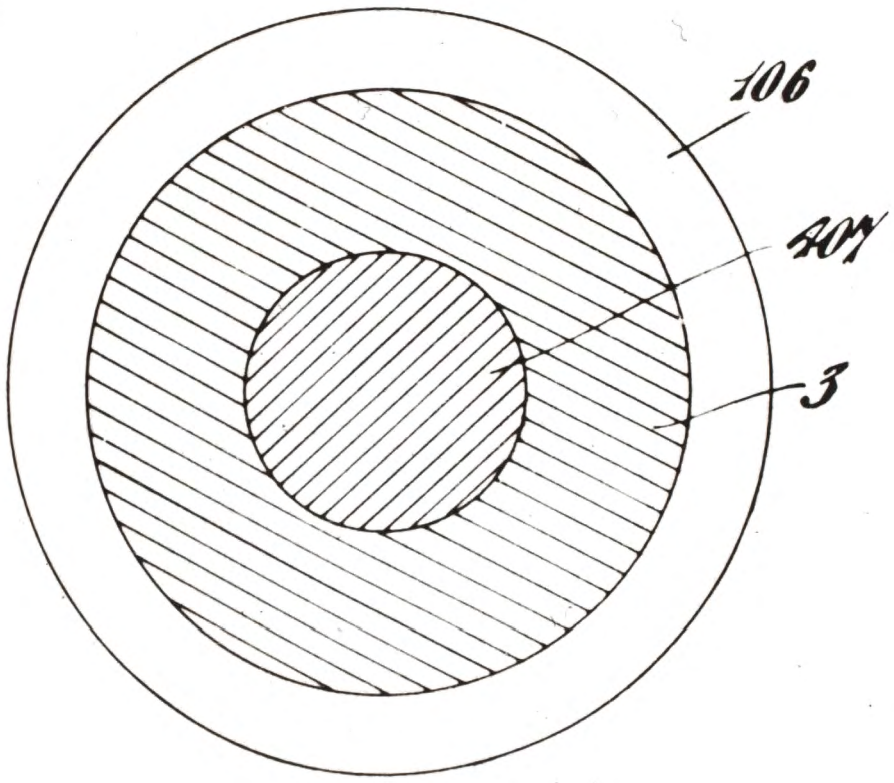


Fig 9