May 25, 1943

VII

SLOWING AGENTS

If we have a lattice of aggregates which contain in sufficient concentration uranium that is sufficiently enriched in U^{235} or element 94^{239} or U^{233} embedded in a slowing agent we may have a chain reacting system for a number of slowing agents. Water can, for instance, be used as a slowing agent in such systems. However, if we have a system in which natural uranium is used or at least if natural uranium is used to start with, the slowing agent has to be selected from a very narrow choice of slowing agents which we have called "efficient" slowing agents. In the following, we characterize this class of slowing agents by their various properties:

First, in order to have a practical system the efficient slowing agent must be of sufficient density and, therefore, must be either liquid or solid at operating temperature and pressure, and preferably also at room temperature and atmospheric pressure.

Secondly, it is necessary that the efficient slowing agent should show a favorable balance between neutron absorption and the slowing down ability of the slowing agent. In this respect it must be about as favorable as or more favorable than impure graphite which has an average capture cross section per carbon atom of $\overline{\sigma_c}$ (C) 0.01×10^{-24} cm^2. The absorption cross section of pure graphite is considerably smaller, about 0.005×10^{-24} cm^2; but if impurities are present in the graphite which raise the average absorption cross section into the neighborhood of 0.01, we can still have a chain reaction in a system of otherwise favorable composition and structure. This is no longer

possible if we allow the cross section of the slowing agent of the carbon atom to rise above approximately 0.01×10^{-24} cm^2.

Carbon and other light elements can be characterized from the point of view of their suitability as slowing down agents in a chain reaction by a dimensionless constant, N, which we may call the characteristic number of the slowing agent. This number, N, is defined by the following formula:

$$N = \frac{\sigma^*_{sc}}{\sigma_c} \ln\left(1 + \frac{2m}{1+m^2}\right)$$

where m is the mass number of the element, σ^*_{sc} is the scattering cross section of the element for neutrons which are above the thermal region and have energies between a few volts and a few hundred volts, and σ_c is the capture cross section of the element in the thermal region. For pure carbon [σ_c (C) $\sim 5 \times 10^{-27}$ cm^2 and m equal 12] the characteristic number N(C) is about N(C)$\sim$160.

As we have stated before, impure carbon for which the absorption cross section σ_c is about twice that of pure carbon is close to the limit at which a chain reaction is no longer possible in graphite. Such impure graphite would have a characteristic number of $N = \frac{1}{2} N(C) = 80$. Accordingly, we may define the class of slowing agents which are capable of sustaining a chain reaction in natural uranium by the requirement that the characteristic number, N, must necessarily be larger than 80 and should preferably be appreciably higher. A value of N = 120 would be already quite satisfactory. This requirement is fulfilled for carbon in the form of pure graphite and for heavy hydrogen in the form of deuterium oxide. In order to compute the approximate value for the characteristic number, N, of deuterium oxide, we have therefore,

$$m = 2; \sigma_c = \sigma_c(D) + \tfrac{1}{2}\sigma_c(O); \sigma^*_{sc} \sim \sigma^*_{sc}(D).$$

The criterion of the characteristic number must, however, not be applied to elements which apart from scattering fast neutrons undergo also other reactions with fast neutrons such as n-2n, n-p, or n-α reactings.

Apart from N, the characteristic number of the slowing agent, another property of the slowing agent has a bearing on the question whether the slowing agent is an efficient slowing agent and suitable for a system in which a lattice of uranium-containing aggregates is used. For a slowing agent, it is of advantage in this connection that the range, A, of the thermal neutrons should be large in comparison with range, B, of the low energy resonance neutrons.

A, the range of the thermal neutrons is defined by the value

$$A = \lambda \sqrt{\frac{\sigma_{sc}}{3\,\sigma_c}} \tag{3}$$

where λ is the mean free-path of thermal neutrons in the slowing agent, and σ_{sc} and σ_c are the scattering cross section of the capture cross section for thermal neutrons of the molecule which acts as the slowing agent.

B, the range of the low energy resonance neutrons is defined by

$$B = \lambda^* \sqrt{\frac{k}{3}} \tag{4}$$

where λ^* is the mean free path for scattering of neutrons having the energy between few volts and a few hundred volts in the slowing agent, and k is defined as follows:

$$k = \frac{\ln 1/10}{\ln \left(1 - \frac{2m}{(1+m)^2}\right)} \tag{5}$$

If, instead of an element like carbon, a compound of a light element and a much heavier element is used as

slowing agent, the value given for k has to be multiplied approximately by the fraction of the scattering cross section of the molecule of the compound which is due to the light element.

For D_2O, for instance, one has to multiply k by approximately

$$\frac{2\ \sigma_{sc}(D)}{2\ \sigma_{sc}(D) + \sigma_{sc}(O)}$$

Accordingly, the requirements that the range A should be large compared to the range B amounts to

$$(6) \qquad A/B = \frac{\lambda}{\lambda^*} \sqrt{\frac{\sigma_{sc}}{\sigma_c k}} \gg 1$$

or writing $\frac{\lambda}{\lambda^*} = \left(\frac{m}{m+1}\right)^2$

$$(7) \qquad A/B \left(\frac{m}{m+1}\right)^2 \sqrt{\frac{\sigma_{sc}}{\sigma_c k}} \gg 1$$

This condition is fulfilled for graphite, heavy water, and beryllium.

For instance, for carbon we have about

$$k \sim 15$$

$$\frac{\sigma_{sc}}{\sigma_c} \sim 1000$$

$$\left(\frac{m}{m+1}\right)^2 = \frac{1}{1.18}$$

$$A/B \sim 7$$

which is a satisfactory ratio.

FUNCTIONAL DESCRIPTION

Of the thermal neutrons which are absorbed in the uranium contained in the lattice-element a fraction will cause fission and will give rise to the emission of fast fission neutrons. Some of these fast fission neutrons will cause fission (in the same lattice element from which they originate) before they are slowed down by collisions with uranium or carbon below the fission threshold of the abundant isotope U^{238}. In this manner, for every thermal neutron absorbed a certain number, δ, of fast neutrons are generated which are slowed down partly by inelastic collisions in uranium, but mostly by elastic collisions in carbon and a fraction $(1 - p)$ of these eventually reaches thermal energies. Another fraction p is absorbed at resonance by uranium before reaching thermal energies.

The neutrons which are absorbed at resonance by uranium are removed from the chain reaction without leading to fission and the generation of neutrons. Some of these neutrons are absorbed at comparatively high energies, between a few hundred volts and some 10,000 volts, while others are absorbed at comparatively low energies, between a few volts and a few hundred volts. The lattice elements are moderately transparent for the high energy resonance neutrons but are practically black for at least part of the low energy resonance neutrons, i.e., a certain fraction of the resonance neutrons is absorbed in a thin surface layer of the lattice element whereas another fraction penetrates. Accordingly, the resonance absorption of the lattice element may be divided into two terms, one of which may be called surface absorption and one of which may be called mass absorption. If the dimensions of the lattice element are small, the fraction of the neutrons

which is removed from the chain reaction by mass absorption is essentially determined by the ratio of uranium to carbon and is independent of the shape and size of the lattice elements. If it were only for this type of resonance absorption it would be immaterial how small we make the lattice elements and we could make them very small indeed and still have a potentially chain reaction system.

In reality a graphite system in which the dimensions of the lattice element are made very small are not potentially chain reacting since too large a fraction of the neutrons would be removed from the chain reaction by the surface resonance absorption. Clearly if for a given ratio of uranium to carbon we decrease the dimensions of the lattice elements we increase the total surface and thereby the fraction of the neutrons which are removed by absorption at resonance.

In most practical cases a lattice element can be fairly well represented by replacing it with an ellipsoid and we may then express the above-mentioned point of view by saying that the smallest of the three axes of that ellipsoid must not be made too short compared with the range of thermal neutrons in the lattice element. This range "U" is proportionate to the density of the U in the lattice element and increases with the temperature of the neutrons somewhat faster than the fourth root of the temperature. For room temperature and uranium metal of density 18 the range is about 1.4 cm.

VII Replacement

May 25, 1943

SLOWING AGENT 5

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REPLACEMENT May 26, 1943

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