In the following, we shall consider the balance of neutron emission and absorption in a system composed of Uranium and a light element, the latter serving the purpose of slowing down the neutrons. The question in which we are interested is whether it is possible under certain conditions to maintain a neuclear chain reaction in such a system. We shall speak of a chain reaction only in connection with a process in which more than one neutron is produced for every neutron absorbed within the system. If less than one neutron is produced it will obviously still be true that a neutron emission in the fission of Uranium will cause further fission and thereby lead to secondary neutrons which in turn will again cause further fission leading to tertiary neutrons but this process will necessarily converge even for infinitely large masses of uranium and will not be referred to by us as a chain reaction.

In a system which is composed of uranium and hydrogen the neutrons emitted from uranium are fast and are slowed down by the hydrogen until they finally reach thermal energies. A fraction of the neutrons will be absorbed before reaching thermal energies, while the neutrons pass through an energy region ΔE, at which the uranium has a resonance absorption. Those neutrons which reach thermal energies are then either absorbed by uranium or captured by hydrogen respectively. If q is the fraction of the fast neutrons emitted by uranium which are absorbed in the thermal region by the uranium alone, and if μ fast neutrons are emitted by uranium for one thermal neutron absorbed by uranium, then the condition for the possibility of a chain reaction is (0) $\mu q > 1$

If μq is only slightly larger than 1, then it may take a very large mass of uranium to approach a divergent chain reaction.

JOLIOT

It should be possible to obtain the value of μ with great accuracy by the following method: let us have a sphere containing a homogeneous mixture of uranium and water having n atoms of Hydrogen per atom of uranium. This sphere of radius l is immersed in a large water tank and a photo neutron source is placed in the center of the sphere. The density ρ of the thermal neutrons is measured along a radius and the integrals of

$$I_{int} = \int_0^l r^2 \rho \, dr \qquad I_{ext} = \int_l^\infty r^2 \rho \, dr$$

are determined giving a measure of the number of slow neutrons which are present within the mixture inside the sphere and in the water outside the sphere. In another experiment the integral

$$I_o = \int_0^\infty r^2 \rho \, dr$$

inspite of the heat flux

is determined for the same neutron source in pure water.

We will now show that it is possible to derive from the value of μ these three integrals the number of neutrons emitted by Uranium per thermal neutron absorbed by Uranium. ~~And it is noteworthy that in order to do this it is not necessary to determine what~~ a large fraction of the fast neutrons emitted by Uranium inside the sphere may escape from the interior of the sphere across its boundary, ~~and which~~ we may leave its value undetermined.

Q_{int} the number of neutrons which reach the thermal region inside the sphere in the mixture per neutron emitted by the photo-neutron source in the center of the sphere is given by

$$Q_{int} = \frac{I_{int}}{I_o} \Big/ \frac{\tau_{int}}{\tau_o}$$

Where

$$\frac{\tau_{int}}{\tau_o} = \frac{n \sigma_s(H)}{\sigma_a(a) + n \sigma_s(H)}$$

~~The radius~~ is the ratio of the life-time of the neutrons in pure water and in the mixture inside the sphere and

Of the neutrons which are slowed down to the resonance region inside the sphere a fraction p is absorbed at resonance and a fraction 1-p reaches the thermal region. Q^*_{int} the number of the neutrons reaching the resonance region per second inside the sphere is therefore given by

$$Q^*_{int} = (1-p) Q_{int}$$

JOLIOT

Q^*_{ext} the number of neutrons reaching the resonance region outside the sphere in the water is the same as Q_{ext} the number of neutrons reaching the thermal region outside the sphere in the water and we have

$$Q^*_{ext} = Q_{ext} = \frac{I_{ext}}{I_0}$$

P, the total number of fast neutrons emitted by the Uranium per neutron emitted by the photo source is equal to the total number of the resonance neutrons produced per second minus the total number of resonance neutrons produced by the photo source alone which is of course 1 per photo-neutron. Therefore, we have

$$P = Q^*_{int} + Q^*_{ext} - 1 = \frac{Q_{int}}{1-p} + Q_{ext} - 1$$

The number of fast neutrons produced by Uranium per thermal neutron produced within the sphere is given by

$$\frac{P}{Q_{int}} = \frac{1}{1-p} + \frac{Q_{ext} - 1}{Q_{int}}$$

And finally since the Uranium absorbs of the neutrons which reach the thermal region within the sphere the fraction

$$\frac{\sigma_a(U)}{\sigma_a(U) + n\sigma_c(H)}$$

We have

$$\mu = \frac{P/Q_{int}}{\left(\frac{n\sigma_a(U)}{\sigma_a(U) + n\sigma_c(H)}\right)}$$

~~Therefore~~
~~For this we can also write~~

$$\mu = \frac{\sigma_a(U) + n\sigma_c H}{\sigma_a(U)} \left\{ \frac{1}{1-p} + \frac{\tau_{int}}{\tau_0} \frac{I_{ext}/I_0 - 1}{I_{int}/I_0} \right\}$$

~~Since we have~~
On putting in the value of τ_{int}/τ_0

We obtain

$$(1) \quad \mu = \frac{1}{1-p} + \frac{n\sigma_c(H)}{\sigma_a(U)} \left\{ \frac{1}{1-p} - \frac{1}{s} \frac{1 - I_{ext}/I_0}{I_{int}/I_0} \right\}$$

This expresion holds strictly if the radius of the sphere is sufficiently large to permit to neglect transition phenomena in the equilibrium of resonance neutrons and thermal neutrons near the surface of the sphere.

The values of $I_{int}/I_0 = 0.72$; $I_{ext}/I_0 = 0.45$ for $n = 3$ $\quad s = 0.42$ gm/cc can be taken from an experiment reported by Halban, Joliot, Kowarski, and Perrin. These authors used a spherical symmetrical experimental arrangement as described above and they determined the value of these 3 integrals by using dysprosium as an indicator of the thermal neutron density. They concluded from their observations that 8 neutrons are generated in their particular arrangement per ~~every~~ primary photo neutron which causes a fission process in Uranium. They emphasized that a considerable number of secondary and tertiary neutrons are created and that the process of the neutron liberation converges in their experiment.

In order to find p for $n=3$ we have to extrapolate the values of p which have been measured only for values of $n \geq 30$. A set of values for such low Uranium concentrations has been obtained by

who reported for instance $p_1 = 0.20 \pm 0.02$ for $n_1 = 30$

In order to find from these values the value of p for $n=3$ one may make use of the relationship

$$\frac{\log(1-p_1)}{\log(1-p)} = \frac{\sqrt{n}}{\sqrt{n_1}} \qquad (2)$$

which should hold for a single line with heavy absorption

~~This relationship is one obtained on theoretical grounds in a manner which is generally known.~~ By applying it to the values of p, which have been measured for $n=30$ one obtains for p_2 for $n=3$ one finds $p = 0.5$

This is also the value attributed by Halban, Joliot, Kowarski and Perrin to p for the concentration $n=3$ and I assume for the same reason.

By attributing the value of $\frac{1}{4}$ or $\frac{1}{9}$ to $\frac{n\sigma_c(H)}{\sigma_a(U)}$ we obtain $\mu = 2.05$ or $\mu = 2.02$ respectively. We see that μ is very insensitive to uncertainties in the value of these absorbing cross-sections and this should make it possible to

obtain a very accurate value for μ merely by measuring very accurately the value of p. This is now being done by a new method which has been devised for the purpose.

HOMOGENEOUS MIXTURES

For a value of p=.5 and $\mu = 2$. it appears very doubtful if there is any homogeneous mixture of water and Uranium at which $\mu\nu$ will exceed 1. But even if there should be a concentration for which this product slightly exceeds unity it would then take a very large part of Uranium to have an arrangement in which a chain reaction can approach the the divergent point.

As we will discuss later in great detail it appears that in order to make a chain reaction possible Carbon is a much better element to use for slowing down the neutrons than Hydrogen. However, it ~~appears doubtful~~ is uncertain whether a chain reaction can be maintained in homogeneous mixtures of Carbon and Uranium.

In Carbon it takes about 6 collisions to reduce the energy of a neutron by a factor $1/e$ therefore, a neutron which while being slowed down has entered the resonance region, will stay wwithin this region for a comparatively long time. It has, therefore, a large probability to be captured by Uranium unless very low Uranium concentrations are used. Though the captured cross-section of Carbon $\sigma_c(C)$ is small < 0.01 at such very low Uranium concentrations as would have to be used the capture of thermal neutrons in the Carbon might prevent a chain reaction. Since only the above stated upper limit for the Carbon cross-section ~~is known this point has to be left undecided~~ The point has to be left undecided which was determined by Frisch Halban & Koch

~~A new method for measuring very small captured cross-sections has recently been devised and is now being applied to Graphite in the hope of obtaining a better value for~~

SINGLE URANIUM SPHERES

In the present paper we shall show that if non-homogeneous mixtures of Carbon and Uranium are used, Carbon is certainly much more favorable than Hydrogen, and that in the case of Carbon itis possible to make use of certain tricks which greatly reduce the fraction of the neutrons captured by Uranium at resonance. One reason for this difference in favor of Carbon is the fact that the scattering cross-section of Carbon is only slightly higher for thermal neutrons than for resonance neutrons whereas the scattering cross-section of Hydrogen varies by a large factor.

Uranium has a strong resonance absorption line at energies of the order of magnitude of 10 volts. If we have to deal with an absorption line having a maximum at some energies $E = E_0$ in this energy region and if this line obeys the B-W formula then the following will hold: at thermal energies the absorption will follow the 1/v law, which will then go through a minimum at $E_1 = 0.2\ E_0$ after which the absorption rises until it reaches its maximum at E_0 and falls off again reaching a very small value for $E_2 = 2E_0$

We shall call the energy interval $\Delta E = E_2 - E_1$ the resonance region and for a line obeying the B-W formula we have

(3) $$\frac{E_2}{E_1} = 10$$

In the following we shall refer to the energy region below 0.2 E as the thermal region. We shall deal with bodies composed of Uranium embedded in Carbon and shall assume that every neutron which reaches the surface of a Uranium body while it is in the resonance region as defined above is absorbed by the Uranium by radiative capture without causing neutron emission.

Let us first consider a plane sheet of Uranium embedded inaan infinite space filled with carbon, and assume that the Uranium is "black" not only for resonance neutrons but also for thermal neutrons. i.e. every thermal neutron and every neutron which has an energy within the energy interval ΔE is absorbed by Uranium if it reaches the Uranium layer at all. We are interested in finding ε_0 the ratio of the number of thermal neutrons and the number of resonance neutrons which are absorbedb by the Uranium and then try to find other arrangements for which this ratio is considerably higher.

The diffusion of thermal neutrons and resonance neutrons towards the Uranium layer can both be treated with good approximation in exactly the same manner if the following is born in mind:

A thermal neutron produced in the carbon will disappear in pure carbon

after a certain number of elastic collisions with carbon atoms by being captured. The probability that it survives h elastic collisions is given by $1 - e^{-h/\frac{\sigma_{sc}(C)}{\sigma_c(C)}}$

the expression

(4) $$A = \lambda_{sc}(C)\sqrt{\frac{\sigma_{sc}(C)}{3\sigma_c(C)}}$$

has the dimension of a length, and we shall refer to it for the sake of brevity as the "range" of thermal neutrons in carbon. For graphite of density 1.6 to 1.7 and $\sigma_{sc}(C)/\sigma_c(C) > 500$ we have $A > 31$ cm

Similarly, faster neutrons in pure carbon disappear out of a given energy interval $\Delta E = E_2 - E_1$, because they are slowed down by elastic collisions with carbon atoms. The probability that a neutron survives k collisions within the energy region ΔE is approximately given by

$$1 - e^{-k/k_0}$$

Where

(5) $$k_0 = \frac{\ln \frac{E_2}{E_1}}{\ln\left(1 - \frac{4M}{(1+M)^2}\right)} \cong 6.5 \ln \frac{E_2}{E} \qquad M = 12$$

Again the expression

(6) $$B = \lambda^*_{sc}(C)\sqrt{\frac{k_0}{3}}$$ For graphite of density 1.6 to 1.7 and

which has the dimension of a length will be called the "range" of resonance neutrons in carbon. with $\frac{\lambda^*_{sc}(C)}{\lambda_{sc}(C)} =$ and $\frac{E_2}{E_1} = 10$ we have $B \cong 5.9$ cm

By treating the problem as a diffusion phenomenon we find in the case of a plane sheet of Uranium embedded in an infinite amount of carbon

(7) $$\varepsilon_0 = \frac{A}{B} = \frac{\lambda_{sc}(C)}{\lambda^*_{sc}(C)}\sqrt{\frac{\sigma_{sc}(C)/\sigma_c(C)}{k_0}}$$

This value holds under the assumption that everywhere the same number Q of resonance neutrons and thermal neutrons is produced per c.c. and second and will be corrected later to take into account deviations from this assumption.

Let us now compare this value of ε_0 with the case of a Uranium sphere of radius R embedded in an infinite amount of carbon, which we wish to consider in greater detail. Again we assume that the same number of thermal neutrons Q is produced everywhere in the carbon. In reality this is not so, because, owing to the absorption of resonance neutrons by the sphere a smaller number of thermal neutrons is produced in the neighborhood of the sphere.

A correction factor will be calculated later and applied to the final result.

If R is large compared to $\lambda_{sc}(C)$ the mean free path for scattering of thermal neutrons in carbon, the density of the thermal neutrons in the carbon can be calculated as a function of distance r from th center of the sphere by treating the problem as a diffusion phenomenon. We thus find for $\rho(r)$

(8) the equation $D(C)\frac{d(r\rho)}{dr^2} - S(C)r\rho + Q(r)r = 0$

$D(C) = \frac{w\lambda_{sc}(C)}{3}$; $S(C) = \frac{w}{\lambda_{sc}(C)}\frac{\sigma_i(C)}{\sigma_{sc}(C)}$; assuming $\frac{dQ}{dr} = 0$ (8a)

we find for a black sphere for which $\rho(R) = 0$

(9) $$\rho(r) = \frac{Q}{S(C)}\left(1 - \frac{R}{r}e^{\frac{r-R}{A}}\right)$$

$$A = \sqrt{\frac{D}{S}} = \lambda_{sc}(C)\sqrt{\frac{\sigma_{sc}(C)}{3\sigma_c(C)}}$$

Accordingly the number of thermal neutrons which reaches the uranium sphere per second by diffusion

(10) $$J_0^{th} = D(C)\,4\pi R^2\rho'(R) = 4\pi Q R A^2\{1 + R/B\}$$

Quite similarly, the number of resonance neutrons which reaches the "black" uranium sphere is given by

(11) $$J_0^{res} = 4\pi Q R B^2\{1 + R/B\}$$

The ratio ε_0 is given by

(12) $$\varepsilon_0 = \frac{J_0^{th}}{J_0^{res}} = \frac{A^2}{B^2}\frac{1 + R/A}{1 + R/B}$$

By comparing this expression with the expression obtained in the case of plane uranium sheet, we see that for small values of R the case of the sphere is more favorable by a factor of A/B . This factor might have a value of as much as , though in practice we can hardly have R very much smaller than B and therefore a factor of perhaps two ~~or three~~ is lost, still the case of the sphere remains very much more favorable than the plane sheet.

In order to calculate ε for conditions which can be ~~realized~~ obtained in practice, we have now to consider the case of a uranium sphere which is not "black" for thermal neutrons, but which can be considred "black" for resonance neutrons.

The number of thermal neutrons absorbed by a such a sphere will be smaller than γ_0^{th}. Only a fraction of the thermal neutrons which reach the sphere will be ultimately absorbed by it, and the ratio of the number of the thermal neutrons absorbed by a black resonance neutrons absorbed by the sphere is therefore we write $\gamma^{th} = \varphi \gamma_0^{th}$ and have than for ε

(13) $$\varepsilon = \frac{\gamma^{th}}{\gamma^{res}} = \varphi \varepsilon_0$$

(14)

In order to calculate φ we take into account that inside the uranium sphere the thermal neutron density ρ obeys the equation

(15) $$D(u) \frac{d^2}{dr}(r\rho) - S(u) r\rho = 0$$

$$D(u) = \frac{w \lambda_{sc}(u)}{3} \; ; \; S(u) = w N(u) \sigma_a(u)$$

having as its solution

(16) $$\rho(r) = \frac{C}{r}\left(e^{r/u} - e^{-r/u}\right)$$

where $u = \sqrt{\frac{D(u)}{S(u)}}$ is the "range" of thermal neutrons in the substance of the uranium sphere

From equations and we find that the number of thermal neutrons diffusing per second into the sphere

(17) $$\gamma^{th} = 4\pi Q A^2 (1 + R/A) \frac{\frac{\lambda_{sc}(u)}{u}\left\{\frac{e^{R/u} - e^{-R/u}}{e^{R/u} - e^{-R/u}} - \frac{u}{R}\right\}}{\frac{\lambda_{sc}(u)}{R}(1 + R/A) + \frac{\lambda_{sc}(u)}{u}\left\{\frac{e^{R/u} - e^{-R/u}}{e^{R/u} - e^{-R/u}} - \frac{u}{R}\right\}}$$

Where U is the range of thermal neutrons in the substance of the uranium sphere

And for pure uranium we have

(18) $$u = \sqrt{\frac{D}{S}} = \lambda_{sc}(u) \sqrt{\frac{\sigma_{sc}(u)}{3\sigma_a(u)}}$$

The number of resonance neutrons γ^{res} diffusing into the sphere per second is the same as before

(19) $$\gamma^{res} = \gamma_0^{res}$$

since
~~Therefore~~ we have

$$\varphi = \frac{\zeta^{U}}{\gamma_0^{U}}$$

equation No. 1 No. gives
~~and accordingly~~

$$(21)\quad \varphi = \frac{\frac{\lambda_{sc}(U)}{U}\left\{\frac{e^{R/U}+e^{-R/U}}{e^{R/U}-e^{-R/U}} - \frac{U}{R}\right\}}{\frac{\lambda_{sc}(C)}{R}(1+R/A) + \frac{\lambda_{sc}(U)}{U}\left\{\frac{e^{R/U}+e^{-R/U}}{e^{R/U}-e^{-R/U}} - \frac{U}{R}\right\}}$$

For uranium metal spheres of $R = 5$ cm we have at room temperature with $\frac{\sigma_{sc}(U)}{\sigma_a U} \cong 2$ $\varphi = 0.54$

All expressions for ε were so far obtained from a diffusion equation assuming $R >> \lambda_{sc}(C)$; $R >> \lambda_{sc}(U)$ $U >> \lambda_{sc}(U)$

The expression for φ holds both for Uranium metal and compounds like uranium oxide only the range U is no longer given by the expression No. if a compound of uranium is used.

For small values of R the problem is no longer a trueddiffusion problem and as a matter of fact, we shall refrain from using the expression for φ for values of R of less than five centimeters. We

~~We wish to state, however, that~~ will if we have uranium spheres which are "black" for thermal neutrons and for which $\lambda_{sc}(C) >> R$

we find again as the correct value

$$(22)\quad \varepsilon = \frac{A^2}{B^2}$$ in hydrocarbons in graphite merely

However, uranium do a small sphere not become "black" for thermal neutrons ~~even for infinite~~ by increasing its density. This is due to the scattering of thermal neutrons by uranium in the uranium lumps and ~~the limiting value for seems therefore to be about~~ ε will hardly exceed the value of $1/2\,\frac{A^2}{B^2}$

LATTICE OF URANIUM SPHERES

If we have now a lattice of a large number of uranium spheres embedded in an infinite mass of carbon and want to calculate the ratio of the number of thermal neutrons and resonance neutrons absorbed by the uranium spheres we shall again assume for the time being that everywhere in the carbon the same number Q of neutrons enter the resonance region and the thermal region per c.c. a second.

Equation number 9 shows that a uranium sphere does not affect the thermal neutrons' density appreciably at distances which are large compared to R. At a distance of R from the center ρ is equal to half of $\frac{1}{2}\rho_\infty$ the value of the thermal neutron density at infinity. For this reason the number of thermal neutrons J^{th} absorbed by one uranium sphere within the lattice is only insofar affected by the other uranium spheres as the presence of these other spheres reduce the average value $\bar{\rho}$ of the thermal neutron density by some factor α which is smaller than 1. $\bar{\rho} = \alpha \rho_\infty$ Accordingly, the number of thermal neutrons absorbed J^{th} by one sphere within the lattice J^{th} is by the factor α smaller than J^{th} the number of thermal neutrons which an isolated sphere absorbs under otherwise identical circumstances.

(23)

$$J^{th} = \alpha J^{th}_{\infty} \qquad (24)$$

The number of resonance neutrons on the other hand which one uranium sphere absorbs within the lattice J^{res} is the same as the number of resonance neutrons which a single uranium sphere would absorb. We have, therefore

$$J^{res} = J^{res}_{\infty} \qquad (25)$$

The fraction of the neutrons produced which is absorbed by the carbon is proportionate to the average neutron density $\bar{\rho}$. Since in the absence of uranium ~~xxxxxxxxxxxxxxxxxx~~ in the carbon we have $\bar{\rho} = \rho_\infty$ and since in the absence of uranium <u>all</u> the neutrons are absorbed by the carbon the fraction of the neutrons which are absorbed by the carbon which contains a lattice of uranium spheres is given by α

Correspondingly the fraction of the neutrons which are absorbed by the lattice of uranium spheres by either as resonance neutrons or as thermal neutrons is given by $(1-\alpha)$

From this it follows that q the fraction of all the neutrons which are absorbed by the uranium spheres in the thermal region alone is given by

(26) $$q = \frac{J^{th}}{J^{th} + J^{res}}(1-\alpha)$$

or

(27) $$q = \frac{\varepsilon\alpha}{1+\varepsilon\alpha}(1-\alpha) = \frac{q\varepsilon_0\alpha}{1+q\varepsilon_0\alpha}(1-\alpha)$$

Now this expression has its maximum value for $\alpha = \alpha_m$

(28) $$\alpha_m = \frac{-1+\sqrt{1+\varepsilon}}{\varepsilon}$$

and for the maximum value of q we have $q = q_m$

(29) $$q_m = 1-2\alpha_m$$

(30) $$q_m = 1-2\,\frac{-1+\sqrt{1+\varepsilon}}{\varepsilon}$$

and

(31) $$\varepsilon = \frac{4q_m}{(1-q_m)^2}$$

CORRECTED VALUE

In calculating the value of q we have so far neglected the effect of the absorption of resonance neutrons by the uranium spheres embedded in the lattice on the production on thermal neutrons in the carbon in the neighborhood of the uranium spheres. The absorption of resonance neutrons reduces in reality the value of Q near the spheres below the average value of Q and accordingly the correct number of thermal neutrons absorbed by the uranium spheres per sphere will be smaller than J^{th}. In order to find the value for the difference $(J^{th}(corr) - J^{th})$ we may proceed in the following way:

If the uranium spheres in the lattice, an act of God, stopped absorbing resonance neutrons then a neutron which reaches a given sphere at least once while its energy is in the resonance region would have some probability Y of reaching the sphere at least once after it had been slowed down to the thermal region. On the other hand the probability that the sphere which is

in "black" for thermal neutrons absorbs a thermal neutron which reaches its sphere at least once is given by φ. (This can be readily seen by considering the following: the number of thermal neutrons which reach the non-"black" sphere at least once is the same as J^{th} the number of thermal neutrons absorbed by a "black" sphere, and the number of thermal neutrons absorbed by the non-"black" sphere is by definition φJ_o^{th}). It follows that we have

(32) $$J^{th}_{(korr)} = J^{th} + \nu\varphi J^{res} \qquad 0 < \nu < 1$$

Correspondingly we have

(33) $$q_{(korr.)} = \frac{J^{th} - \nu\varphi J^{res}}{J^{res} + J^{th} - \nu\varphi J^{res}} (1-\alpha)$$

or

(34) $$q_{(korr)} = \frac{\varepsilon\alpha - \nu\varphi}{1 + \varepsilon\alpha - \nu\varphi} (1-\alpha)$$

By introducing the value of q_m from equation No. 31 we find

(35) $$q_{(korr)} = q_m \frac{1 - \frac{\nu\varphi(1-q_m)}{2q_m}}{1 - \frac{\nu\varphi(1-q_m)}{1+q_m}}$$

and if we neglect terms which contain powers higher than the second of

$$\frac{\nu\varphi(1-q_m)}{1+q_m}$$

we obtain finally

(36) $$q_{(korr)} = q_m \left\{ 1 - \frac{\nu\varphi(1-q_m)^2}{2q_m(1+q_m)} \left(1 + \nu\varphi \frac{1-q_m}{1+q_m} \right) \right\}$$

Even for $\nu = 1$ the correction would still remain below 10% for a value of $q_m = 1/2$ and for higher values of q_m the correction is even smaller.

SPACING OF THE LATTICE

It is necessary to estimate how widely the uranium spheres are spaced in the lattice in order to see whether the assumptions under which we derive equations No. , and 26 are fulfilled. In deriving those equations we have assumed that everywhere in the carbon the same number of neutrons Q are slowed down to enter the resonance region. This assumption would not hold if the distance of the uranium spheres in the lattice became large compared with the distance to which a fast neutron diffuses away from its point of origin before it is slowed down to a few volts. Our formulae imply that this would happen for extremely small values of $\sigma_c(C)$. Those values of $\sigma_c(C)$ are so small that we can leave them for the present out of consideration. (However, the values of for which this would happen is much smaller than the value of which is required to make).

The value for q_m given by equation No. 31 can therefore be used for studying the prospects of a chain reaction.

In order to estimate the spacing of the uranium spheres in the lattice we estimate the volume V of carbon per one uranium sphere in the lattice.

In the lattice of uranium spheres from the QV neutrons which are slowed down per second within the volume V to resonance energies, the carbon absorbs αQV neutrons and the uranium sphere in the lattice absorbs $(1-\alpha)QV$ neutrons. Accordingly we have

$$J^{th} + J^{res} = (1-\alpha)QV \tag{37}$$

On the other hand we can write

$$J^{th} + J^{res} = \alpha\varphi J_0 + J^{res} = \alpha\varphi J_0 \frac{\varphi\varepsilon_0\alpha + 1}{\varphi\varepsilon_0\alpha} =$$

$$= \varphi J_0 \frac{\alpha}{q}(1-\alpha)$$

Equation No. and give

$$\varphi J_0 \frac{\alpha}{q} = QV$$

and putting in from Equation No. the value of J_0

$$V = 4\pi\varphi\frac{\alpha}{q}A^2R\left(1+\frac{R}{A}\right) \tag{38}$$

if q has its maximum value q_m

$$\frac{\alpha}{q} = \frac{1-q_m}{2q_m}$$

SPACING OF THE LATTICE

AND IT IS

$$v = 4\pi \rho \frac{1-q_m}{2q_m} A^2 R \left(1 + R/A\right) \tag{39}$$

For large values of ε we can write

$$\frac{1-q_m}{2q_m} \cong \frac{1}{\sqrt{\varepsilon}} = \frac{1}{\sqrt{q\varepsilon_0}} \tag{40}$$

so that we have (rewritten) for $\varepsilon >> 1$

$$v \cong 4\pi \sqrt{(1+R/A)(1+R/B)}\, ABR \tag{41}$$

For the ratio of the volumes of carbon and uranium we have from Eqn 39

$$\frac{v}{\frac{4\pi}{3}R^3} = 3q \frac{(1-q_m)}{2q_m} \frac{A^2}{R^2} \left(1 + R/A\right) \tag{42}$$

and for the mean distance between neighboring uranium spheres very roughly

$$L \cong 2\sqrt[3]{v} \tag{43}$$

STABILIZING THE CHAIN REACTION

If a chain reaction could be maintained in a homogeneous mixture of water and uranium or carbon and uranium it would have a certain natural stability in the sense that with rising temperature there would be a decrease in the neutron production. The reason for this is the fact that the absorption of both uranium and hydrogen obey 1/v law in the thermal region and thus the range of thermal neutrons in the mixture is larger at higher temperatures. Accordingly, at higher temperatures a larger fraction of the thermal neutrons will escape across the boundary of the mixture without having reacted with the uranium in the mixture. X (in great detail like the one which we have discussed)

A system on the other hand in which "black" uranium bodies are embedded in carbon has a natural thermal instability. This is due to the fact that with rising temperature the captured cross-section of the carbon decreases whereas the "blackness" of the uranium spheres remains unchanged. Accordingly, a larger fraction of the thermal neutrons is absorbed by uranium leading to an increase in q.

It is, however, quite easy artifically to stabilize the chain reaction by slowly shifting the position of absorbing bodies within the system in such a way as to reduce the average value of q whenever the density of the neutron radiation emanating from the chain reaction can be stabilized without difficulty. ~~Since~~, statements found their way into print which give the impression that the time within which such controlling action would have to take place is very short (i.e.) of the order of magnitude of t', where t' is the time required for a fast neutron which is emitted by the splitting uranium atoms and which slowed down and absorbed within the uranium water or uranium carbon mixture to produce two fast neutrons. ~~The real situation is very different~~.

This natural stability could even be ~~well~~ enhanced by ~~inserting~~ having sheets of strong thermal neutron absorbers (inserted) in the mixture, ~~which may be practically~~ Fairly thin sheets of strong absorbers like boron are practically "black" for thermal neutrons. Whether very thin sheets of these thermal neutron absorbers follow the 1/v law like boron or have a constant absorption in the thermal region like cadmium is quite irrelevant in this connection since there is no need of using very thin sheets. and accordingly any strong thermal neutron absorber ~~will~~ would stabilize equally well ~~as cadmium~~.

Of the neutrons which are emitted in the chain reaction by uranium only a fraction ψ is absorbed within the system and $1-\psi$ escapes across the boundary of the system without reacting with uranium a stationary state can be maintained as long as

$$\mu q \psi < 1$$

We write

$$q\psi = \Gamma(T, x(t))$$

In order to indicate that this product is a function of the temperature T and also depends on a parameter such as the position of some absorbing or scattering body near or within the system which can be shifted by the control and which can thus be made the functions of time t.

In order to have a large neutron production we must maintain a chain reaction near the point

$$\mu \Gamma_0 = 1$$

If this product becomes larger than one, as it may well happen, then there is an exponential rise in the neutron production and accordingly also in the temperature. In case of a sudden small deviation from one ~~and in the absence of sufficient thermal stability~~

$$\Gamma = \Gamma_0 (1+\xi)$$

the time t_2 in which the number of neutrons doubles is given by

$$t_2 \cong \xi t_1$$

(44)

Where t_1 is the time which a fast neutron which is emitted by a uranium atom in the mixture and which is slowed down within and absorbed by the mixture would require to produce two fast neutrons. *

In arriving at these conclusions we did not take into consideration the the fact that ~~the~~ a fraction of the neutrons is emitted by uranium with a time delay of about 10 seconds. ~~Thus~~ Though this ~~small~~ fraction is small it ~~has an effect in considerations of this type and leads~~ will have a marked effect in leading to longer times than those we have estimated. Since, however, the time ~~which we have given~~ t_2 is already long enough for all practical purposes we may refrain for the present from including the delayed neutron emission in the treatment of this subject.

* For instance if we have a sudden change in Γ of 1/1000 as we well may if $t_1 \cong 10^{-3}$ sec it would take one second for the neutron production to double its value and accordingly there will be an insignificant rise in

PROPOSED MEASUREMENTS

Since a small ~~error~~ variation in the value of μ causes great change in the value of q it is of importance to obtain an accurate value for p in order to have a dependable value of μ for purposes of such computations as we have carried out. ~~It is, therefore, necessary to have a trustworthy and accurate value for .~~ For this reason the value of p will ~~therefore~~ now be determined for n=3 and this will be done in the following way: The activities of an iodine indicator which is due to resonance neutrons which are strongly absorbable in iodine has to be compared for a uranium water mixture and for another mixture of a similar hydrogen concentration in which uranium is replaced, for instance, by bismuth. Since iodine resonance seems to be above the uranium resonance the ratio of these two activities

$$J(U)/J(Bi)$$

should not be affected by the resonance absorption of uranium and is a measure of the slight dissimilarity of the two systems. Further the ratio of the activities induced in an indium indicator by indium resonance neutrons which are strongly absorbable in indium has to be determined for the uranium water and the bismuth-water mixtures. Since the indium resonance appears to be far below the uranium resonance but well outside the thermal region this ratio

$$J_{In}(U)/J_{In}(Bi)$$

is also affected by the resonance absorption of uranium . It can be shown that an accurate determination of p can be based on such measurements and we have

$$p = \frac{J_{In}(U)}{J_{In}(Bi)} \times \frac{J(Bi)}{J(U)} \qquad (44)$$

PROPOSED MEASUREMENTS 2

Another value which has to be measured is the capture cross-section of carbon for thermal neutrons or, even better, directly the range A. This will be determined in the following way: let us consider a sphere of carbon and a neutron source in the center of the sphere. The thermal neutron density inside the carbon will then obey equation No. only in this case Q is a function of r of which we must not assume $\frac{dQ}{dr} = 0$. Let the carbon sphere be immersed in a water tank or surrounded by paraffin wax. The thermal neutron density will then have a certain fairly high value at the surface of the sphere and inside the sphere it will be some function $\rho_1(r)$ of r. If in a second experiment the surface of the sphere in the water is covered by cadmium the thermal neutron density at the surface of the sphere is then reduced to zero and inside the sphere it will be another function $\rho_2(r)$ of r. The difference $\rho(r) = \rho_1 - \rho_2$ obeys the homogeneous equation

(45) $$D(r)\frac{d^2(r\rho)}{dr} - S(r)(r\rho) = 0$$

which has the solution . We can thus find A by determining the value of for two values of r, for instance, $r = 0$ and $r = r$ It is

(47) $$\frac{\rho(r)}{\rho(0)} = \frac{e^{r/A} - e^{-r/A}}{2R/A}$$

Or for small values of r/A

(48) $$\frac{\rho(r)}{\rho(0)} \simeq 1 + \frac{1}{6}(R/A)^2$$

Using a sphere of graphite of fifty to seventy centimeters of radius it should be possible to measure the range A with sufficient accuracy.

(46) $$\rho(r) = C\frac{e^{r/A} - e^{-r/A}}{r}$$

CONCLUSION

We wish to conclude that it is possible to maintain a chain reaction in a lattice of spheres of uranium embedded in graphite and that it does not take enormous masses of uranium to reach the point of divergence at which neuclear transmutation will go on with an intensity which is limited only by the necessity of avoiding overheating. Enormous quantities of radioactive elements will be produced both directly by the splitting uranium and indirectly by the intense neutron emission. The radiations emanating from the chain reaction will (retard and hinder) practical applications for purposes of power production but it is difficult to imagine that far reaching practical applications should not follow in due course of time such a revolutionary development in the field of physics.

The necessity of protecting human beings from deadly irradiations will undoubtedly slow down the technical development but will hardly preven it. It may be doubtful (for instance) whether these radiations will even make it possible to drive naval vessels with an atomic engine which is carried by the same boat as the crew but a naval vessel could tow a boat in which an atomic engine operates without the help of a crew generating steam and electricity and supplying the electric power by cable if necessary half a mile long to the naval vessel.

It is impossible though to have an opinion on the value of such products unless it is known which of the two uranium isotopes is being split by thermal neutrons. If it is uranium 135 which splits then a ton of uranium will hardly supply more power than could be obtained for about 5,000 tons of coal. Although this might enable a vessel to stay on the high seas without additional fuel supply for a long time, as a method for power production even for diving boats the disintegration of uranium by thermal neutrons would appear rather limited (in scope). The picture is very different if it is the abundant isotope of uranium which is responsible for the neutron emission.

At present it is not known which of the two isotopes is the active agent in the reactions (slow neutrons which) we have discussed, although interesting arguments in favor of the view that it is the rare isotope which is responsible have been put forward by Bohr.

This question could obviously be decided by performing an experiment on a small sample in which the rare isotope has been concentrated to an appreciable extent, perhaps by the time this paper appears in print such

separation will have been achieved by using the method of the mass spectograph. It would be, however, quite feasible to obtain a sample of a few milligrams of uranium per day in which the rare isotope is concentrated by a factor of one hundred by a ~~new modification of the method of centrifuging~~ using a centrifuge and this ~~method~~ will be described in another paper. ~~which makes it necessary to work with gaseous~~ a centrifuge in a somewhat modified way. This method which can be applied to ordinary U salts and makes it unnecessary to work with vapor. Like U6 will be described in another paper. —

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