

Nuclear Chain Reaction in a System Composed of Uranium, Beryllium and Carbon.

In a previous paper dated *Febr 14th 1940* I have attempted to show that we may expect to be able to maintain a nuclear chain reaction in a system composed of uranium and carbon. The purpose of the present paper is to point out that we may perhaps obtain a considerable improvement of the efficiency of the system for the purpose of a chain reaction by introducing beryllium into the system. An appreciable fraction of the neutrons emitted from the uranium which is split by thermal neutrons appear ¹⁾ to have energies above 1.7 MEV., ~~the~~ ^{the} binding energy of neutrons in beryllium, ~~has about this value,~~ and hence we may expect that an appreciable fraction of the fission neutrons can produce an additional neutron by knocking out a neutron from beryllium. In the circumstances, by introducing beryllium into the system in such a way that it is exposed to the fast neutrons emitted from uranium we may obtain a significant increase from this knock-out process in ~~the~~ ^{the} total number of neutrons generated in the system per thermal neutron which is absorbed by the uranium in the system.

In the above mentioned paper particular attention was given to a system consisting of a lattice of uranium spheres embedded in a large mass of graphite. Formulae were derived for a lattice in which the distance between two uranium spheres is large compared to the radius of a single uranium sphere. Under these conditions, and within the limits of the approximation used in deriving these formulae, one finds the optimal radius for the uranium spheres by determining the value of R for which the expression

$$(20) \quad \epsilon = \frac{A^2}{B^2} \frac{1}{1 + R/B} \frac{1}{\frac{RG \sqrt{\frac{302(u)}{52c(u)}} - \lambda(u)}{1 + R/A} + \frac{1}{1 + R/A}}$$

becomes a maximum. Using uranium at a density of 16 gm per cc and graphite at a density of 1.7 gm per cc we take at room temperature the values involved as follows: $A = 53.5$ cm corresponding to $\sigma_c(C) = 0.0033$; $B = 6.5$ cm; $\lambda(C) = 2.44$ cm; $\sigma_a(U) = 5.5$; $\sigma_c(U) = 11$ corresponding to $\lambda(U) = 2.25$ cm

For a value of $R = 5$ cm we have $G \approx 1$ and we find from (20) $\xi \approx 24$ which is a value close to the maximum. The corresponding value for the fraction of the neutrons which are absorbed as thermal neutrons by the uranium spheres in the lattice is given by

$$(26) \quad q_m \approx 1 - 2 \frac{-1 + \sqrt{1 + \xi}}{\xi} \approx 0.67$$

and for the ratio of the volumes of uranium and carbon we have

$$(33a) \quad \frac{4\pi R^3}{3} / V \approx \frac{1 - q_m}{6} \frac{R^2}{B^2} \frac{1}{1 + R/B} \approx \frac{1}{40}$$

giving a ratio of weights of uranium to carbon about 1 to 4

Beryllium may now be introduced into such a system by surrounding each uranium sphere with a spherical shell of beryllium metal 4 - 5 cm thick. The density of beryllium is about 1.8 gm per cc, and the amount required would be about equal in weight to the amount of uranium and perhaps one tenth of the amount of graphite.

Thus the beryllium would be located at a site where the thermal neutron density is low, ~~and the average thermal neutron density is low~~ and the average thermal neutron density within the beryllium would be less than one half of the average thermal neutron density in the graphite. Moreover, the number of beryllium atoms would be about one tenth of the number of carbon atoms, and in the circumstances a much larger thermal neutron absorption cross-section per beryllium atom can be tolerated for beryllium metal with its impurities than can be tolerated per carbon atom for graphite. Since the fraction of neutrons absorbed is given by q_m , an absorption cross-section of

$$\sigma_c(Be) = \xi \sigma_c(C)$$

would lead to a loss of $\frac{\xi}{20} \alpha_m$ neutrons. Since we have

$$(25) \quad \alpha_m \cong \frac{1 - g_m}{2} \cong$$

we would have a loss of perhaps 5% if we had an absorption in beryllium six times as large per beryllium atom as the absorption in graphite per carbon atom, i.e. if we had $\sigma_c(\text{Be}) =$

A fast neutron emitted from an uranium atom within the sphere will go through the beryllium shell once and may pass through the shell again after one or more collisions with carbon atoms. During its passage through the beryllium shell it will suffer collisions with beryllium atoms. The energy of such a fast neutron will decrease by every collision with either beryllium or carbon. This process of slowing down will limit the total number of neutrons which may be liberated by a fission neutron moving in beryllium.

In order to get a better picture of this limitation we may assume for the sake of argument that one half of the fission neutrons has an initial energy above the dissociation energy of beryllium, and that the cross-section for the disintegration of beryllium is one third of its total cross-section (and one half of its elastic collision cross-section). A fission neutron would then in its first collision with a beryllium nucleus on the average knock out 0.166 neutrons. If we further assume, rather arbitrarily, that the fission neutrons withstand two elastic collisions with beryllium with undiminished capacity for the disintegration of beryllium, but that after the third elastic collision their energy is below the threshold, we find that a fission neutron moving entirely in beryllium would liberate about $0.5 \left(\frac{1}{3} + \frac{2}{9} + \frac{4}{27} \right) = 0.35$ neutrons and not more.

In our arrangement collisions will take place with carbon atoms

as well as beryllium atoms, and accordingly the total number of neutrons liberated from beryllium by one fission neutron would be smaller. It should be emphasized though that a value of 0.2 would already be very significant since it would raise μ , the value of the neutrons generated in the system per thermal neutron absorbed in uranium, from a value between 1.5 and 2 to a value between 1.8 and 2.4. The data available at present do not permit to estimate the increase in μ which we may expect from the introduction of beryllium into a system composed of uranium and carbon. Experiments using 75 to 150 lbs. of beryllium are in preparation for the purpose of clearing up this point.

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Nuclear Chain Reaction in a System Composed of Uranium, Carbon and Beryllium.

In a previous paper dated _____ I have attempted to show that we may expect to be able to maintain a nuclear chain reaction in a system composed of uranium and carbon. The purpose of the present paper is to point out that we may ^{perhaps} obtain a considerable improvement of the efficiency of the system for the purpose of a chain reaction by introducing beryllium into the system. A ^{certain appreciable} ~~considerable~~ fraction of the neutrons emitted from the uranium which is split by thermal neutrons appear ^{to} to have energies above 1.7 MEV. The binding energy of neutrons in beryllium has about this value, and hence we may expect that a ^{appreciable} ~~substantial~~ fraction of the fission neutrons can produce an additional neutron by knocking out a neutron from beryllium. ~~X~~ ^{overall} The ~~average~~ cross-section of fission neutrons ^{for} by this process may not be very large, but even a cross-section of 10^{-25} cm² would give a significant contribution by materially increasing the number of neutrons which are ~~are~~ generated in the system for one thermal neutron which is absorbed by uranium within the system.

~~Using the formulae which were derived in the above mentioned paper under the assumption that the radius of the~~ ^{This would lead to an increase}

In the above mentioned paper particular attention was given to a system consisting of a lattice of uranium spheres embedded in a large mass of graphite. Formulae were derived for ~~the case~~ a lattice in which the distance between two uranium spheres is large compared to the radius of a single uranium sphere. Under ^{and within the limits of the approx.} these conditions one finds the optimal radius for the uranium spheres by ~~making the expression~~ determining the value of R for which the expression

(20)
$$E = \frac{A^2}{B^2} \times \frac{1}{1 + R/B} \times \frac{1}{\frac{RG \sqrt{30a(u)}}{0.5d(u)} - \lambda(u)} + \frac{1}{1 + R/A}$$

function used in deriving these formulae

becomes a maximum. Using uranium at a density of 16 gm per cc and graphite at a density of 1.7 gm per cc we take at room temperature the values involved as follows: $A = 53.5$ cm corresponding to $\sigma_c(C) = 6.0033$;

$B = 6.5$ cm ; $\lambda(C) = 2.44$ cm ; $\sigma_a(U) = 5.5$ $\sigma_{sc}(U) = 11$ corresponding to $\lambda(U) =$

For a value of $R = 5$ cm we have $G \approx 1$ and we find from (20)

$\xi =$ which is ^a ~~the~~ value close to the maximum. The corresponding value for the fraction of the neutrons which are absorbed as thermal neutrons by the ~~xxx~~ uranium spheres in the lattice is given by

$$(26) \quad q_{cm} = 1 - 2 \frac{-1 + \sqrt{1 + \xi'}}{\xi} =$$

and for the ratio of the volumes of uranium and carbon we have

$$(33a) \quad \frac{4\pi R^3}{3} / V = \frac{1 - q_{cm}}{6} \frac{R^2}{B^2} \frac{1}{1 + R/B} =$$

giving a ratio of weights of uranium to carbon of about .

Beryllium may now be introduced into such a system by surrounding each uranium sphere with a spherical shell of beryllium metal 4 - 5 cm thick. The density of beryllium ~~x~~ is about 1.8 gm per cc, and the amount required would be about equal in weight to the amount of uranium and perhaps one tenth of the amount of graphite.

Thus the beryllium would be located at a site where the thermal neutron density is low, and the average thermal neutron density within the beryllium would be less than one half of the average thermal neutron density in the graphite. Moreover, the number of beryllium atoms would be about one tenth of the number of carbon atoms, and in the circumstances a much larger thermal neutron absorption cross-section per beryllium atom can be tolerated for beryllium metal with its impurities than can be tolerated per carbon atom for graphite. Since the fraction of neutrons absorbed is given by α_n , an absorption cross-section of

$$\sigma_a(\text{Be}) = k \sigma_a(\text{C})$$

would lead to a loss of $\frac{k}{20} L_m$ neutrons. Since we have

(25)

$$L_m \approx \frac{1 - \alpha_n}{\Sigma} =$$

~~an absorption cross-section of beryllium which is six times that of carbon~~

we would have a loss of perhaps 5% if we had an absorption in beryllium six times as large per beryllium atom as the absorption in graphite per carbon atom, i.e. if we had $\sigma_a(\text{Be}) = 0.02$

A fast neutron emitted from an uranium atom within the sphere will go through the beryllium shell once and may pass through the shell again after one or more collisions with carbon atoms. During its passage through the beryllium shell it will suffer collisions with beryllium atoms. The energy of such a fast neutron will decrease by every collision with either beryllium or carbon. The process of slowing down ^{will} limits the total number of neutrons which may be liberated by a fission neutron moving in beryllium.

November 22, 1940

Note to Physical Review

In a previous letter to the Physical Review dated _____ are both
and a more detailed paper dated _____ which ~~is~~ in press
I have shown that we may expect to be able to maintain a
divergent nuclear chain reaction in a system composed of
uranium and carbon. The particular system discussed was so
designed as to require a comparatively small amount of uranium
and this end was achieved by having a lattice of comparatively
small uranium spheres embedded in a large mass of graphite.
The spheres of uranium of 4-8 cm. radius may thus be used and
the total amount of uranium required may be expected to fall
almost within the square of the radius in this interval.

The particular system which was discussed in detail in these
papers consisted of a lattice of uranium spheres, the individual
spheres having a high density and ^a the radius (of the order of
magnitude of 5 cm.) embedded in a large mass of graphite. ^{small com} It was
proved to be the distance between neighboring spheres
shown that even if carbon had a noticeable absorption which could
be measured with the method ^{described in more detail} ~~indicated~~ in the paper, a divergent
chain reaction could be maintained in such a system. [The purpose
of the present note is to communicate a method by which the number
of fast neutrons generated in the system is one thermal neutron
which is absorbed by uranium within the system can be increased
thereby making conditions for maintaining a chain reaction more
favorable. This method consists in introducing beryllium into
the system in such a manner that the beryllium would be exposed
to the fast neutrons emitted from the uranium before these
neutrons have been appreciably slowed down by collisions with

carbon. Since the ~~above~~ ^{if was found} found "that a considerable fraction of neutrons emitted in fission which is caused by thermal neutrons have energies above the disassociation energy of beryllium, we may expect that a fraction of the fast neutrons emitted from uranium will liberate further neutrons by disintegrating the the beryllium. ~~It appears worth emphasizing that~~ ^{Even} if the over all cross section of such a disintegration process of beryllium were as low as ~~10 to 25~~ ^{10⁻²⁵} sq. cm.² we still ~~would~~ ^{may} obtain an increase in neutrons of ~~10 to 25%~~ ^{10 to 25%} by surrounding each uranium sphere with a spherical layer of metallic beryllium having a thickness of 5 cm. The total amount of beryllium required would in this arrangement be about ~~and~~ in weight of the amount of uranium used ~~and~~ ~~about~~ ⁷ for this reason and also because the beryllium is concentrated in location where the thermal neutrons density is lowest the capture cross section of beryllium for thermal neutrons could be considerably higher than that of carbon without appreciably decreasing the efficiency of the arrangement for the purpose of maintaining a chain reaction,