January 31, 1956

The attached report A-55 of the Uranium Committee is a copy of a paper which I sent to the Physical Review in February, 1940. This paper was accepted for publication, but publication was deferred at my request because of the nature of the subject.

Leo Szilard

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"Divergent Chain Reactions in Systems composed of Uranium and Carbon"
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1. SUMMARY
2. NOTATIONS of Symbols (4 pp.)
3. Introduction 1 - 4
4. Conclusions
5. Rep. of paper 1 - 28
6. Drawing (1)
7. Conclusion 1 page

1

Divergent Chain Reaction in Systems Composed of Uranium and Carbon

A-55

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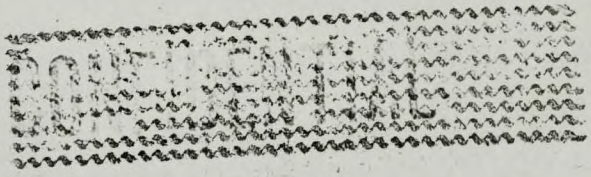
By L. Szilard

SUMMARY

It is shown that a divergent chain reaction may be maintained in a system composed of uranium and carbon. Conditions particularly favorable for a chain reaction are obtained if instead of using a homogeneous mixture of uranium and carbon a large number of rather small spheres of uranium metal are used embedded in a mass of graphite. The small uranium spheres may form a close-packed hexagonal or cubic lattice embedded in a large sphere of graphite. The average number of fast neutrons emitted by uranium for one thermal neutron absorbed by uranium is calculated from known experimental data and is found to be about 2. In our system conditions for a chain reaction become more and more favorable as the temperature increases and it is shown that we could expect a chain reaction to be self- generating in such a system at about 900° C. even if the cross-section of carbon were as high as 0.01, its present experimental upper limit. As the intensity of the chain reaction increases with increasing temperature the system is thermally unstable. It can be controlled artificially. The time within which the control would have to respond is found to be longer than one second. As much as 100 tons of graphite and 30 tons of uranium might perhaps be required in order to reach the point of divergence at which nuclear transmutation will go on with an intensity limited only by the necessity of avoiding over heating. But in so far as the capture cross-section of carbon is likely to be below 0.01 the amount of material required will probably be smaller.



[redacted] notations.

All numerical values of nuclear cross- sections given in this paper are in units of 10^{-24} cm.

1.) $\sigma_c(C)$ is the capture cross section of carbon for thermal neutrons.

2.) $\sigma_{sc}(C)$ is the scattering cross- section of carbon for thermal neutrons.

3.) ε is the ratio of the number of thermal neutrons and the number of resonance neutrons absorbed by a sinlge uranium sphere and under the following circumstances: A single uranium sphere is embedded in an infinite space filled with carbon. Neutrons are generated in the carbon and the numbers of thermal neutrons and resonance neutrons produced per cc and sec are equal and have the same value trhoughout the whole infinite mass of carbon.

4.) E_0 is the energy at which the resonance absorption line of uranium has its maximum.

5.) v is the velocity of thermal neutrons .

6.) E_1 is the lower end of the resonance region of uranium below which we consider the ratio of the absorption coefficients for thermal neutrons and for resonance neutrons as constant.

7.) E_2 is the upper end of the resonance region of uranium above which we neglect the resonance absorption in uranium.

8.) k^{th} is the average number of collisions which a neutron moving "diffusing" in carbon survives within the energy region

9.) k^{Th} is the average number of collision which a thermal neutron diffusing in carbon survives before being captured by carbon.

10.) $\lambda(C)$ is the mean free path for scattering of thermal neutrons in carbon .

11.) ρ is the density of thermal neutrons .

12.) Q is the number of thermal neutrons produced per cc and sec. in carbon.

13.) R is the radius of a sphere of uranium or uranium oxide which is embedded in carbon.

14.) A is the range of thermal neutrons in carbon defined by

$$A = \frac{\lambda(C)}{\sqrt{3}} \sqrt{\frac{\sigma_{sc}(C)}{\sigma_c(C)}}$$

15.) J_0^{th} is the number of thermal neutrons absorbed by a single uranium sphere embedde d in carbon provided that the uranium sphere absorbs each thermal neutron which reaches its surface.

16.) B is the range of the resonance neutrons in carbon defined by

$$B = \frac{\lambda^{res}(C)}{\sqrt{3}} \sqrt{R^{res}}$$

17.) J^{res} is the number of resonance neutrons absorbed by a single uranium sphere embedded in carbon.

18.) $\lambda^{res}(C)$ is the mean free path for scattering for neutrons of energy E_0 in carbon.

19.) ε_0 is the value of ε for a uranium sphere which absorbs each thermal neutron which reaches its surface.

20.) J^{th} is the number of thermal neutrons absorbed by a single uranium sphere embedded in carbon.

21.) φ is defined by $J^{th} = \varphi J_0^{th}$ or $\varepsilon = \varphi \varepsilon_0$

22.) $\lambda(U)$ is the mean free path for scattering in the substance of the uranium sphere.

23.) N_U is the number of uranium atoms per c.c. in the substance of the uranium sphere.

24.) $\sigma_a(U)$ is the absorbing cross-section of uranium for thermal neutrons which inclusdes both the cross-section for fission and radiative capture but which does not include the cross-section for scattering.

25.) $\sigma_s(U)$ is the cross-section of uranium for scattering of thermal neutrons.

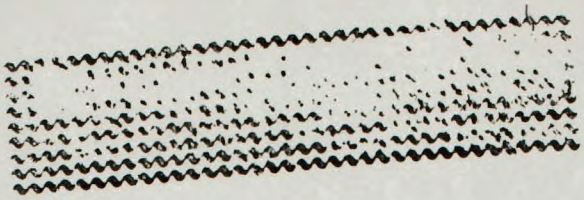
26.) U is the range of thermal neutrons in the substance of the uranium sphere defined by

$$U = \frac{1}{\sqrt{3}} \sqrt{\frac{\lambda(U)}{N_U \sigma_a(U)}}$$

and in the particular case of pure uranium metal

$$U = \lambda(U) \sqrt{\frac{\sigma_s(U)}{3\sigma_a(U)}}$$

Notations 3



27.) α — is the fraction of the thermal neutrons absorbed by carbon in an infinite mass of carbon which contains a lattice of uranium spheres.

28.) J^{th} — is the number of thermal neutrons absorbed per sec by a uranium sphere within a lattice of uranium spheres embedded in carbon.

29.) J^{res} — is the number of resonance neutrons absorbed per sec by a uranium sphere within a lattice of uranium spheres embedded in carbon.

30.) q — is the fraction of the resonance neutrons produced in carbon which is absorbed as a thermal neutron by the lattice of uranium spheres if the number of thermal neutrons and the resonance neutrons produced per cc and sec. in the carbon are equal and have the same value throughout the whole infinite mass of carbon.

31.) α_m — is the value of α for which q becomes maximum.

32.) q_m — is the maximum value of q

33.) $q_{corr.}$ — is the correct value of the fraction of the resonance neutrons produced in carbon which is absorbed as a thermal neutron by the lattice of uranium spheres if obtained by taking into account that the number of thermal neutrons produced per cc and sec. in the carbon near the uranium spheres is reduced due to the absorption of resonance neutrons by the uranium.

34.) V — Is the volume of carbon per uranium sphere.

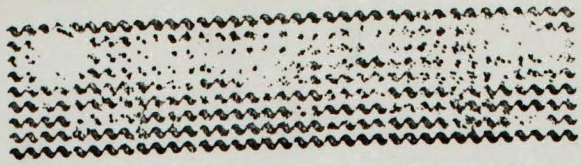
35.) L — is the distance between neighbouring uranium spheres in a cubic or hexagonal close packed lattice.

36.) n — is the number of H atoms per uranium atom in a mixture of uranium oxide and water.

37.) τ_0 and τ_{int} — is the mean lifetime of a thermal neutron in water or a mixture of uranium oxide and water,

38.) $\sigma_c(H)$ — is the capture cross section of hydrogen for thermal neutrons.

39.) p — is the fraction of the resonance neutrons generated in a homogenous mixture of uranium oxide and water which is captured by uranium at resonance.



40.) δ is the density of water in gm per cc in a homogeneous mixture of uranium oxide and water.

41.) $\dot{c}$ is the first factor in

$$\varepsilon = \left\{\frac{A^2}{B^2}\right\} \times \left\{\frac{1 + R/A}{1 + R/B}\,\varphi\right\}$$

42.) $\dot{f}$ is the second factor in the same expression.

43.) l is the critical radius ~~xxxxxxx~~ of a graphite sphere which contains a lattice of uranium spheres giving the value of the radius for which the chain reaction becomes divergent.

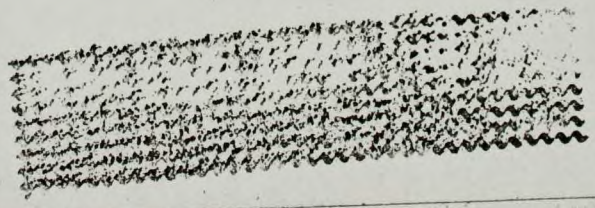
A-55

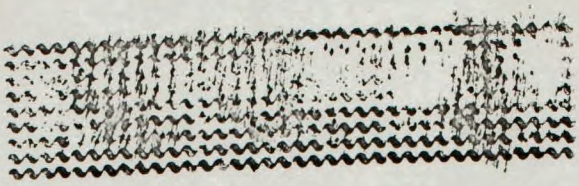
Divergent Chain Reaction In Systems Composed Of Uranium And Carbon

INTRODUCTION 1

As early as 1913 H. G. Wells forecast the discovery of induced radio-activity for the year 1933 and described the subsequent advent of nuclear transmutations on an industrial scale. It was not possible for physicists to realize the potentialities and the limitation of nuclear physics in this direction until after the discovery of the neutron in 1932. Owing to the discovery of artificial radio-activity in 1933 by Joliot and Irene Curie, and Fermi's pioneer work on neutron reactions in 1934, progress in the field of nuclear physics was greatly accelerated. By the middle of 1934 it could be clearly seen that within the framework of modern nuclear physics transmutation of elements on an industrial scale might be achieved by means of a chain reaction in which neutrons form the links of the chain. If there is an element or a mixture of elements which interacts with neutrons and from which a neutron liberates on the average more than one neutron for one neutron which is absorbed within the mixture we have a chain reaction and may bring about nuclear transmutations on a large scale. If a neutron source is placed in the center of a large sphere which is composed of such an element or mixture the number of neutrons emerging out of the sphere will be larger than the number of neutrons emitted by the source in the center of the sphere. If the radius of the sphere approached a certain critical radius the number of neutrons generated in the chain would tend to become infinite. If the radius of the sphere is larger than the critical radius there is no stationary solution of the equation and the number of neutrons would increase exponentially with time.

By simplifying the problem so as to be able to apply the theory of diffusion to the motion of the neutrons which are liberated in the chain reaction it is easy to get an approximate picture of the general type of behaviour of a chain reaction within a finite sphere of matter.





Such a simplified treatment has been applied to the problem as early as 1934 particularly with regard to the potential possibility of setting up a chain reaction in beryllium. At that time it appeared from Bainsbridges's value for the mass of beryllium, and Aston's value for the mass of helium, that beryllium ought to be unstable and that neutrons might be liberated from beryllium by slow neutrons in sufficient numbers to make a chain reaction possible. This idea had to be abandoned, as far as beryllium was concerned, when Aston's value for the mass of helium proved to be in error.

In 1934 the transmutation of uranium by neutrons was discovered along that of other elements by Amaldi, D' Agostino, Fermi, Rasetti, and Segre who found that a number of radioactive elements are generated from uranium by neutrons. An important advance was made by Irene Curie and P. Savitch, who found that an element which behaved chemically apparently like radium was among those produced and later Irene Curie and P. Savitch discovered radio active rare earths among the disintegration products of uranium. Finally Hahn and Strassman announced in December 1938 that in reality uranium splits into a large number of elements of medium atomic weight if irradiated by neutrons.

As soon as this became known it must have been evident to all those who had been thinking previously of the potential possibilities of a chain reaction, as well as to others, that it neutrons should be emitted from splitting uranium atoms in sufficient numbers a chain reactions might be set up in a large mass of uranium. Independently of each other a small number of physicists began therefore to prepare experiments with the aim to discover whether or not neutrons are emitted from splitting uranium.

Ideas along these lines found wider circulation in America through a semi-private meeting held in Washington in January, 1939 under the auspices of the George Washington University and the Carnegie Institute for Terrestrial Magnetism. At this meeting I understand, Fermi drew attention to the potential possibility of a neutron emission from splitting

uranium and some of its possible consequences. For a short time afterwards it seemed that the absorption of uranium would over compensate any conceivable neutron emission since it was found[1] that the cross-section of uranium for splitting by thermal neutrons is about 2 whereas the currently accepted value for the absorption of uranium for the thermal neutrons was 43. However the absorption of uranium for thermal neutrons was thereupon remeasured by Fermi who discovered that its value was only about 5.

A delayed emission of neutrons from uranium under the action of neutrons bombardment was discovered in February 1939 by Roberts, Meyer, and Wang.[2]

A much stronger instaneous emission of neutrons by uranium under the action of thermal neutrons was discovered independently and almost simultaneously by Halban, Joliot, and Kowarski;[3] Anderson and Fermi;[4] and Szilard and Zinn[5] who reported their observations in letters to Nature and to Physical Review dated March 8, March 16, and March 16 (1939) respectively. An observation pointing in the same direction was also reported by v. Droste[6] in a letter to Die Naturwissenschaften dated March 17, (1939).

The first paper to appear in print was that of Halban, Joliot, and Kowarski which was published on Nature on March 18, (1939). A number of papers dealing with the instantaneous neutron emission of uranium were sent to periodicals after this date some of which presumably reporting experiments which were started independently at an earlier date.[7-10]

The balance of neutron absorption and emission by uranium was subsequently studied by Halban, Joliot and Kowarski;[11] Anderson, Fermi and Szilard;[12] and Halban, Joliot, Kowarski, and Perrin.[13] This work demonstrates beyond doubt that, on the average, more than one fast neutron is emitted for one thermal neutron absorbed by uranium. It shows therefore, that in certain circumstances a chain reaction might be maintained in a mixture of uranium and an element which slows down to thermal energies the fast neutrons emitted by the splitting uranium. If it is possible to avoid that too large a fraction of the fast neutrons emitted from the splitting uranium be captured (without causing fission) at resonance by uranium and if it is possible at the same time to avoid that too large a

uranium and some of its possible consequences. For a short time afterwards it seemed that the absorption of uranium would over compensate any conceivable neutron emission since it was found (1) that the cross-section of uranium for splitting by thermal neutrons is about 2 whereas the currently accepted value for the absorption of uranium for the thermal neutrons was 43. However the absorption of uranium for thermal neutrons was thereupon remeasured by Fermi who discovered that its value was only about 3.

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fraction of the thermal neutrons be captured by the element which is used for the purpose of slowing down, then we would have a chain reaction which may diverge for a sufficiently large mass of uranium.

This does not necessarily mean that a chain reaction can be maintained in a mixture of uranium oxide and water. In a homogeneous mixture of uranium oxide and water a considerable fraction of the neutrons is absorbed by uranium at resonance and it remains doubtful whether in such a mixture the number of neutrons produced may exceed the number of neutrons absorbed. (9) Fermi has investigated the question whether more favorable conditions can be obtained in mixtures of uranium oxide and water by keeping the uranium oxide and water in separate layers and found that a slight improvement can thus be obtained. But even so, for the present the question whether a chain reaction can be obtained in a system composed of uranium and water is being left in abeyance.

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QUOTATIONS

0) H. G. Wells, The World Set Free, (1913)

1) Anderson, Booth, Dunning, Fermi, Glasoe and Slack, Phys. Rev. 55, 511, (1939)

2) Roberts, Meyer and Wang, Phys. Rev. 55, 510 (1939)

3) Halban, Joliot and Kowarski, Nature 143, 479, (1939)

4) Anderson, Fermi and Hanstein, Phys. Rev. 55, 797 (1939)

5) Szilard and Zinn, Phys. Rev. 55, 799, (1939)

6) v. Droste, Naturwissenschaften, 27, 198, (1939)

7) Haeny and Rosenberg, C. R. 208, 898, (1939)

8) Michiels, Perry, and G. P. Thomson, Nature, 143, 760, (1939)

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10) Rotblatt, Nature, 143, 852, (1939)

11) Halban, Joliot and Kowarski, Nature 143, 680, (1939)

12) Anderson, Fermi, and Szilard, Phys. Rev. 56, 284, (1939)

13) Halban, Joliot, Kowarski and Perrin, Journ. de Phys. 10, pp. 428-429, (1939)

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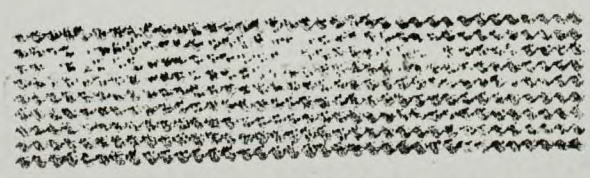
15) Adler and Halban, Nature 143, 793, (1939)

A-55

The purpose of the present paper is to show that a chain reaction can be achieved by using an element like carbon for slowing down the neutrons in certain particular systems composed of uranium and carbon. The theory which is given in the present paper can be applied to elements other than carbon but it does not give any useful information for systems composed of uranium and hydrogen.

Though one might think that carbon should be much less efficient for slowing down neutrons than hydrogen from several points of view it would be preferable to use carbon in the form of graphite rather than hydrogen in the form of water. The capture cross-section of carbon for thermal neutrons $\sigma_c(C)$ is small. An upper limit of $\sigma_c(C) > 0.01 \times 10^{-24}$ cm^2 has been reported by Frisch, Halban and Koch, but this upper limit is not sufficiently low to allow us at present to conclude that a chain reaction could be maintained in homogeneous mixtures of uranium and carbon. For neutrons it takes about 6.5 collisions with carbon atoms to reduce their energy by a factor of e. Thus a neutron which is being slowed down by carbon stays for a long time within the resonance absorption region of uranium. Consequently, very low uranium concentrations would have to be used in order to avoid that a large fraction of the fast neutrons emitted by uranium is absorbed at resonance by uranium. At such very low uranium concentrations, on the other hand, the fraction of the thermal neutrons which is absorbed by carbon might perhaps be too large to permit a chain reaction.

It will be shown, however, in the present paper that if instead of using a homogeneous mixture of uranium and carbon a large number of spheres of uranium which may form, for instance, a close-packed hexagonal or cubic lattice are embedded in carbon, the ratio of the number of thermal neutrons and the number of resonance neutrons absorbed by the uranium can be so much increased that a chain reaction will become possible. It will be seen that this ratio strongly depends on the radius of the uranium spheres and that a rather small radius must be chosen in order to obtain most favorable conditions.



Single spheres of uranium.

We wish to first calculate ε the ratio of the number of thermal neutrons, and the number of resonance neutrons which are absorbed by a single uranium sphere, which is embedded in an infinite space, filled with carbon, in the special case in which the number of resonance neutrons and thermal neutrons produced per c.c. and second are equal and have the same value throughout the whole infinite mass of graphite. In order to obtain a conservative estimate for the value of ε we shall assume that all neutrons which have an energy between $E = 0.2\ E_0$ and $E = 2E_0$ where E_0 is the energy at which the resonance absorption of uranium has its maximum, are absorbed by uranium at resonance if they reach the surface of the uranium sphere by diffusion. That this is indeed a conservative assumption can be seen by considering an absorption line which obeys the Breit-Wigner formula and has its maximum at E_0. For such an absorption line the absorption falls off with 1/v in the thermal region. It reaches a minimum at $0.2\ E_0$ then it rises up to E_0 and falls again so rapidly that if E_0 is not too close to the thermal region the absorption becomes negligible for $2\ E_0$. If E_0 is higher than five volts and if the temperature of the thermal neutrons does not exceed 1/10 of a volt then the absorbing cross-section beyond $2\ E_0$ is less than 1/10 of the absorbing cross-section for the thermal neutrons.

A neutron which is slowed down by elastic collisions with carbon atoms and which enters the resonance at $E = E_2$ will survive on the average $k^{res.}$ collisions within the resonance region between E_2 and E_1 and we have

(1)

Under the assumption which we have made above i.e. $E_2 = 2E_0$; $E_1 = 0.2\ E_0$

$$k^{res} = 6.5 \ln 10 = 15$$

This may be compared with the average number of elastic collisions k which a thermal neutron will survive in carbon before being captured by a carbon atom. Since the capture cross-section of carbon is small compared with $\sigma_{sc}(C)$ the scattering cross-section of carbon for thermal neutrons the probability $g_1(h)$ that a thermal neutron will survive h collisions in carbon will be given by

$$g_1(h) = e^{-h\frac{\sigma_c(C)}{\sigma_{sc}(C)}}$$

A-55

Single spheres of uranium .

We wish to first calculate ε , the ratio of the number of thermal neutrons, and the number of resonance neutrons which are absorbed by a single uranium sphere, which is embedded in an infinite space filled with carbon, in the special case in which the numbers of resonance neutrons and thermal neutrons produced per c.c. and second are equal and have the same value throughout the whole infinite mass of graphite. In order to obtain a conservative estimate for the value of ε we shall assume that all neutrons which have an energy between $E = 0.2\ E_0$ and $E = 2E$ where E_0 is the energy at which the resonance absorption of uranium has its maximum, are absorbed by uranium at resonance if they reach the surface of the uranium sphere by diffusion. That this is indeed a conservative assumption can be seen by considering an absorption line which obeys the Breit - Wigner formula and has its maximum at E_0 . For such an absorption line the absorption falls off with 1/v in the thermal region. It reaches a minimum at $0.2\ E_0$ then it rises up to E_0 and falls again so rapidly that if E_0 is not too close to the thermal region the absorption becomes negligible for $2\ E_0$. If E_0 is higher than five volts and if the temperature of the thermal neutrons does not exceed 1/10 of a volt then the absorbing cross- section beyond $2\ E_0$ is less than 1/10 of the absorbing cross- section for the thermal neutrons.

A neutron which is slowed down by elastic collisions with carbon atoms and which enters the resonance at $E = E_2$ will survive on the average k^{res} collisions within the resonance region between E_2 and E_1 and we have

$$k^{res} = 6.5 \ln E_2/E_1 \qquad (1)$$

Under the assumption which we have made above i.e. $E_2 = 2E_0$; $E_1 = 0.2\ E_0$

$$k^{res} = 6.5 \ln 10 = 15$$

This may be compared with the average number of elastic collisions k^{el} which a thermal neutron will survive in carbon before being captured by a carbon atom. Since the capture cross- section of carbon is small compared with $\sigma_{el}(C)$ the scattering cross - section of carbon for thermal neutrons the probability $g_1(h)$ that a thermal neutron will survive h collisions in carbon will be given by

$$g_1(h) = e^{-h \frac{\sigma_c(C)}{\sigma_{sc}(C)}}$$

Accordingly, k^{IR} the average number of collisions which a thermal neutron will make with carbon atoms before being captured is given by

$$k^{H} = \int_0^\infty h\, g_1(h)\, dh$$

so that

$$k^{H} = \frac{\sigma_{sc}}{\sigma_c} \qquad (2)$$

Let us now first determine the number of thermal neutrons which are absorbed by a single uranium sphere of radius R embedded in an infinite space filled with carbon if Q thermal neutrons are produced per cc and sec. in the carbon. If R is large compared to $\lambda(C)$, themean free path for elasic scattering of thermal neutrons in carbon, the density ρ of the thermal neutrons in the carbon can be calculated by treating the problem as a diffusion phenomenon. We thus find for ρ as a function of the distance r from the center of the sphere

3 $$D(C)\frac{d^2(r\rho)}{dr^2} - S(C)\, r\rho + Q(r)\, r = 0$$

$$D(C) = \frac{v\lambda(C)}{3} \quad , \quad S(C) = \frac{v}{\lambda(C)\sigma_{sc}(C)}$$

If the same number of thermal neutrons are produced everywhere in the carbon per cc and sec. we have

4 $$\frac{dQ}{dr} = 0$$

For a sphere which absorbes each thermal neutron which reaches it surface i.e. for a " black" sphere we have $\rho(R) = 0$ and find (from 3 and 4) for $r > R$

5 $$\rho(r) = \frac{Q}{S(C)}\left(1 - \frac{R}{r}\, e^{-(r-R)/A}\right)$$

6 $$A = \sqrt{\frac{D}{S}} = \frac{\lambda(C)}{\sqrt{3}}\sqrt{\frac{\sigma_{sc}(C)}{\sigma_c(C)}} = \frac{\lambda(C)}{\sqrt{3}}\sqrt{R^{IR}}$$

γ^{th} the number of thermal neutrons which is absorbed by a single uranium sphere per sec. is given by

7 $$\gamma^{th} = D(C)\, 4\pi R^2 \rho'(R)$$

and for a black uranium sphere we find from No. 5 $\gamma^{th} = \gamma_0^{th}$

8 $$\gamma_0^{th} = 4\pi Q R A^2 (1 + R/A) \quad ; A = \frac{\lambda(C)}{\sqrt{3}}\sqrt{R^{th}}$$

where A has the dimension of a length and will be called the range of thermal neutrons in carbon.

A-55

Quite similarly since we assume that the uranium sphere is "black" for resonance neutrons we can write for J^{res} the number of resonance neutrons absorbed by the sphere per second with good approximation

(9) $$J^{res} = 4\pi Q R B^2 (1 + R/B)$$

where

(10) $$B = \frac{\lambda^{res}}{\sqrt{3}} \sqrt{k^{res}}$$

is the mean free path of resonance neutrons for scattering and B has the dimension of a length and will be called the range of the resonance neutrons in carbon.

Expression No. 8 is identical with the expression No. 9 which holds for a sphere which is "black" for thermal neutrons. Only the values for λ the mean free path for scattering and k the average number of collisions which a neutron survives within the category called thermal or resonance are different for these two categories of neutrons. It would be strictly true that J the number of neutrons belonging to a category which will reach the sphere by diffusion per second is determined in the same way for different categories by λ the mean free path and $g(h)$ the function giving the probability of surviving h collisions with carbon atoms. In reality the function $g(h)$ is different for thermal and for resonance neutrons and expression No. 9 holds in so far as we may assume that J is determined with sufficient accuracy by λ and $k = \int_0^\infty h\, g(h)\, dh$ the first moment of $g(h)$.

From No. 8 and No. 9 we find as the value of ε for a sphere which is "black" for thermal neutrons $\varepsilon = \varepsilon_0$

(11) $$\varepsilon_0 = \frac{A^2}{B^2} \frac{1 + R/A}{1 + R/B}$$

Assuming, for example, $\sigma_{sc} = 4.8$; $\sigma_c(C) > 0.005$ and $\frac{\lambda^*(C)}{\lambda(C)} = 1.18$ we find for graphite of density 1.7:

$$A \geqq 43.5 \text{ cm} \qquad B = 6.5 \text{ cm}$$

and so obtain for small values of R

$$\varepsilon_0(0) = \frac{A^2}{B^2} \geqq 45$$

and for large values of R

$$\varepsilon_0(\infty) = \frac{A}{B} \geqq 6.7$$

Large values of R correspond to plane layers of uranium and a comparison of these two values for ε_0 illustrates how very much superior small spheres of uranium are to plane layers.

A real sphere of uranium having a finite radius ~~below 8 centimeters~~ is not "black" for thermal neutrons and the number of thermal neutrons absorbed per second by the sphere is smaller than J_0^R. We write

(12) $$J^R = J_0^R \varphi$$

and accordingly we have

(13) $$\varepsilon = \varepsilon_0 \varphi \quad \text{and} \quad (14)\ \varepsilon = \frac{A^2}{B^2} \frac{1 + R/A}{1 + R/B}$$

In order to calculate φ we take into account that inside the uranium sphere the thermal neutron density ρ obeys the equation

(15) $$D(U) \frac{d^2(r\rho)}{dr^2} - S(U) r\rho = 0$$

$$D(U) = \frac{v \lambda(U)}{3} \qquad S(U) = v N_U \sigma_a(U)$$

having as its solution

(16) $$\rho(r) = \frac{C}{r}\left(e^{r/U} - e^{-r/U}\right); \quad r < R$$

(17) where $U = \sqrt{\frac{D}{S}}$ ~~. for pure uranium we have~~ $= \sqrt{\frac{\lambda(U)}{3 N_U \sigma_a(U)}}$

and for ~~[redacted]~~ pure uranium metal we have

(18) $$U = \lambda(U) \sqrt{\frac{\sigma_s(U)}{3\sigma_a(U)}}$$

From equations 3, 4 and 16 we find that J^R the number of thermal neutrons diffusing into the sphere per second is given by

$$J^R = J_0 \varphi$$

where

(19) $$\varphi = \frac{\frac{\lambda_{tr}(U)}{U}\left\{\frac{e^{R/U} + e^{-R/U}}{e^{R/U} - e^{-R/U}} - \frac{U}{R}\right\}}{\frac{\lambda_{tr}(C)}{R}(1 + R/A) + \frac{\lambda_{tr}(U)}{U}\left\{\frac{e^{R/U} + e^{-R/U}}{e^{R/U} - e^{-R/U}} - \frac{U}{R}\right\}}$$

[redacted]

A-55

For uranium in its pure state we have from No. 14, 18, and 19

$$(20a) \quad \varepsilon = \left\{\frac{A^2}{B}\right\} \times \left\{\frac{1}{1+R/B} \times \frac{1}{\left(\frac{\lambda(C)}{RG\sqrt{\frac{3\sigma_a U}{\sigma_s(U)}} - \lambda(U)} + \frac{1}{1+R/A}\right)}\right\}$$

where G stands for

$$G = \frac{e^{R/U} + e^{-R/U}}{e^{R/U} - e^{-R/U}} \quad ; \quad R/U = \frac{R}{\lambda(U)}\sqrt{\frac{3\sigma_a(U)}{3\sigma_s(U)}}$$

For $R/U > 2$ we can write $G \cong 1$, the difference being about 3.5% for $R/U = 2$.

The first factor in expression No. 20 a increases proportionately with the reciprocal value of the capture cross-section of carbon. The second factor is practically independent of the carbon cross-section, since we have $R/A << 1$. Its value is determined by the density of graphite and uranium and the nuclear values $\sigma_s(U)$, $\frac{\sigma_a(U)}{\sigma_s(U)}$. The value of R may be so chosen as to make this factor a maximum.

All expressions for ε were so far obtained from diffusion equations involving the assumptions

$$R >> \lambda(C) \; ; \; R >> \lambda(U) \; ; \; U >> \lambda(U)$$

For small values of R the problem can no longer be treated as a diffusion phenomenon and we shall, therefore, refrain from using expression No. 20 or 20 a for values of R of less than 5 cm.

LATTICE OF URANIUM SPHERES

If we have now an infinitely large number of uranium spheres forming a lattice embedded in an infinite mass of carbon and want to calculate the ratio of the number of thermal neutrons and resonance neutrons absorbed by the uranium spheres, we shall again assume for the time being that everywhere in the carbon the same number Q of neutrons enter the resonance region and the thermal region per c.c and second.

In the absence of uranium the thermal neutron density in the graphite is given by $\rho_0 = \frac{Q}{\sigma(C)}$. If a lattice of uranium spheres is embedded in the carbon, the average neutron density $\bar{\rho}$ in the carbon is reduced by some factor α .

$$\bar{\rho} = \alpha \rho_0$$

Since the number of neutrons captured per second by carbon is proportionate to the average neutron density, and since in the absence of uranium all the neutrons produced are captured by carbon, the fraction of the neutrons which is captured by carbon in the presence of the uranium lattice is given by α . Correspondingly, the fraction of the neutrons which are absorbed by the uranium lattice is given by $1-\alpha$.

In order to determine the number of thermal neutrons J^{th} absorbed per second by one uranium sphere within the lattice, we may consider the following: a single uranium sphere, which is embedded in carbon, does not appreciably affect the thermal neutron density at distances which are large compared to R. Equation No. 5 shows that even for a "black" uranium sphere at a distance of 2 R from the center of the sphere ρ has already reached the value of $1/2 \rho_0$. For this reason, the uranium spheres within the lattice affect each other with respect to their thermal neutron absorption only in so far as the presence of these spheres in the carbon determines the average neutron density, and we have

$$J^{th} = \alpha J^{th} \qquad (20)$$

Further, since the distance L between neighboring uranium spheres within the lattice will be large compared to B, the range of the resonance neutrons in carbon, we have for J^{res} , the number of resonance neutrons absorbed by a uranium sphere within the lattice

$$J^{res} = J^{res} \qquad (21)$$

From this it follows that q the fraction of all the neutrons which are absorbed by the uranium spheres in the thermal region alone is given by

(22) $$q = \frac{J^{th}}{J^{th} + J^{res}} (1-\alpha)$$

or

(23) $$q = \frac{\varepsilon\alpha}{1+\varepsilon\alpha}(1-\alpha)$$

This expression has its maximum value for $\alpha = \alpha_m$

(24) $$\alpha_m = \frac{-1+\sqrt{1+\varepsilon}}{\varepsilon}$$

and for the maximum value for q we have $q = q_m$

(25) $$\alpha_m = \frac{1-q_m}{2}$$

(26) $$q_m = 1 - 2\frac{-1+\sqrt{1+\varepsilon}}{\varepsilon}$$

or

(27) $$\varepsilon = \frac{4q_m}{(1-q_m)^2}$$

CORRECTED VALUE

By calculating the value of q from ε we have neglected the effect of the absorption of resonance neutrons (by the uranium spheres ~~embodd~~ in the lattice) on the production on thermal neutrons in the carbon in the neighborhood of the uranium spheres. The absorption of resonance neutrons reduces in reality the value of Q near the spheres below the average value of Q and accordingly the correct number J^{th}_{corr} of thermal neutrons absorbed by the uranium spheres per sphere will be smaller than J^{th}. In order to find the value for the difference $(J^{th}_{corr} - J^{th})$ we may proceed in the following way:

If the uranium spheres in the lattice, ~~[illegible]~~, stopped absorbing resonance neutrons then a neutron which reaches a given sphere at least once while its energy is in the resonance region would have some probability of reaching the sphere at least once after it had been slowed down to the thermal region. On the other hand, the probability that the sphere which is

A-55

.... Not "black" for thermal neutrons absorbs a thermal neutron which reaches the sphere at least once is given by φ . (φ was originally defined as the ratiom of the number of thermal neutrons which a sphere of uranium absorbs and the number of thermal neutrons which an equally "black" sphere of uranium would absorb under the same previously specified conditions. It is easy to see though that φ can also be defined as the ratio of the number of thermal neutrons which are absorbed by the uranium sphere and the number of thermal neutrons which reach the surface of the uranium sphere at least once during their life time. That these two definitions of are identical simply follows from the fact that a "black" uranium sphere absorbs all the thermal neutrons which reach its surface during their lifetime.)

It follows that we have

$$J^{th}_{corr} = J^{th} - \nu\varphi J^{res} \quad ; \quad 0 < \nu < 1$$

correspondingly we have

$$q_{corr} = \frac{J^{th} - \nu\varphi J^{res}}{J^{res} + J^{th} - \nu\varphi J^{res}} (1-\alpha)$$

or

$$q_{corr} = \frac{\varepsilon\alpha - \nu\varphi}{1 + \varepsilon\alpha - \nu\varphi} (1-\alpha)$$

and by introducing the value of q_m from equation No 25 and 27 we find

$$(28) \qquad q_{corr} = q_m \frac{1 - \frac{\nu\varphi(1-q_m)}{2q_m}}{1 - \frac{\nu\varphi(1-q_m)}{1+q_m}}$$

If we neglect terms which contain powers higher than the second of $\frac{\nu\varphi(1-q_m)}{1+q_m}$ we obtain

$$(29) \qquad q_{corr} = q_m \left\{ 1 - \frac{\nu\varphi(1-q_m)^2}{2q_m(1+q_m)} \left(1 + \nu\varphi \frac{1-q_m}{1+q_m}\right) \right\}$$

Taking into consideration that we have $\nu < 1$ and that for the uranium spheres which we shall consider φ has a value of ab ut $\varphi \cong 0.5$ we may assume $\nu\varphi \leqq 0.5$. and find then for $q_m > 0.5$ from No. 28 :

$$(30) \qquad q_{corr} > q_m \times 0.9$$

With increasing q_m q_{corr}/q_m approaches 1; for $\varphi \leqq 0.667$; $q_m \geqq 0.68$. we have $q_{corr}/q_m > 0.97$. for $\varphi \leqq 0.645$; $q_m \geqq 0.73$ we have $q_{corr}/q_m > 0.98$

Spacing of the lattice.

Formula No. 26 giving the value of q_m was derived under the assumption that there is a uniform production of resonance neutrons throughout the whole mass of carbon into which the lattice of uranium spheres is embedded. We have to verify that this assumption is correct. For this reason we have to compare the distance $\sqrt{\tau^2}$ to which a fast neutron "diffuses" away from a uranium sphere from which it is emitted before it is slowed down to thermal energies with the distance L between two neighboring uranium spheres in a close-packed hexagonal or cubic lattice. If the value of L which corresponds to the maximum value of q were large compared to $\sqrt{\tau^2}$ then obviously equation No. 26 giving the value of q_m would not be used.

In graphite of density 1.7 we have $\sqrt{\tau^2}$ about 50 cm. and we shall see that the values of L which correspond to the maximum value of q are smaller for all values of $\sigma_c(c)$ which we are going to discuss in this paper.

In order to estimate L as well as for other reasons we shall calculate the volume V of carbon per uranium sphere in the lattice.

In the lattice of uranium spheres from the q V neutrons which are slowed down per second within the volume V to resonance energies, the carbon absorbs $\alpha Q V$ neutrons and the uranium spheres in the lattice absorb $(1-\alpha) Q V$ neutrons. Accordingly we have $J^{th} + J^{res} = (1-\alpha) Q V$

and from this we find

$$31 \qquad V = 4\pi \varphi \frac{\alpha}{q} A^2 R (1 + R/A)$$

and if q has its maximum value q_m we have $\frac{\alpha}{q} = \frac{1-q_m}{2 q_m}$

and it is

$$32 \qquad V = 4\pi \varphi \frac{1-q_m}{2 q_m} A^2 R (1 + R/A)$$

This gives for the ratio of the volumes of carbon and uranium

$$33 \qquad \frac{V}{\frac{4\pi}{3} R^3} = 3 \varphi \frac{1-q_m}{2 q_m} \frac{A^2}{R^2} (1 + R/A)$$

and for L the distance between neighboring uranium spheres in a hexagonal or cubic close-packed lattice we have

$$34 \qquad L^3 = \left(V + \frac{4\pi}{3} R^3\right) \sqrt{2}$$

or

$$(35)\quad L = \lambda_0 R \sqrt[3]{1 + 3\varphi \frac{1-g_m}{2g_m} \frac{A^2}{R^2}\left(1 + \frac{R}{A}\right)}$$

For large values of ε we can write

$$\frac{1-g_m}{2g_m} \simeq \frac{1}{\sqrt{\varepsilon}} = \frac{1}{\sqrt{\varphi \varepsilon_0}}$$

so that we would have for $\varepsilon >> 1$

$$(36)\quad \gamma \cong 4\pi A B R \sqrt{\varphi\left(1 + \frac{R}{A}\right)\left(1 + \frac{R}{B}\right)}$$

From No. 35 we find for R = 8 cm., $\sigma_c(c) = 0.0033$, A = 75.5, $g_m = 0.68$
$\varphi = 0.65$, [illegible].

$$L = 51 \text{ cm}$$

BALANCE OF EMISSION AND ABSORPTION OF NEUTRONS BY URANIUM

A direct comparison of the number of fast neutrons emitted by uranium with the number of thermal neutrons absorbed by uranium may be obtained from different types of experiments carried out by Halban, Joliot and Kowarski,[11] Anderson, Fermi and Szilard,[12] and Halban, Joliot, Kowarski and Perrin.[13] These experiments show that on the average more than one fast neutron is emitted by uranium for one thermal neutron absorbed by uranium.

Halban, Joliot and Kowarski[8] reported that 3.5 ± 0.7 fast neutrons are emitted per fission. Anderson, Fermi and Szilard[9] reported that about $\mu = 1.5$ fast neutrons are emitted on the average for one thermal neutron absorbed by uranium.

Halban, Joliot, Kowarski and Perrin[13] carried out an experiment of the following type: a sphere of radius r is filled with a homogeneous mixture of uranium and water which contains n atoms of hydrogen per atom of uranium. This sphere is immersed in a large water tank and a photo neutron source is placed in the center of the sphere. The density ρ of the thermal neutrons is measured along a radius and the integrals of

$$I_{int} = \int_0^{r_0} r^2 \rho \, dr \quad ; \quad I_{ext} = \int_0^{\infty} r^2 \rho \, dr$$

are determined, giving a measure of the number of slow neutrons which are present within the mixture inside the sphere and in the water outside the sphere. In another experiment the integral

$$I_0 = \int_0^{\infty} r^2 \rho \, dr$$

is determined for the same neutron source in pure water.

From the values of these three integrals which these authors determined for n = 1, 2 and 3 and r = 25 cm. they concluded that on the average eight neutrons are emitted from the uranium within the sphere for one photo neutron which is slowed down to thermal energies and causes a fission process in uranium. This was interpreted to show that a considerable number of secondary and tertiary neutrons were generated in this experiment.

Since the value of the fission cross-section is not well known and rather difficult to determine, it seems necessary to avoid expressing results in terms of this cross-section and to use the magnitude μ rather than the number of neutrons emitted per fission. An accurate knowledge of the value of μ is of greatest importance from the point

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A-55

of view of estimating the chances and finding the optimum conditions for a chain reaction. In the following we shall therefore indicate in what way an accurate value for ρ can be obtained, from an experiment of the type performed by Halban, Joliot, Kowarski and Perrin:

F_{int}, the average number of neutrons which reach the thermal region inside the sphere within the uranium water mixture for one neutron emitted by the source in the center of the sphere, is given by

$$F_{int} = \frac{\tau_{int}/\tau_0}{\tau_{int}/\tau_0}$$

where

$$\frac{\tau_{int}}{\tau_0} = \frac{n\sigma_s(H)}{s\{\sigma_a(U) + n\sigma_s(H)\}} \quad (37)$$

is the ratio of the life-time of the thermal neutrons in the mixture inside the sphere and in pure water.

Of the neutrons which are slowed down to the resonance region inside the sphere a fraction p is absorbed at resonance and a fraction $(1 - p)$ reaches the thermal region. F^*_{int} the number of the neutrons reaching the resonance region per second inside the sphere is therefore given by

$$(1-p) F^*_{int} = F_{int}$$

F^*_{ext} the number of neutrons reaching the resonance region outside the sphere in the water is the same as F_{ext} the number of neutrons reaching the thermal region outside the sphere in the water and we have, therefore

$$F^*_{ext} = F_{ext} = \frac{I_{ext}}{I_0}$$

P, the total number of fast neutrons emitted by the uranium in the mixture for one neutron emitted by the neutron source in the center of the sphere is equal to the total number of the resonance neutrons produced minus 1, the neutron emitted by the source in the center. Therefore, we have

$$P = F^*_{int} + F^*_{ext} - 1 = \frac{F_{int}}{1-p} + F_{ext} - 1$$

The total number of fast neutrons produced by the uranium divided by the number of thermal neutrons produced within the sphere in the mixture is

$$\frac{P}{F_{int}} = \frac{1}{1-p} + \frac{F_{ext} - 1}{F_{int}}$$

And finally since the uranium absorbs of the neutrons which reach the thermal region within the sphere the fraction $\frac{\sigma_a(U)}{\sigma_a(U) + n\sigma_c(H)}$

we have

$$\mu = \frac{P/F_{int}}{\left(\frac{\sigma_a(U)}{\sigma_a(U) + n\sigma_c(H)}\right)}$$

or

$$\mu = \frac{\sigma_a(U) + n\sigma_c(H)}{\sigma_a(U)} \left\{ \frac{1}{1-p} + \frac{\tau_{int}}{\tau_0} \frac{I_{int}/I_0 - 1}{I_{int}/I_0} \right\}$$

Introducing the value of τ_{int}/τ_0 from No. we obtain

$$3\delta \quad \mu = \frac{1}{1-p} + \frac{n\sigma_c(H)}{\sigma_a(U)} \left\{ \frac{1}{1-p} + \frac{1}{\delta} \frac{I_{int}/I_0 - 1}{I_{int}/I_0} \right\}$$

From this equation we see that we can determine μ by determining the value of p.

A considerable fraction of the fast neutrons emitted by uranium may of course escape from within the sphere but the value of this fraction need not be determined for our purpose. On the other hand, the formula given for μ holds with good approximation only if the sphere is sufficiently large to permit us to neglect transition phenomena in the equilibrium between resonance neutrons and thermal neutrons near the surface of the sphere. Since the uranium inside the sphere absorbs resonance neutrons the density of the neutrons which have an energy below the upper end of the resonance region of uranium and above the thermal region is smaller. This has the consequence that neutrons of this category will diffuse from the water across the sphere into the inside of the sphere. If the radius of the sphere were not sufficiently large so that this phenomenon could not be neglected the expression $F_{int}/1-p$ would then give a too large value for the number of resonance neutrons produced inside the sphere. Correspondingly the expression No. would give a too large value for μ. But with the proviso that a sufficiently large sphere is used we shall obtain an accurate value for μ if only the value of p is measured accurately.

near the surface inside the sphere than near the surface outside the sphere.

The value of p has so far not been measured for hydrogen concentrations as low as n = 3 but we can give a lower limit for p for this concentration as we are able to extrapolate by means of theoretical considerations from values of p which have been measured for higher hydrogene concentrations i.e. n = 30.

In order to do this we shall make use of formulae relating to homogeneous mixtures of hydrogen and uranium which will be published shortly by G. Placzek. A general treatment of the process of the slowing down of neutrons in water has been given by E. Fermi.* In 1936 In a uranium oxide - water mixture p, the fraction of the neutrons which is absorbed by uranium between the energies E_1 and E, is given by*

39 $$\ln(1-p) = -\int_{E_1}^{E_2} \frac{f(E)}{E}\, dE \quad ; \quad E_1 > 1\,Volt$$

where

40 $$f(E) = \frac{1}{1 + \frac{nH}{g(E)}}$$

$g(E)$ is the cross- section of uranium for radiative capture and H is the scattering cross- section of hydrogen for the resonance neutrons of uranium. ($H \cong 17 \times 10^{-24}\ cm^2$) If the hydrogen concentration is high enough so that the energy region in which there is an appreciable absorption by uranium is small compared with the resonance energy E_0 one may write

41 $$\ln(1-p) = -\frac{1}{E_0}\int_{-\infty}^{\infty} f(E)\, dE$$

for a single absorption line of the form

42 $$g(E) = \frac{g_0}{1 + \left(\frac{E-E_0}{\Gamma}\right)^2} \quad ; \quad g_0 = g(E_0)$$

giving*

43 $$\ln(1-p) = -\frac{\pi\Gamma}{E_0}\,\frac{1}{\sqrt{\frac{nH}{g_0}\left(1+\frac{nH}{g_0}\right)}}$$

For $n \leq 30$ we have $\frac{nH}{g_0} \ll 1$ and No 44 gives**

44 $$\ln(1-p) = -\frac{\pi\Gamma}{E_0}\sqrt{\frac{g_0}{nH}}$$

From this we see that for such hydrogen concentrations for which No 44 holds we have for two different hydrogen concentrations n and n_1,

45 $$\ln(1-p) = \sqrt{\frac{n_1}{n}}\,\ln(1-p_1)$$

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*Private communication by G. Placzek June, 1939

**Private communication independently by G. Placzek and E. Fermi June, 1939

This relationship No. 45 does no longer hold for hydrogen concentrations which are as low as n = 3. But we may write in the case of uranium

(46) $$\ln(1-p) = \int_{0.2E_0}^{1.2E_0} \frac{L(E)}{E}\, dE$$

and it is then easy to see that for an absorption line of the form given by No. 42 which is symmetrical in $(E - E_0)$ or even more so for a similar line which deviates from symmetry in the sense that the absorption is larger for smaller energies (as it is the case for a line which obeys the Breit-Wigner formula) we have

(47) $$\ln(1-p) < \sqrt{\frac{n'}{n}}\, \ln(1-p') \quad \text{if } n < n'$$

The value of p has been measured by Halban, Joliot, Kowarski and Perrin for n = 30 and was found to be $p = 0.2 \pm 0.02$; using equation No. 47 we find from this

$$p_3 > 0.5 \pm 0.04 \quad \text{for } n = 3$$

According to No. 38 μ increases with increasing values of p and in the circumstances we should obtain a conservative value for μ by using the value of p = 0.5.

Using this value and the values

$$\frac{I_{int}}{I_0} = 0.72 \quad ; \quad \frac{I_{ext}}{I_0} = 0.45$$

reported by Halban, Joliot, Kowarski and Perrin for a sphere of 25 cm. radius filled with a uranium oxide--water mixture for which they had n = 3 and s = 0.42 gm./c.c. we find from No. 38

(48) $$\mu = 2 + 0.2\, \frac{3\sigma_a(C)}{\sigma_a(U)}$$

We see that the value obtained for μ if calculated from this equation is scarcely affected by the wide limits of error of the present experimental values of $\sigma_a(U)$. By attributing the value of $\frac{1}{4}$ or 1/8 to $3\sigma_a(H)/\sigma_a(U)$ we obtain $\mu = 2.05$ or $\mu = 2.02$ respectively.

In these circumstances we shall use for the present as a presumably conservative value and as the best value at the present available:

$$\mu = 2$$

A-55

Conditions For A Chain Reaction

If q denotes the fraction of fast neutrons emitted by uranium which are slowed down to the thermal region and are absorbed as thermal neutrons by uranium and if μ denotes the number of fast neutrons produced on the average by uranium for one thermal neutron absorbed by uranium then obviously

(49) $\mu q > 1$

is the condition for the possibility of a chain reaction. If this condition is fulfilled then a divergent chain reaction can be maintained in a sufficiently large system from which only a small fraction of the neutrons emitted by the uranium within ~~the system~~ can escape across the boundary of the system without being absorbed within ~~the system~~.

Accordingly, the condition for the possibility of a chain reaction in a system composed of a lattice of uranium spheres embedded in carbon

(50) $\mu q_{conv} > 1$

and using equation No. 30 we find

$\mu q_m \times 0.9 > 1$ or $q_m > 1.11/\mu$

From which we find by using equation No. 27 as a sufficient condition for the possibility of a chain reaction

(51) $$\frac{4.44\mu}{(\mu - 1.1)^2} < \varepsilon$$

Using the value of $\mu = 2$ as a presumably conservative value we have as a sufficient condition

$$11.3 < \varepsilon .$$

In order to see now whether a chain reaction is possible we have to calculate from our formulae the numerical value of ε . We shall do that in the following under the assumption that the energy liberated in the chain reaction will maintain the carbon at a temperature of about 900°. and in order to be on the conservative side we shall assume that the temperature of the uranium spheres in which most of the energy is liberated is , in spite of efficient cooling, about the same.

Since we have at room temperature $\sigma_a(C) > 0.01$ we shall have at 900 C. a capture cross-section of carbon half of this value. The scattering cross-section of uranium for thermal neutrons we take to be $\sigma_s(U) \cong 9$. Finally, at room temperature we take $\frac{\sigma_a(U)}{\sigma_s(U)} = 1/2$ and correspondingly we take at 900 C. $\frac{\sigma_a(U)}{\sigma_s(U)} = 1/4$. For a density of graphite of 1.7 and a density of uranium of 15 we then obtain from No. 14 for $R = 8$ cm

$$\varepsilon = 14$$

This being larger than the value required by No. 51 we conclude that in the circumstances we can expect a divergent chain reaction to take place in the system which we have investigated.

In reality the capture cross-section of carbon is perhaps much smaller than the upper limit which has so far been established and consequently there is hope that conditions will be much more favorable for a chain reaction than would seem from the values so far quoted.

The amount of carbon and uranium required to reach the point of divergence at which nuclear transmutation will proceed at a rate limited only by the necessity of avoiding over-heating is essentially determined by the value of

$$(\mu q - 1)$$

[illegible]

A-55

In the following we shall calculate how the value of this expression depends on the value of the carbon capture cross-section at room temperature.

We shall take the density of graphite to be 1.7; the density of uranium metal to be 15 and choose R = 8cm.

We then obtain for a capture cross-section of carbon at room temperatur of 0.005 the following set of values: at 900 C° $A = 64\,cm$ $\frac{A^2}{B^2} = 90$; $\varphi = 0.666$ $\varepsilon = 27$; $\eta_m = 0.601$; $\eta_{corr} = 0.66$ and for $\mu = 2$

$$(\mu\eta - 1) = 0.32$$

If the capture cross-section of carbon at room temperature were .003 then at 900 C. our equations would give the following set of values: $A = 75.5\,cm$; $\frac{A^2}{B^2} = 135$; $\varphi = 0.645$; $\varepsilon = 39$; $\eta_m = 0.728$ $\eta_{corr} = 0.71$ and for $\mu = 2$

$$(\mu\eta - 1) = 0.42$$

A-55

CRITICAL DIMENSIONS

For a large sphere of graphite which contains a large number of small spheres of uranium 1, the critical value for the radius of the graphite sphere for which the chain reaction becomes divergent may be calculated for various distributions of uranium within the graphite sphere. The optimum distribution of uranium is not uniform within the graphite sphere and will either decrease or increase with r according to whether we want to have a minimum amount of uranium or a minimum value for 1. The treatment of this question may as well be postponed until the value of the carbon capture cross-section is known. It will then be possible to find the optimum distribution of uranium as a function of the distance from the center of the graphite sphere and give a value for 1. In the meantime, a very rough approximation may give an idea of the order of magnitudes which are involved. In graphite of 1.7 density the average distance $\sqrt{\overline{r^2}}$ to which a fast neutron emitted by uranium diffuses away from its point of origin until it becomes a thermal neutron and reacts with uranium or carbon is about 50 cm. For $(\mu q - 1) \cong \frac{1}{\rho}$ we find for the critical radius 1 from

$$l \sim \sqrt{\frac{3\overline{r^2}}{\mu q - 1}}$$

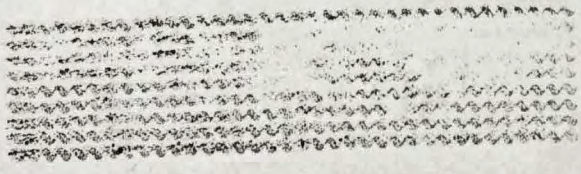
$l \sim 250$ cm. This corresponds to about 100 metric tons of graphite. If the carbon capture cross-section is lower, then 1 will be smaller and the amount of graphite required might perhaps be as low as 20 tons.

The amount of uranium required can be calculated from equation No. 33a

$$\frac{4\pi}{3} R^3 / v = \frac{1 - q_m}{6} \frac{R^2}{B^2} \frac{1}{1 + R/B}$$

It may be ~~minimum~~ kept down by choosing a smaller value for R than the value corresponding to the maximum value of ε. For $R = 5$cm. and $q_m = 0.6$ 33a gives $\frac{4\pi}{3} R^3/v = 0.0336$ corresponding to 30 tons of uranium for 100 tons of graphite For larger values of q_m we find a smaller ratio of uranium to carbon.

~~For q we would have as the ratio of weights~~



In order to determine the critical dimenstions and the most favorable distribution of the small uranium spheres wihin a large graphite sphere it is essential to have an accurate value for μ and for the capture cross-section of carbon for thermal neutrons.

In order to have an accurate value for μ it will be necessary to have a direct measurement of p for small hydrogen concentrations preferably for $n = 3$. In the following we shall indicate in what way an accurate value for p can be obtained.

Let us consider a box filled with a homogeneous mixture of uranium oxide and water and let the box be so large that a neutron which has an energy of less than 1000 volts in the center of the box be slowed down below 1 volt before it reaches the boundary of the box by diffusion. If such a box is then irradiated with neutrons the energy distribution of the neutrons below 1000 volts in the center of the box will be the same as it would be in an infinite space filled with the same uranium water mixture in case of a uniform generation of neutrons throughout the space. If a radio-active indicator like rhodium or indium which has its lowest dominant resonance absorption line below the dominant resonance absorption line of uranium is placed in the center of the box the activity induced in the rhodium indicator by the resonance neutrons of rhodium will be ~~called~~ caused only by those neutrons which are slowed down below the resonance region of uranium without being captured by uranium at resonance. We shall call this activity the resonance activity of the indicator and designate it if a rhodium indicator is used with $(Rh^{*}$ in $U)$. If, on the other hand, an indicator like, for instance, iodine is used which has its dominant absorption lines above the uranium resonance the resonance activity of this indicator $(J^{*}$ in $U)$ will, at least in the ideal case, not be affected by the resonance absorption of uranium. The ratio of $(Rh^{*}$ in $U)$ and J^{*} in $U)$ would therefore give some measure of 1 - p if the activity of the indicators as measured by an ionization chamber or electron counter gave a real measure of the number of the neutrons which pass through the resonance region of rhodium or iodine. This, of course, is by no means the case. Therefore, a second experiment has to be performed in which the resonance activities of the rhodium and iodine indicators are determined in the center of another box which contains water. ~~(It is perhaps even better to use a mixture of water and bismuth having approximately the same water density as the mixture used in the first experiment. We then find~~

(Preferably but not necessarily water of the same density as the water contained in the uranium water mixture used in the first experiment) We then find

$$1-p = \frac{(Rh^{*} \text{ in } U) \times (J^{*} \text{ in } H_2O)}{(Rh^{*} \text{ in } H_2O) \times (J^{*} \text{ in } U)}$$

If there are uranium resonance lines of some importance above the lowest resonance lines of iodine then we have

$$1-p < \frac{(Rh^{*} \text{ in } U) \times (J^{*} \text{ in } H_2O)}{(Rh^{*} \text{ in } H_2O) \times (J^{*} \text{ in } U)}$$

giving a reliable upper limit for 1 - p and consequently a lower limit for p which used in conjunction with equation No. 38 will give a lower limit for the value of μ .

The upper limit which has been reported for the carbon cross-section by Halban, Frisch, and Koch is already so low that it would be difficult to improve upon it unless a method were used which is specifically designed to measure extremely small catura cross-sections. Such a method will be described in the following:

A-55

~~CAPTURE CROSS SECTION OF CARBON~~

~~Since the capture cross section of carbon determines the critical dimensions and the mass of uranium which has to be used in order to reach the point of divergence of the chain reaction it is necessary to measure this value beyond the upper limit which has been reported by Halban, Frisch and Koch. This upper limit is already so low that it would be difficult to improve upon it unless some method is used specifically designed for measuring extremely small capture cross sections. In the following we wish to describe such a method.~~

Let us consider a sphere of carbon and a neutron source in the center of the sphere. The thermal neutron density inside the carbon will then obey equation No. only in this case Q is a function of r of which we must not assume $\frac{dQ}{dr} = 0$. Let the carbon sphere be immersed in a water tank or surrounded by paraffin wax. The thermal neutron density will then have a cerfairly high value at the surface of the sphere and inside the sphere it will some function of r, $\rho_1(r)$ If in a second experiment the surface of the sphere in the water is covered by cadmium the thermal neutron density at the surface of the sphere is then reduced to zero and inside the pphere it will be another function of r, $\rho_2(r)$. The difference $\rho(r) = \rho_1 - \rho_2$ obeys the homogeneous equation

$$D(C)\frac{d^2(r\rho)}{dr^2} - \delta(C)\, r\rho = 0$$

which has the solution

$$\rho(r) = C\frac{e^{r/A} - e^{-r/A}}{r}$$

We can thus find A by determining the value of ρ for two values of r, for instance, r=0 and r = r . It is

$$\frac{\rho(r)}{\rho(0)} = \frac{e^{r/A} - e^{-r/A}}{2\, r/A}$$

Or for small values of r/A

$$\frac{\rho(r)}{\rho(0)} \cong 1 + \frac{1}{6}\left(\frac{r}{A}\right)^2$$

Using a sphere of graphite of fifty to seventy centimeters of radius it should be possible to measure the range A with sufficient accuracy. There is a limit to using very large spheres which arises out of the fact that for a very large sphere most of the neutrons emitted in the center of the sphere will be slowed down to the thermal region within the carbon and the thermal neutron density near the surface of the carbon may become very low. If that

A-55

happens, then the difference $\rho = \rho_1 - \rho_2$ will become small and will therefore set a limit to the accuracy of the measurement.

A-55

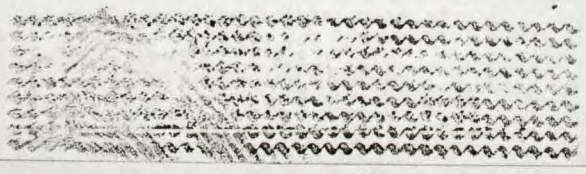
Stabilizing the chain reaction

Soon after the discovery of an abundant neutron emission from uranium ~~and when before it was known whether a way would be found to set up a chain reaction~~ the question of stabilizing such a reaction was a subject of discussion, (1415) but the situation as we see it appears to be rather different in practice:

If a chain reaction could be maintained in a homogeneous mixture of water and uranium or carbon and uranium it would have a certain natural stability in the sense that with rising temperature there would be a decrease in the neutron production. The reason for this is the fact that the absorption of both uranium and hydrogen obey the 1/v law in the thermal region and thus at higher temperatures the range of thermal neutrons in the mixture is larger. Correspondingly, at higher temperatures a larger fraction of the thermal neutron will escape across the boundary of the mixture without having reacted with the uranium in the mixture. This natural stability could even be enhanced by having bodies of strong thermal neutron absorbers inserted in the mixture. Fairly thin sheets of such absorbers as boron, for instance, are practically "black" for thermal neutrons and any strong thermal neutron absorber would stabilize equally well.

A system, on the other hand, in which uranium bodies which are almost "black" for thermal neutrons are embedded in carbon, like the system which we have considered in great detail in the present paper, has no such stability. This is due to the fact that with rising temperature the capture cross-section of the carbon decreases whereas the absorption by the uranium spheres remains almost unchanged. Accordingly, at higher temperatures, a larger fraction of the thermal neutrons is absorbed by uranium and a smaller fraction is absorbed by carbon and this leads to an increase in q. and thermal instability.

It is, however, quite easy artificially to stabilize the chain reaction by slowly shifting the position of absorbing bodies within the system in such a way as to reduce the average value of q whenever the intensity of the neutron radiation emanating from the chain reaction increases. One might perhaps think that the time within which such controlling action would have to take place is very short. We shall therefore now show that this is not so.



Of the neutrons which are emitted in the chain reaction by uranium only a fraction ψ is absorbed within the system and $1-\psi$ escapes across the boundary of the system without reacting with uranium. A stationary state can be maintained as long as

$$\mu q \psi < 1$$

We write

$$q\psi = \delta(T, x(t))$$

In order to indicate that this product is a function of the temperature T and also depends on a parameter such as the position of some absorbing or scattering body near or within the system which can be shifted by some controlling mechanism and thus be made a function of time t.

In order to have a large neutron production we must maintain a chain reaction near the point

$$\mu \delta_0 = 1$$

If this product becomes larger than one, as it may well happen then there is an exponential rise in the neutron production and accordingly also in the temperature. In case of a sudden small deviation from one

$$\Gamma = \delta_0 (1+\xi)$$

the time t_2 in which the number of neutrons doubles is given by

$$t_2 = \xi t_1$$

where t_1 is the time which a fast neutron ~~xxxxx~~ emitted by a uranium atom in the system would require to produce two fast neutrons if it is slowed down and absorbed (as a thermal neutron) within the system.

For instance if we had a sudden change in δ of 1% as we well may have and if we have $t_1 = 4 \cdot 10^{-3}$ sec then it would take 4 seconds for the neutron production to double its value and accordingly there would be only an insignificant rise ~~xxxx~~ in the temperature if the control responded within 4 seconds.

It is easy to see that for a lattice of uranium spheres in carbon τ_1, the mean life-time of a thermal neutron within the system is given by

$$\tau_1 = \lambda \frac{\sigma_c(C)}{\sigma_c(C)} \frac{\lambda(C)}{v}$$

For $\sigma_c(C) = 0.005$ at room temperature, for instance, we have at 900 C.

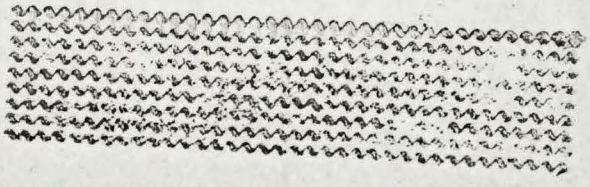
$\eta_m = 0.66$ and $\lambda_m = \frac{1-\eta_m}{2} = 0.17$ giving at 900 C.

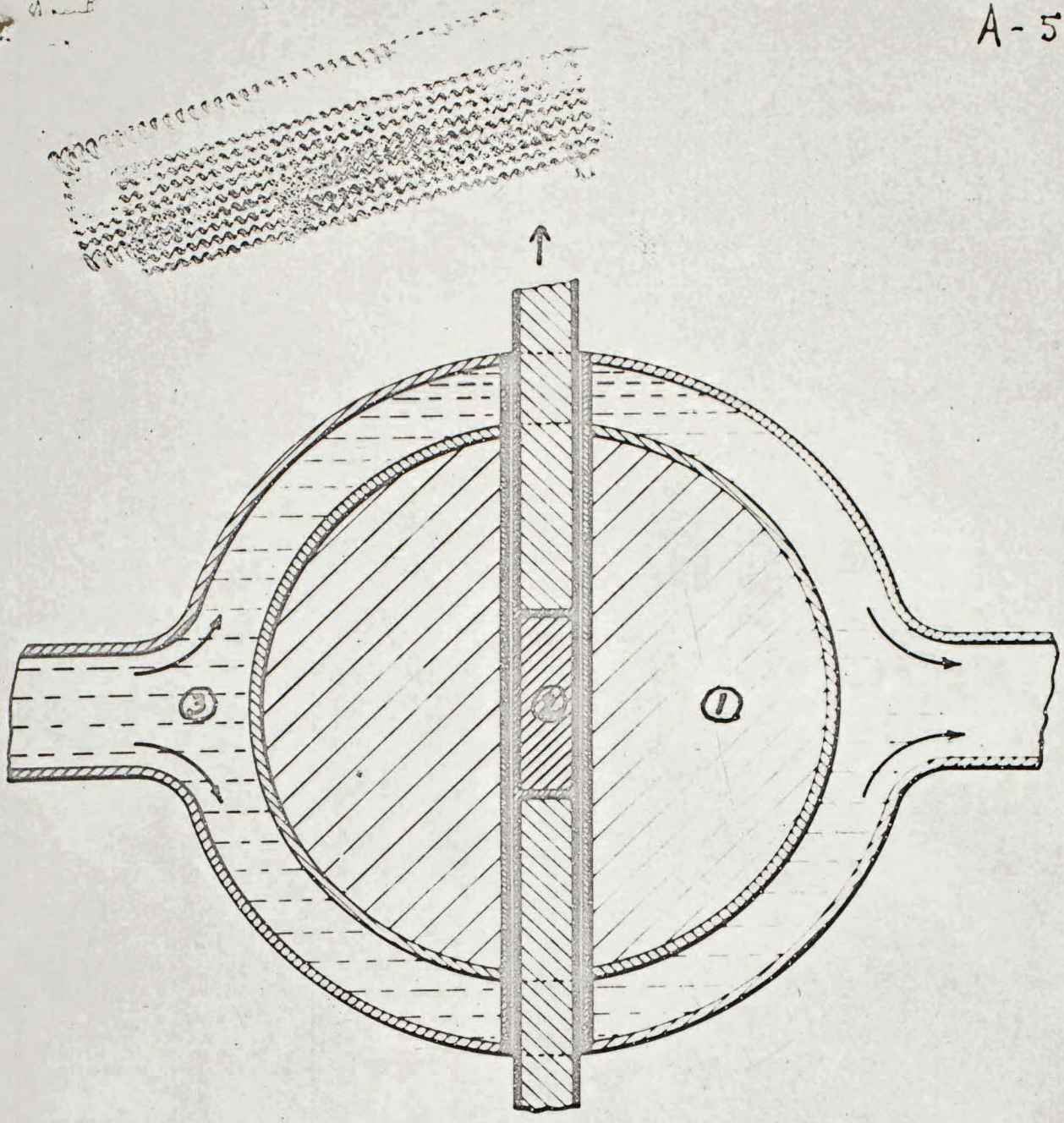
$$\tau_1 = 4 \times 10^{-3} \text{ sec}$$

A-55

In calculating t_2 we did not take into consideration the fact that a fraction of the neutrons is emitted by uranium with a time delay of about ten seconds. Though this fraction is small it has a marked effect in leading to still longer times than those which we have estimated. But as for all practical purposes the time t_2 which we have found is already long enough we need not include for the present the delayed neutron emission in the treatment of the subject.

A way to stabilize the nuclear chain reaction in a lattice of uranium spheres which is "economical" from the point of view of the nuclear phenomena involved is illustrated in figure 1. The uranium sphere which is surrounded by a spherical layer of liquid bismuth (serving the purpose of cooling the uranium in a way which does not reduce the nuclear efficiency of the arrangement as expressed by φ is shown in this figure. A short rod or disc composed of an element which strongly absorbs thermal neutrons is near the center of the uranium sphere and is shielded by the uranium from thermal neutrons. This rod or disc can move within a tube or slit and its position may be controlled by the intensity of the neutron radiation emitted by the chain reaction. If the intensity of this radiation increases the rod or disc may be automatically moved away from the center of the uranium sphere and ultimately if required entirely out of the uranium sphere. It will then absorb larger and larger numbers of thermal neutrons thereby reducing the value of q and thus stabilizing the chain reaction.





1 represents a sphere of uranium metal. 2 represents a rod, composed of a thermal neutron absorber moving in a vertical bore, or a circular disc having a horizontal axis moving in a vertical slit in the uranium sphere 1. 3 is a spherical shell of liquid bismuth which surrounds the uranium sphere.

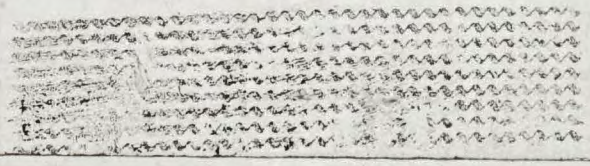
A-55

CONCLUSIONS

We conclude that we can expect a chain reaction to take place in a sufficiently large mass of graphite which contains, for instance, a close-packed hexagonal or cubic lattice of uranium spheres. The capture cross-section of carbon is likely to be smaller than the upper limit so far established and consequently there is hope that moderately large masses of graphite and uranium or uranium oxide will be sufficient to reach the point of divergence at which nuclear transmutation can be maintained at an intensity which is limited only by the necessity of avoiding over heating.

Large quantities of radio-active elements will be produced directly from the splitting uranium atoms and indirectly by the intense neutron emission. The necessity of protecting human beings from deadly irradiations emanating from the chain reaction will undoubtedly limit the scope of practical applications and perhaps will slow down the industrial development of this field but it is difficult to imagine that practical applications should not follow in due course of time the present turn of events in physics.

In so far as the production of power for practical purposes is concerned the crucial question which will determine the scope of applications is now whether the rare isotope of uranium 235 or the abundant isotope 238 is the active agent in the thermal neutron reaction. If the rare isotope is the active agent ten tons of uranium may become exhausted by the chain reaction after having supplied as much power as can be obtained from about fifty thousand tons of coal. In case of the other alternative, ten tons of uranium could supply more power than five million tons of coal without being used up. Though N. Bohr put forward interesting arguments in support of the view that it is the rare isotope which is split by thermal neutrons this question will have to be decided by direct observations performed on small samples of the separated isotopes. Only after some such observation shall we be able to express a well balanced opinion upon the immediate future of "atomic engineering."



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B-19

All postmarked Feb 21, 1940

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