This memorandum deals with lead or bismuth reflector outside a uranium lattice in light water. We wish to estimate how many neutrons leak out from the lattice per $\mathrm{cm}^{2}$ if q fission neutrons are produced per $\mathrm{cm}^{3}$ near the boundary. The leak will consist of two terms, one relating to fast neutrons and one relating to thermal neutrons. We shall first estimate the leak of the fast neutrons as follows:

In order to slow down from a million volts to $1 / 10$ volt we need 16 collisions with hydrogen, We shall consider the density of the fast neutrons in each of the logarithmic intervals and we on the conservative side if we assume that the density of neutrons in these intervals is not affected by the boundary, ie us aroume that the density is not reduced due to the fact that an appreciable fraction of the neutrons leak. out. Our result for the reflector will therefore give a lower limit for its effect.

We shall write for this density in the nth interval from the bottom up

where $\lambda_{\text {is }}$ the mean free path for an elastic collision with hydrogen. This for $q=1$, that is if one fission neutron is produced per cc and second. The leak of these neutrons into an infinite plane reflector is determined from the following consideration: The density of the neutrons originating from the nth logarithmic interval will fall off in the lead reflector according to the law


#### Abstract

In order to calculate the we assume here that if a neutron is slowed down to $1 / 10$ volt it is absorbed by the lead and does not return to the water lattice boundary. a is then determined by the consideration that in 100 collisions with lead the neutron is slowed down by one logarithmic interval so that for the neutrons originating from the nth logarithmic interval it takes $n \times 100$ colLisions to become thermal. Therefore we may write


The number of neutrons originating from the nth interval which diffuses into the lead scatterer is then given in the plane case by
or the total leak of fast neutrons into the lead given by

We shall distinguish two groups of neutrons from one to $n$ resonance neutrons having a mean free path of one in water with respect to collision with hydrogen and above resonance neutrons having a mean free path of 4. We shall therefore write

General expression for saving in the plane case we have
where

This gives for extrapolation length

# 0.854 <br> For instance, for leak $=$ we would get 

Please note for $a=0.0243$ and $M^{2}=40 \mathrm{~cm}^{2}$ the critical radius of a free sphere is 130 cm .

zxxyxxxxxxxuxx<br>Finite reflector, plane case, with no slowing down in lead. Here we may write for the leakage

For one fission neutron per cc produced near the boundary. By assuming for the bottom 12 energy intervals a mean free path in lead of 3 cm , for the 4 top energy intervals a mean free path in lead of 3 cm and a mean free path in water of 4 cm

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From this we can, by means of (8), (9) and (10) calculate the
extrapolation distance, which comes out to be }35\textrm{cm}
    No absorption in the reflector, spherical case.--We have
to write, in order to obtain the leakage,
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where $D$ is the radius of the outer layer of the reflector and $R$ is
the radius of the lattice sphere. From this and
we obtain the relation
the expression
and this gives for $D-R=50 \mathrm{~cm}$, for $a R=2.30$, or $R=95 \mathrm{~cm}$. For infinite reflector $a R$ would correspond to 1.61 giving $R=66 \mathrm{~cm}$ 。 Absorbing infinite reflector, spherical case.--Here we can write
In this expression we have added the leak coefficient for thermal neutrons which we estimate to be 4/11. Taking for the thermal leak coefficient $4 / 11$, this gives for $a R=$

SUPPLEMENT TO MUC-IS-28 of $7-7-44$ addressed to W . W . Watson
The following contains considerations referring to the constructional and functional problems which arise in connection with the composite heavy water unit discussed in the above mentioned memorandum.

## Insert I

Most of the uranium in the composite unit is in the light water lattice. The constructional problems of this light water lattice are obviously the same whether or not we have a heavy water core and some of the problems connected with it have been discussed before our attention was focussed on the possibility of using a heavy water core. In the following considerations we shall assume that the volume ratio of water to uranium is about 2 to 3, i.e., about one-third of the space is occupied by uranium. If all the water were allowed to move through the lattice with a velocity which is needed for good heat transfer we would have to circulate a rather large water volume. The water volume which has to be circulated for a good amount of heat transfer can be cut by about a factor 2 if we use a construction which is illustrated in Fig. 1. In Fig. I we see a quadratic lattice of aluminum tubes (2,5,4, etc.); every second aluminum tube contains a uranium rod ( 2,4 , etc.), and every other aluminum tube (5) is filled with water. The whole lattice of aluminum tubes is of course immersed in water but while the water between the aluminum tubes circulates rapidly through the lattice the water within the aluminum tubes (5) flows only slowly. In this way slightly less than half of the water in the lattice moves fast and the amount of water which we have to circulate is cut by a factor 2. The aluminum tubes which do not contain uranium have slits near the top and bottom of the tube so that a slow circulation within the tubes is maintained.

The Canalized Water Reflector.--The top and bottom of the aluminum tubes in the lattice can be so conttructed as to form a structure which might be called a canalized water reflector. For a certain length from

the top and from the bottom (about 30 cm ) the aluminum tubes may be left free from uranium and in place of the uranium rod may be simply left empty or preferably may contain an aluminum can filled with bismuth. This is indicated in Figs. 2 a and 2 b showing the top and bottom of the lattice respectively. The average water density within the reflector is about $1 / 3 \mathrm{gm} \mathrm{cm}^{2}$ and the heat of the reflector must be sufficiently large to prevent the escape of a substantial fraction of fast neutrons. The basic idea of such a canalized reflector is the following: If we have an ordinary water reflector the efficiency of the reflector is low because the ratio of the diffusion length for slowing down and the diffusion length for thermal neutrons is large. In a canalized reflector as considered above where we have cylinders of about 2 cm in diameter free from water occupying two-thirds of the space, this ratio is considerably smaller and therefore a larger fraction of the thermal neutrons produced in the reflector will reach the uranium in the light water lattice and will be absorbed there. A gas like helium, or a solid like graphite or beryllium or bismuth may be used to fill the aluminum tubes widen reflector.
For the time being preference is being given to bismuth which on account of its lower thermal absorption cross section is preferable to lead. The slit (8) and (Bb) in Figs. $2 a$ and $2 b$ show the water inlet and outlet respectively from the aluminum tube (5).

Support of the Light Water Lattice.--The light water lattice has to be supported in such a manner as to permit the outflow of the water. One solution is shown in Figs. 3, 8 and 9 and a slightly different solution in Figs $3 C, 8$, and 9. In Fig. 3 (12, etc.) are parallel aluminum plates standing vertically forming the support 41 at the bottom of Fig. 3. A top view of these plates, section A, B of Fig. 8 is shown in Fig. 9. These vertical aluminum plates must be high enough to be able to support the weight of the uranium rod lattice.

A sightly different way of holding the aluminum tubes which form the lattice is shown in Fig. 30 which is an alternative construction to the construction shown in Fig. 3B. In the center of the composite unit the top part (10I) of the center portion of the support (4I) is of a different construction since this part of the support has to fit in with the construction of the heavy water core.

Heavy Water Core.--The heavy water cores which we may wish to consider fall into two classes: (a) the heavy water flask or mosaic core which is light water cooled and in which the light water freely communicates with the light water in the light water lattice, and (b) the inclosed heavy water core which may be cooled either by light water or heavy water circulation.

A mosaic type core is shown in Figs. 11, 12a, b and 12c. In this type of core the heavy water is contained in cylindrical aluminum flasks having spherical tops and bottoms. The pressure of the heavy water is kept higher within the heavy water flask than in the light water lattice throughout the whole length of the composite unit. This keeps the aluminum cylinders from collapsing and a wall thickness of

1 mm will be sufficient for a maximum pressure difference of 3 atmospheres for a flask radius of 3 cm . Such aluminum flasks are arranged as shown in Fig. ll in a hexagonal pattern. The spaces left free by the flasks communicate with the light water lattice and are therefore filled with light water. The problem for, which we must find a satisfactory solution is the following: The aluminum flask which is exposed to the pressure in the heavy water must have a circular shape in order to be able to withstand the pressure. The cross section exposed to the light water, however, must have a shape which permits to keep constant distances between neighboring elements. This cross section can for instance be a hexagon as shown in Fig. 11. One possible solution for this problem would be to fasten beryllium plate of a suitable shape on the outside of the aluminum flasks but before we move in this direction it would be desirable to have the result of corrosion experiments on the aluminum beryllium couple.

Another solution which is indicated in the drawings consists of having a thin, say $\frac{1}{2} \mathrm{~mm}$, aluminum sheet bent in the form of a hexagon and forming a cylindrical container which incloses the heavy water flask. The space between the two containers can be filled with a bismuth-lead alloy. The pressure in the space containing the bismuth lead alloy may be lower than the hydraulic pressure in the heavy water lattice and the thin aluminum wall of the hexagon is supported by the lead-bismuth alloy. Aluminum tubes containing uranium rods are arranged in the light water gap between the hexagon as shown in Fig. 12a and in greater detail in Fig. 13.

## Approximate Data.--The follawing data may serve for a quick

 orientation concerning the magnitudes which are involved. Fpr a standard but otherwise too small radius of 10 cm of the heavy water flasks and for a heavy water to uranium volume ratio of $1: 15$, uranium rods in the heavy water core would have about a $I \cdot \mathrm{~cm}$ diameter. The average thickness of the lead-bismuth alloy surrounding such a cylinder of 10 cm radius would be about .5 cm whereas the average thickness of the uranium (if it were uniformly smeared around the circle of 10 cm radius) would amount to .3 cm , i.e., the weight of the bismuth-lead alloy would be about equal to the weight of the uranium. For the eutectic alloy of bismuthlead the absorption of the alloy would amount to a loss in multiplication factor of about $1 \%$. This loss would be smaller if the alloy were richer in bismuth. The I mm thick aluminum wall of the heavy water flask of 10 cm radius has a volume corresponding to .3 of the volume of the uranium in the heavy water core. This may be compared with the familiar case of a uranium slug of cm diameter covered with 1 mm of aluminum where the ratio of the volume of aluminum to the volume of uranium is.2. It may be seen therefore that the absorption of neutrons in the aluminum flask is not too serious.Amount of Light Water in the Core.--A reasonable amount of light water in the core would be about $1 / 30$ of the volume of heavy water (3.3\%) or $1 / 2$ of the volume of uranium. Since a cross section of one slug of $l \mathrm{~cm}$ of diameter is $14 \mathrm{~cm}^{2}$, this would correspond to a ring of water around the uranium rod of about 2 mm thickness. This would leave 4 mm between adjacent uranium rods which is more than is needed. On the
other hand, in view of inaccuracies of manufacture, it appears safer not to count on less than $3.3 \%$ of light water in the heavy water core and the amount may have to be increased to perhaps $5 \%$ if tolerance required should prove to be difficult to meet.

