BALANCE OF EMISSION AND ABSORPTION OF NEUTRONS BY URANIUM

A direct comparison of the number of fast neutrons emitted by uranium with the number of thermal neutrons absorbed by uranium may be obtained from different types of experiments carried out by Halban, Joliot and Kowarski,[11] Anderson, Fermi and Szilard,[12] and Halban, Joliot, Kowarski and Perrin.[13] These experiments show that on the average more than one fast neutron is emitted by uranium for one thermal neutron absorbed by uranium.

Halban, Joliot and Kowarski[8] reported that 3.5 ± 0.7 fast neutrons are emitted per fission. Anderson, Fermi and Szilard[9] reported that about $\mu = 1.5$ fast neutrons are emitted on the average for one thermal neutron absorbed by uranium.

Halban, Joliot, Kowarski and Perrin[13] carried out an experiment of the following type: a sphere of radius r is filled with a homogeneous mixture of uranium and water which contains n atoms of hydrogen per atom of uranium. This sphere is immersed in a large water tank and a photo neutron source is placed in the center of the sphere. The density ρ of the thermal neutrons is measured along a radius and the integrals of

$$I_{int} = \int_0^{r_0} r^2 \rho \, dr \quad ; \quad I_{ext} = \int_0^{\infty} r^2 \rho \, dr$$

are determined, giving a measure of the number of slow neutrons which are present within the mixture inside the sphere and in the water outside the sphere. In another experiment the integral

$$I_0 = \int_0^{\infty} r^2 \rho \, dr$$

is determined for the same neutron source in pure water.

From the values of these three integrals which these authors determined for n = 1, 2 and 3 and r = 25 cm. they concluded that on the average eight neutrons are emitted from the uranium within the sphere for one photo neutron which is slowed down to thermal energies and causes a fission process in uranium. This was interpreted to show that a considerable number of secondary and tertiary neutrons were generated in this experiment.

Since the value of the fission cross-section is not well known and rather difficult to determine, it seems necessary to avoid expressing results in terms of this cross-section and to use the magnitude μ rather than the number of neutrons emitted per fission. An accurate knowledge of the value of μ is of greatest importance from the point

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[redacted]

of view of estimating the chances and finding the optimum conditions for a chain reaction. In the following we shall therefore indicate in what way an accurate value for ρ can be obtained, from an experiment of the type performed by Halban, Joliot, Kowarski and Perrin:

F_{int}, the average number of neutrons which reach the thermal region inside the sphere within the uranium water mixture for one neutron emitted by the source in the center of the sphere, is given by

$$F_{int} = \frac{\tau_{int}/\tau_0}{\tau_{int}/\tau_0}$$

where

$$37) \quad \frac{\tau_{int}}{\tau_0} = \frac{n\sigma_t(H)}{s\{\sigma_a(U) + n\sigma_t(H)\}}$$

is the ratio of the life-time of the thermal neutrons in the mixture inside the sphere and in pure water. [redacted]

Of the neutrons which are slowed down to the resonance region inside the sphere a fraction p is absorbed at resonance and a fraction $(1 - p)$ reaches the thermal region. F^*_{int} the number of the neutrons reaching the resonance region per second inside the sphere is therefore given by

$$(1-p) F^*_{int} = F_{int}$$

F^*_{ext} the number of neutrons reaching the resonance region outside the sphere in the water is the same as F_{ext} the number of neutrons reaching the thermal region outside the sphere in the water and we have, therefore

$$F^*_{ext} = F_{ext} = \frac{I_{ext}}{I_0}$$

P, the total number of fast neutrons emitted by the uranium in the mixture for one neutron emitted by the neutron source in the center of the sphere is equal to the total number of the resonance neutrons produced minus 1, the neutron emitted by the source in the center. Therefore, we have

$$P = F^*_{int} + F^*_{ext} - 1 = \frac{F_{int}}{1-p} + F_{ext} - 1$$

The total number of fast neutrons produced by the uranium divided by the number of thermal neutrons produced within the sphere in the mixture is

$$\frac{P}{F_{int}} = \frac{1}{1-p} + \frac{F_{ext} - 1}{F_{int}}$$

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And finally since the uranium absorbs of the neutrons which reach the thermal region within the sphere the fraction

$$\frac{\sigma_a(U)}{\sigma_a(U) + n\sigma_c(H)}$$

we have

$$\mu = \frac{P/F_{int}}{\left(\frac{\sigma_a(U)}{\sigma_a(U) + n\sigma_c(H)}\right)}$$

or

$$\mu = \frac{\sigma_a(U) + n\sigma_c(H)}{\sigma_a(U)} \left\{ \frac{1}{1-p} + \frac{\bar{\tau}_{int}}{\tau_0} \frac{I_{int}/I_0 - 1}{I_{int}/I_0} \right\}$$

Introducing the value of $\bar{\tau}_{int}/\tau_0$ from No. we obtain

$$\mu = \frac{1}{1-p} + \frac{n\sigma_c(H)}{\sigma_a(U)} \left\{ \frac{1}{1-p} + \frac{1}{s} \frac{I_{int}/I_0 - 1}{I_{int}/I_0} \right\} \quad 38$$

From this equation we see that we can determine μ by determining the value of p.

A considerable fraction of the fast neutrons emitted by uranium may of course escape from within the sphere but the value of this fraction need not be determined for our purpose. On the other hand, the formula given for μ holds with good approximation only if the sphere is sufficiently large to permit us to neglect transition phenomena in the equilibrium between resonance neutrons and thermal neutrons near the surface of the sphere. Since the uranium inside the sphere absorbs resonance neutrons the density of the neutrons which have an energy below the upper end of the resonance region of uranium and above the thermal region is smaller A This has the consequence that neutrons of this category will diffuse from the water across the sphere into the inside of the sphere. If the radius of the sphere were not sufficiently large so that this phenomenon could not be neglected the expression $F_{int}/1-p$ would then give a too large value for the number of resonance neutrons produced inside the sphere. Correspondingly the expression No. would give a too large value for μ . But with the proviso that a sufficiently large sphere is used we shall obtain an accurate value for μ if only the value of p is measured accurately.

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The value of p has so far not been measured for hydrogen concentrations as low as n = 3 but we can give a lower limit for p for this concentration as we are able to extrapolate by means of theoretical considerations from values of p which have been measured for higher hydrogene concentrations i.e. n = 30.

In order to do this we shall make use of formulae relating to homogeneous mixtures of hydrogen and uranium which will be published shortly by G. Placzek. A general treatment of the process of slowing down of neutrons in water has been given by E. Fermi.* In 1936 In a uranium oxide - water mixture p, the fraction of the neutrons which is absorbed by uranium between the energies E_2 and E, is given by

39 $$\ln(1-p) = -\int_{E_1}^{E_2} \frac{f(E)}{E}\, dE \quad ; \quad E_1 > 1\,Volt$$

where

40 $$f(E) = \frac{1}{1 + \frac{nH}{g(E)}}$$

$g(E)$ is the cross- section of uranium for radiative capture and H is the scattering cross- section of hydrogen for the resonance neutrons of uranium. ($H \cong 17 \times 10^{-24}\ cm^2$) If the hydrogen concentration is high enough so that the energy region in which there is an appreciable absorption by uranium is small compared with the resonance energy E_0 one may write

41 $$\ln(1-p) = -\frac{1}{E_0}\int_{-\infty}^{\infty} f(E)\, dE$$

for a single absorption line of the form

42 $$g(E) = \frac{g_0}{1 + \left(\frac{E-E_0}{\Gamma}\right)^2} \quad ; \quad g_0 = g(E_0)$$

giving*

43 $$\ln(1-p) = -\frac{\pi\Gamma}{E_0} \frac{1}{\sqrt{\frac{nH}{g_0}\left(1 + \frac{nH}{g_0}\right)}}$$

For $n \leq 30$ we have $\frac{nH}{g_0} \ll 1$ and No 44 gives**

44 $$\ln(1-p) = -\frac{\pi\Gamma}{E_0}\sqrt{\frac{g_0}{nH}}$$

From this we see that for such hydrogen concentrations for which No 44 holds we have for two different hydrogen concentrations n and n_1,

45 $$\ln(1-p) = \sqrt{\frac{n_1}{n}}\ln(1-p_1)$$

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* Private communication by G. Placzek June, 1939

** Private communication independently by G. Placzek and E. Fermi June, 1939

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This relationship No. 45 does no longer hold for hydrogen concentrations which are as low as n = 3. But we may write in the case of uranium

(46) $$\ln(1-p) = \int_{0.2E_0}^{1.8E_0} \frac{f(E)}{E}\, dE$$

and it is then easy to see that for an absorption line of the form given by No. 42 which is symmetrical in $(E - E_0)$ or even more so for a similar line which deviates from symmetry in the sense that the absorption is larger for smaller energies (as it is the case for a line which obeys the Breit-Wigner formula) we have

(47) $$\ln(1-p) < \sqrt{\frac{n'}{n}}\, \ln(1-p') \quad \text{if } n < n'$$

The value of p has been measured by Halban, Joliot, Kowarski and Perrin for n = 30 and was found to be $p = 0.2 \pm 0.02$; using equation No. 47 we find from this

$$p_3 > 0.5 \pm 0.04 \quad \text{for } n = 3$$

According to No. 38 μ increases with increasing values of p and in the circumstances we should obtain a conservative value for μ by using the value of p = 0.5.

Using this value and the values

$$\frac{I_{int}}{I_0} = 0.72 \quad ; \quad \frac{I_{ext}}{I_0} = 0.45$$

reported by Halban, Joliot, Kowarski and Perrin for a sphere of 25 cm. radius filled with a uranium oxide--water mixture for which they had n = 3 and s = 0.42 gm./c.c. we find from No. 38

(48) $$\mu = 2 + 0.2\, \frac{3\sigma_c(C)}{\sigma_a(U)}$$

We see that the value obtained for μ if calculated from this equation is scarcely affected by the wide limits of error of the present experimental values of $\sigma_a(U)$. By attributing the value of $\frac{1}{4}$ or 1/8 to $3\sigma_c(H)/\sigma_a(U)$ we obtain $\mu = 2.05$ or $\mu = 2.02$ respectively.

In these circumstances we shall use for the present as a presumably conservative value and as the best value at the present available:

$$\mu = 2$$