In the following, we shall consider the balance of neutron emission and absorption in a system composed of Uranium and a light element, the latter serving the purpose of slowing down the neutrons. The question in which we are interested is whether it is possible under certain conditions to maintain a neuclear chain reaction in such a system. We shall ~~expect~~ mark of a chain reaction only in connection with ~~the~~ a process in which more than one neutron is is produced for every neutron absorbed within the system. If less ~~th~~ than one neutron is produced it will obviously still be true that a neutron emission in the fission of Uranium will cause further fission and thereby lead to secondary neutrons which in turn will again cause further fission in tertiary neutrons, but this ~~we will not call a chain reaction.~~ A chain reaction may be called convergent if more neutrons escape across the boundary of the system than are emitted by the Uranium which it contains, and otherwise it may be called divergent. Whether or not a chain reaction takes place depends essentially on the composition of the system. Whether the chain reaction is convergent or divergent depends further on the shape and size of the space which is filled by the system.

About 1.5 neutrons have been found to be emitted on the average for every thermal neutron absorbed by Uranium. From certain quantities ~~which~~ measured by Halban, Joliot, Kovarski and Perrin I derive a value of $\nu = 2$ which is probably more accurate.

If $q$ is a fraction of neutrons which reach the thermal region and the absorption in the thermal region by Uranium, then, on the average $\nu q$ neutrons are produced in the system for every neutron absorbed in the system and the chain reaction is

In a system which is composed of uranium and ~~either~~ hydrogen ~~or carbon~~, the neutrons emitted from uranium are fast and are slowed down by the hydrogen ~~or carbon~~ until they finally reach thermal energies. A fraction of the neutrons ~~may~~ will be absorbed before reaching thermal energies, while the neutrons pass through an energy region $\delta$ E, at which the uranium has a resonance absorption. Those neutrons which reach thermal energies are then either absorbed by uranium or captured by hydrogen ~~or carbon~~ respectively. If $q$ is the fraction of the fast neutrons emitted by uranium which are absorbed in the thermal region by the uranium alone, and if $\mu$ fast neutrons are emitted by uranium for one thermal neutron absorbed by uranium, then the condition for the possibility of a chain reaction is

$$\mu q > 1$$

If $\mu q$ is only slightly larger than 1, then it may take a very large mass of uranium to approach a divergent chain reaction.

According to ~~Fermix~~ Anderson, Fermi and Szilard, about 1.5 neutrons are emitted for every thermal neutron absorbed by uranium. I find that I can derive a more accurate value ~~for~~ $\mu = 2$ from certain quantities measured by Halban, Joliot, Kovarsky and Perrin, and thus obtain

$$\mu = 2$$

Using this value, ~~I find~~ by taking into ~~the~~ account the known cross-sections of hydrogen and uranium, that in homogenous mixtures of water and uranium $\mu q$ will certainly not exceed considerably 1. Whether it can reach this value is not even certain. This means that even if a chain reaction should be possible in homogenous mixtures of uranium and water, it would require very large masses of uranium.

Insertion to page 14

This holds under the well founded assumption that the Uranium resonance rise between the Indium resonance and the Iodine resonance. This method appears promising insofar as the Iodine activities need not be measured in the same units as the Indium activities and accordingly the ranges of the B rays can be left out of consideration. An experiment of this type is now being performed for n 3.

In a chem. mixt. of C and U appears doubtful.

It appears that in order to make a chain reaction possible Carbon is a very much better element for slowing down the neutrons than Hydrogen.

Whether a chain reaction can be maintained in Carbon it takes about 6 collisions to reduce the energy of a neutron by a factor $1/e$ therefore, a neutron which while being slowed down has entered the resonance region, will stay within this region for a comparatively long time. It has, therefore, a large probability to be captured by Uranium unless very low Uranium concentrations are used. The captured cross-section of Carbon $\sigma_a(C)$ is small ($\sigma_a(C) < 0.01$).

~~But~~ at such very low Uranium concentrations ~~is must be considered~~ (as must have to be used) the capture of the Carbon ~~might be considered and may~~ (might) be sufficient to prevent a chain reaction from taking place. ~~in a homogeneous mixture of Uranium and Carbon.~~ At present, we have only the above stated upper limit for the Carbon cross-section.

A new method for measuring very small captured cross -sections has recently been devised and is now being applied to Graphite in the hope of obtaining a beeter value for .

It appears that in order to make a chain reaction possible, carbon is a very much better element for slowing down the neutrons than hydrogen, Though it takes about 6 collisions in carbon to reduce the neutron energy by a factor $e$. It is true that a fast neutron produced in carbon will diffuse further away from its point of origin before it is slowed down to energies of the order of volts. Yet this distance in graphite is only of the order of magnitude of 50 cm. On the other hand, since the capture cross-section of carbon for thermal neutrons is small, uranium-carbon mixtures could be used, which contain only a small amount of uranium. This has the double advantage that the distances ~~required~~, which a neutron has to travel in the mixture in order to reach thermal energies, is not materially increased by the presence of uranium, and that the amount of uranium required ~~is~~ may be a small ~~fraction of the amount of carbon used.~~ (than .01 The capture of neutron of carbon is another)

Since only an upper limit of the capture cross-section of carbon is at present known, it is not possible to state yet whether a chain reaction is possible in homogenous mixtures of uranium and carbon. ~~Though this cross-section is certainly very small, it may not be small enough. Since~~ However if a heterogeneous mixture of a single collision with a carbon atom slows the neutron only very little, a neutron which has entered the resonance region $\delta$ E stays within that region for a comparatively long time, and has therefore a large probability to be captured by uranium atoms. This compels to use mixtures which have a very low concentration of uranium, and in the circumstances even a small cross-section of carbon might prove fatal, and the carbon + might prove to be too large after all

Insert in Page 1 C

Uramium has a strong resonance absorption line at energies of the order of magnitude of 10 volts. If we have to deal with an absorption line having a maximum at some energies $E = E_0$ in this energy region and if this line obeys the B - W formula then the following will hold: At thermal energies the absorption will follow the 1/v law which will then go through a minimum at $0.2\,E_0$ after which the absorption rises until ~~at energies it is~~ reaches maximum at $E_0$ and ~~follows~~ falls off again reaching a very small value for $E_2 = 2E_0$

We shall ~~consider~~ call the energy interval $\Delta E = E_1 - E$ the resonance region and for a line obeying the B - W formula we have

$$\frac{E_2}{E_1} = 10$$

In the following we shall refer to the energy region below ~~points~~ 0.2E as the thermal region. ~~Whenever~~ We shall ~~consider~~ deal with bodies composed of Uranium embedded in Carbon ~~we~~ and shall assume that every neutron which reaches the surface of a Uranium body while it is in the resonance region as defined above is absorbed by the Uranium by radiative capture without causing neutron emission.

A new method for measuring very small capture cross-section has been deviced and is now being applied to graphite in the hope of obtaining a reliable value for the carbon capture cross-section. In the present paper I wish to show that if non-homogenous mixtures of carbon and uranium are used, carbon is certainly much more favorable than hydrogen, and that in the case of carbon it is possible to make use of certain tricks which greatly reduced the fraction of the neutrons captured by uranium at resonance. One reason for this difference in favor of carbon is the fact that the scattering cross-section of carbon is about the same for thermal neutrons as for resonance neutrons whereas the scattering cross-section of hydrogen varies by a factor of about 3.

Let us first consider a plane sheet of uranium embedded in an infinite space filled with carbon, and assume that the uranium is black both for thermal neutrons and for resonance neutrons, i.e. every thermal neutron and every neutron which has an energy within an energy interval which contains the resonance energy $E_0$, is absorbed by uranium if it reaches the uranium layer at all. We are interested in finding the ratio of the number of thermal neutrons and the number of resonance neutrons which are absorbed by the uranium, and then try to find other arrangements for which this ratio is higher.

The diffusion of thermal neutrons and resonance neutrons towards the uranium layer can both be treated with good approximation in exactly the same manner if the following is born in mind:

A thermal neutron produced in the carbon will disappear in pure carbon after a certain number of elastic collisions with carbon atoms by being captured. The probability that it survives h elastic collisions

is given by $1 - e^{-k \frac{\sigma_c(C)}{\sigma_{sc}(C)}} = 1 - e^{-\frac{k}{\sigma_{sc}(C)/\sigma_c(C)}}$

the expression $A = \lambda_{sc} \sqrt{\frac{\sigma_{sc}(C)}{3\sigma_c(C)}}$

has the dimension of a length, and we shall refer to it for the sake of brevity as the "range" of thermal neutrons in carbon.

Similarly, faster neutrons in pure carbon disappear out of a given energy interval $\Delta E = E_2 - E_1$, because they are slowed down by elastic collisions with carbon atoms. The probability that a neutron survives k collisions within the energy region is approximately given by

$$1 - e^{-\frac{k}{k_0}}$$

Where $k_0$ $k_0 = 6.5 \ln \frac{E_2}{E_1} \left( = \frac{\ln \frac{E_2}{E_1}}{\ln\left(1 - \frac{4M}{(1+M)^2}\right)} \right)$

Again the expression $B = \lambda^*_{sc} \sqrt{\frac{k_0}{3}}$

which has the dimension of a length will be called the "range" of resonance neutrons in carbon.

By treating the problem as a diffusion phenomenon we find in the case of a plane sheet of uranium embedded in an infinite amount of carbon

$$\varepsilon_0(\text{Plane}) = \frac{A}{B} = \frac{\lambda_{sc}}{\lambda^*_{sc}} \sqrt{\frac{\sigma_{sc}(C)}{\sigma_c(C)} k_0}$$

This value holds under the assumption that everywhere the same number Q of resonance neutrons and thermal neutrons is produced per cc. and sec. and will be corrected later to take into account deviations (value of $\varepsilon_0$ from this assumption

Let us now compare this with the case of a uranium sphere of radius R embedded in an infinite amount of carbon, which we wish to consider in greater detail. Again we assume that the same number of thermal neutrons Q is produced everywhere in the carbon. In reality this is not so, because, owing to the absorption of resonance neutrons by the sphere,

a smaller number of thermal neutrons is produced in the neighborhood of the sphere. A correction factor will be calculated later and applied to the final result.

If R is large compared to the mean free path for scattering of thermal neutrons in carbon, the density of the thermal neutrons in carbon can be calculated as a function of distance r from the center of the sphere by treating the problem as a diffusion phenomenon, and we can calculate $\rho(r)$ from the equation

$$D(C)\frac{d^2}{dr^2}(r\rho) - S(C)\,r\rho + Q\,r = 0 \qquad D(C) = \frac{W\lambda_{sc}(C)}{3}$$

and for a "black" sphere, for which $\rho(R) = 0$

$$S(C) = \frac{W}{\lambda_{sc}(C)}\,\frac{\sigma_c(C)}{\sigma_{sc}(C)}$$

$$\rho = \frac{Q}{S(C)}\left(1 - \frac{R}{r}\,e^{-\frac{(r-R)}{A}}\right)$$

For $R = \infty$ we have $\rho = \rho_0 =$

Accordingly, the number of thermal neutrons which reaches the uranium sphere per second by diffusion

$$\gamma_0^{th} = \alpha(C)\,4\pi R^2\,\rho'(R) = 4\pi Q\,RA^2\left\{1 + R/A\right\}$$

Quite similarly, the number of resonance neutrons which reaches the "black" uranium sphere is given by

$$\gamma_0^{res.} = 4\pi Q\left\{BR^2 + RB^2\right\} RB^2\left\{1 + \frac{R}{B}\right\}$$

The ratio $\varepsilon$ is given by

$$\varepsilon(R) = \frac{U}{U^*} = \frac{A^2}{B^2}\,\frac{1 + R/A}{1 + R/B}$$

By comparing this expression with the expression obtained in the case of the plane uranium sheet, we see that for small values of R the case of the sphere is more favorable by a factor of $\frac{A}{B}$. This factor might have a value of as much as , Though in practice we cannot have R smaller than $\frac{1}{2}\frac{A^2}{B^2}$, and therefore a factor of perhaps 2 again is lost, still the case of the sphere remains very much more favorable than the plane sheet.

In order to calculate $\varepsilon$ for conditions which can be realized in practice, we have now to consider the case of a uranium sphere which is not "black" for thermal neutrons, but which can be considered "black" for resonance neutrons. Only a fraction $\varphi$ of the thermal neutrons which reach the sphere will be ultimately absorbed by it, and the ratio $\varepsilon(R)$ of the thermal neutrons and the resonance neutrons absorbed by the sphere is therefore

$$\varepsilon(R) = \varphi \varepsilon_0(R)$$

In order to calculate $\varphi$ we take into account that inside the uranium sphere the thermal neutron density obeys the equation

$$D(u)\frac{d^2(r\rho)}{dr^2} - S(u)(r\rho) = 0 \qquad D(u) = \frac{w\lambda_{sc}(u)}{3}$$

inside the uranium sphere we have $\rho(r) = \frac{c}{r}\left(e^{r/u} - e^{-r/u}\right)$ $\qquad S(u) = \frac{w}{\lambda_{sc}(u)}\left(\sigma_c(u)+\sigma_p(u)\right)$

where $u$ is the range of thermal n in uranium $u = \lambda_{sc}(u)\sqrt{\frac{\sigma_c(u)}{\sigma_c(u)+\sigma_p(u)}}$ $\quad 3\varepsilon$

From equations and we find that the number of thermal neutrons diffusing into the sphere

$$\gamma^{th} = 4\pi Q R A^2 \qquad \sqrt{\frac{3(\sigma_c+\sigma_p)}{\sigma_{sc}(u)}}\,R\left\{\frac{}{}\right. \qquad \gamma^{res} = \gamma_0^{res}$$

for resonance neutrons we have

and by comparing this with expression No. we find for $\varphi$

$$\varphi = \frac{N_0 \gamma^{th}}{N_0 \gamma_0}$$

$$\varepsilon = \varphi \varepsilon_0 =$$

All the calculated values of so far hold only if $R \gg \lambda_{sc}(u)$ otherwise the problem cannot be treated as a diffusion problem, but the values can of course be derived by other methods.

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If one finds for a sphere which is black for thermal neutrons as well as for resonance neutrons

and for a sphere which is black for resonance neutrons only, and faintly scattering and absorbing for thermal neutrons

and by comparing this with expression No. we find for $\varphi$

$$\varphi = \frac{y^u}{y_o^u}$$

$$\varphi = \frac{\lambda_{sc}(u)/u \left\{ \frac{e^{R/u} + e^{-R/u}}{e^{R/u} - e^{-R/u}} - \frac{u}{R} \right\}}{\frac{\lambda_{sc}(C)}{R}\left(1 + \frac{R}{A}\right) + \frac{\lambda_{sc}(u)}{u} \left\{ \frac{e^{R/u} + e^{-R/u}}{e^{R/u} - e^{-R/u}} - \frac{u}{R} \right\}}$$

This expression would also hold if for inst. Uranium oxide is used instead of Uranium metal

All the calculated values so far hold only if $R > \lambda_{sc}$ otherwise the problem cannot be treated as a diffusion problem, but the values can of course be derived by other methods.

If $R \ll \lambda_{sc}(C)$ one finds for instance for a sphere which is "black" for thermal neutrons as well as for resonance neutrons

$$\varepsilon = \frac{A^2}{B^2}$$

and for a sphere which is black for resonance neutrons only, and and faintly scattering and absorbing for thermal neutrons,

$$\varepsilon = \frac{A^2}{B^2} \frac{\lambda_{sc}^*(C)}{\lambda_{sc}(C)} \frac{4}{9} \frac{R}{u} \frac{\lambda_{sc}(u)}{u}$$

where $R \ll u$

If we put in the "range" of thermal neutrons in Uranoxide in place of $u$ as defined by No. "

In order to calculate $\varepsilon$ for conditions which can be realized ~~in practice~~, we have now to consider the case of a Uranium sphere which is not "black" for thermal neutrons, but which can be considered "black" for resonance neutrons. Only a fraction $\varphi$ of the thermal neutrons which reach the sphere will be ultimately absorbed by it, and the ratio $\varepsilon$ of the thermal neutrons and the resonance neutrons absorbed by the sphere is therefore

$$\varepsilon = \varphi \varepsilon_0$$
$$\varphi < 1$$

In order to calculate $\varphi$ we take into account that inside the Uranium sphere the thermal neutron density $\rho$ obeys the equation

$$D(u)\frac{d^2}{dr^2}(r\rho) - \delta(u) r\rho = 0 \qquad D(u) = \frac{w\lambda_{sd}(u)}{3}$$

and inside the Uranium sphere we have $\rho(r) = \frac{C}{r}(e^{r/u} - e^{-r/u})$ $\qquad \delta(u) = \frac{w}{\lambda_{sc}}\frac{\{\sigma_c(u)+\sigma_p(u)\}}{\sigma_{sc}(u)}$

where $u$ is the range of thermal neutrons in the substance of the Uranium sphere

From equations $\gamma^{th}$ and we find that the number of thermal neutrons diffusing into the sphere per sec

$$\gamma = 4\pi Q A^2 (1+\tfrac{\gamma}{A}) \frac{\frac{\lambda_{sd}(u)}{u}\left\{\frac{e^{R/u}+e^{-R/u}}{e^{R/u}-e^{-R/u}} - \frac{u}{R}\right\}}{\frac{\lambda_{sc}(C)}{R}(1+\tfrac{R}{A}) + \frac{\lambda_{sc}(u)}{u}\left\{\frac{e^{R/u}+e^{-R/u}}{e^{R/u}-e^{-R/u}} - \frac{u}{R}\right\}}$$

The number of res. neutrons $\gamma^{res}$ diffusing into the sphere per sec is the same as before

$$\gamma^{res} = \gamma_0^{res}$$

$$u = \sqrt{\frac{D}{\delta}} = \lambda_{sc}(u)\sqrt{\frac{\sigma_{sc}}{(\sigma_c(u)+\sigma_p(u))3}}$$

Lattice (1)

If we have now a lattice of a large number of Uranium spheres embedded in an infinite mass of Carbon and want to calculate the ratio of the number of thermal neutrons and resonance neutrons absorbed by the Uranium spheres we shall again assume for the time being that everywhere in the Carbon the same number Q of neutrons enter the resonance region and the thermal region per c.c. a second.

Equation number shows that a Uranium sphere does not affect the thermal neutrons' density appreciably at distances which are large compared to R. At a distance of 2R from the center ~~to~~ ~~is~~ $\rho$ is equal to ~~of another~~ half of $\rho_0$ ~~is not~~, the value ~~but is~~ of the thermal neutron density at infinity. For this reason the number of thermal neutrons $J^{th}$ absorbed by one Uranium sphere within the lattice is only insofar affected by the other Uranium spheres as the presence of these other spheres reduce the average value of the thermal neutron density by some factor $\alpha$ which is smaller than 1. Accordingly the number of thermal neutrons absorbed by one sphere within the lattice $J^{th}$ (lattice) is by the factor $\alpha$ smaller than the number of thermal neutrons which an isolated sphere absorbs under otherwise identical circumstances.

The number of resonance neutrons on the other hand which one Uranium sphere absorbs within the lattice ($J^{res}$ lattice) is the same as the number of resonance neutrons which an isolated Uranium sphere would absorb.

$$J^{th} = \alpha J^{th}$$

Lattice

We have, therefore,

$$\frac{J^{th}}{J^{res}} = \frac{U^{th}(lattice)}{U^{res}(lattice)} = \varepsilon\alpha = \rho\varepsilon_0\alpha$$

The fraction of the neutrons produced which is absorbed by the Carbon is proportionate to the average neutron density $\bar{\rho}$ Since in the absence of Uranium in the Carbon we have $\bar{\rho} = \rho_0$ and since in the absence of Uranium all the neutrons are absorbed by the Carbon the fraction of the neutrons which are absorbed by the Carbon which contains a lattice of Uranium spheres is given by

$1 - \alpha$

Correspondingly the fraction of the neutrons which are absorbed by the lattice of Uranium spheres by either as resonance neutrons or as thermal neutrons is given by

$$(1-\alpha)\ (1-\varepsilon)$$

From this it follows that Q the fractio of all the neutrons which are absorbed by the Uranium spheres in the thermal region alone is given by

$$q = \frac{J^{th}}{J^{th} \times J^{res}}(1-\alpha)$$

$$q = \frac{U^{th}(lattice)}{U^{th}(lattice) + U^{res}(lattice)} \times (1-\alpha)$$

or

$$q = \frac{\rho\varepsilon_0\alpha}{1+\rho\varepsilon_0\alpha}(1-\alpha)$$

lattice

Now this expression has its maximum value for $\alpha = \alpha_m$

$$\alpha_m = \frac{-1 + \sqrt{1+\varepsilon}}{\varepsilon}$$

and for the maximum value of q we have $q = q_m$

$$q_m = 1 - 2\alpha_m$$

~~and~~

$$q_m = 1 - 2\frac{-1 + \sqrt{1+\varepsilon}}{\varepsilon}$$

and ~~it is~~

$$\varepsilon = \frac{4 q_m}{(1-q_m)^2}$$

CORRECTION

In calculating the value of q we have so far neglected the effect of absorption of resonance neutrons by the Uranium sphere on the production on thermal neutrons in the neighborhood of the Uranium sphere. ~~Due to this effect~~ this effect reduces the production of thermal neutrons ~~will be smaller than Q in the neighborhood of the Uranium spheres.~~ never the value below the value and thereby reduces the number of thermal ~~Accordingly, we have to write for the correct value of the ratio of the thermal and the resonance neutrons absorbed by a Uranium sphere within the lattice of Uranium spheres~~ neutrons which a sphere of the lattice absorbs from $J_{th}$ to a value $J_{th}$ (corr) $J_{th}$ (corr) < $J_{th}$

In order to find an upper limit for the ~~the resonance neutrons~~ with how much the thermal neutron absorption of a Urnium sphere in the lattice would be increased if ~~it~~ (this sphere stopped to absorb the resonance neutrons which reach its surface. Such a resonance neutron which the Uranium sphere failed to absorb as

Correction



resonance neutron would go on diffusing in the Carbon and would finally become thermal. A fraction of such neutrons would then be absorbed as a thermal neutron by the same Uranium sphere which failed to absorb it as a resonance neutron. But this fraction can certainly not exceed the value of $\varphi$ which we have defined further above. We have, therfore, the corrected number $J^{th(corr)}$ of thermal neutrons absorbed by a Uranium sphere within a lattice

$$J^{th} < J^{th(corr)} < J^{th} - \varphi J^{res}$$

~~and we have further~~

~~by making use of the equation No.~~

and accordingly for the corresponding corrected value of $\varphi$ we have

$$\varphi(corr) = \frac{J^{th(corr)}}{J^{th(corr)} + J^{res}} (1-\alpha)$$

This gives

$$\varphi_{corr} = \varphi(1-\eta)$$

with $$0 < \eta < \frac{\varphi}{\varepsilon_0 \alpha (1 + \varphi \varepsilon_0 \alpha)}$$

for $\sigma_c(C) = 0.01 \times 10^{-24}$ we have for instance $\eta \leq \alpha\eta < \frac{\varphi}{\frac{2q_m}{1-q_m} + \left(\frac{2q_m}{1-q_m}\right)^2}$

L 1

## SPACING OF THE LATTICE

It is necessary to estimate how widely the Uranium spheres are spaced in the lattice in order to see whether the assumptions under which we derive equation No. are fulfilled. In deriving ~~this~~ equation we have assumed that everywhere in the Carbon the same number of neutrons Q are slowed down to enter the resonance region. This assumption ~~is already correct as long as~~ the distance of the Uranium spheres in the lattice ~~does not~~ become large compared with the distance to which a fast neutron diffuses away from its point of origin before it is slowed down to a few volts. ~~In principle~~, our formulae imply that this would happen for ~~very~~ small values of $\sigma_c(C)$. However, the values of $\sigma_c(C)$ for which this would happen is much smaller than the value of $0.01 \times 10^{-24}$ which is required to make $\mu q > 1$. The value for q given by equation No. can therefore be used for studying the prospects of a chain reaction.

In order to estimate the spacing of the Uranium spheres in the lattice we estimate the volume ~~of the mixture~~ per 1 Uranium sphere ~~of~~ the lattice.

In the lattice of Uranium spheres from the ~~per~~ second within the volume to resonance energies, the Carbon absorbs neutrons

and ~~the~~ Uranium sphere in the lattice absorbs so that we have

$$\mathcal{J}^{th} + \mathcal{J}^{*} = (1-\alpha)\, Q\, V$$

neutrons

~~on the other hand~~ (according to No and No) a single Uranium sphere within the lattice ~~since it is~~ On the other hand we can write obviously absorbs

$$\mathcal{J}^{th} + \mathcal{J}^{*} = \alpha \varphi \mathcal{J}_0 + \mathcal{J}^{*} = \alpha \varphi \mathcal{J}_0 \frac{\mathcal{J}\varepsilon_0 \alpha + 1}{\varphi \varepsilon_0 \alpha} =$$

~~thus we have~~ $= \varphi \mathcal{J}_0 \frac{\alpha(1-\alpha)}{q}$

~~it is~~

$$\varphi \mathcal{J}_0 \frac{\alpha}{q} = Q V$$

and putting in from eqn V the value of $\mathcal{J}_0$

$$V = 4\pi \varphi \frac{\alpha}{q} A^2 R (1 + R/A)$$

~~so that we have~~

~~and further~~

if q has its maximum value $q_m$:

$$\frac{\alpha_m}{q_m} = \frac{1 - q_m}{2 q_m}$$

and it is

L 3.

$$V = 4\pi \sqrt{\frac{1-\varphi_m}{2\varphi_m}}\, A^2 R\left(1 + R/A\right)$$

For large values of $A$ and $\varepsilon$ we can write

$$\frac{1-\varphi_m}{2\varphi_m} \cong \frac{1}{\sqrt{\varphi \varepsilon_0}}$$

so that we have for $\varepsilon >> 1$

$$V \cong 4\pi \sqrt{\varphi\left(1 + A/R\right)\left(1 + B/R\right)}\, A B R$$

$$\left( \sqrt{\varphi\left(1+A/B\right)\left(1+B/R\right)} \cong 2 \right)$$

2. B for $\sigma_c(C)$ 0.0025
i.e. $\varepsilon = 116$.)

Ratio of Volumes

$$\frac{V}{\frac{4\pi R^3}{3}} = 3\sqrt{\frac{1-\varphi_m}{2\varphi_m}}\,\frac{A^2}{R^2}\left(1+\frac{A}{R}\right)$$

$$\frac{V}{\frac{4\pi R^3}{3}} \cong 3\sqrt{\frac{AB}{R^2}}$$

for large $\varepsilon$

The mean distance $\bar{L}$ between neighbouring $A$ spheres is

$$\bar{L} = x\sqrt[3]{V} \cong$$

$$\sqrt[2]{\quad}$$

Appendix 7

# STABILIZING THE CHAIN REACTION

If a chain reaction could be maintained in a homogeneous mixture of water and Uranium or Carbon and Uranium it would have a certain natural stability in the sense that with rising temperature there would be a decrease in the neutron production. The reason for this is the fact that the absorption of both Uranium and Hydrogen obey 1/v law in the thermal region and thus the range of thermal neutrons in the mixture is larger at higher temperatures. Accordingly, at higher temperatures a larger fraction of the thermal neutrons will escape accross the boundary of the mixture without having reacted with the Uranium in the mixture.

A system on the other hand in which "black" Uranium bodies are embedded in Carbon has a natural thermal instability. This is due to the fact that with rising temperature the captured cross-section of the Carbon decreases whereas the "blackness" of the Uranium spheres remains unchanged. Accordingly, a larger fraction of the thermal neutrons is absorbed by Uranium leading to an increase in q.

It is, however, quite easy artificially to stabilize the chain reaction by slowly shifting the position of absorbing bodies within the system in such a way as to reduce the average value of q whenever the density of the neutron radiation emanating from the chain reaction increases. By controlling the position of such absorbing bodies automatically by the intensity of the radiation the chain reaction can be stabilized without difficulty. Since statements found their way into print which give the impression that the time within

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# STABILIZING THE CHAIN REACTION

~~As~~ If a chain reaction could be maintained in a homogeneous mixture of water and Uranium or Carbon and Uranium it would ~~be~~ have a certain natural stability in the sense that with rising temperature there would be a decrease in neutron production. ~~If we have, for instance, a sphere filled with a Uranium and water mixture standing free in space so that thermal neutrons can escape from its surface the fraction of thermal neutrons which escapes across the surface without reacting with Uranium will increase with increasing temperature. This is due to the fact that the absorption of both Uranium and Hydrogen obeys the 1/V law in the thermal region and accordingly the range of thermal neutrons is larger at higher temperatures.~~

A system, however, in which black Uranium bodies are embedded in ~~in~~ Carbon ~~lacks such natural~~ has a strong thermal stability since with rising temperature the ~~absorption of~~ Carbon decreases whereas the absorption of Uranium remains unchanged. ~~Nor can such thermal stability be produced by inserting some other absorbing material like, for instance, Cadmium.~~ It is, however, quite easy to ~~control~~ a chain reaction artificially, ~~as contrary to popular belief.~~ ~~It would be sufficient slowly~~ to increase or to decrease the absorption in the system, for instance, by inserting or removing ~~slow~~ neutron absorbers, ~~the motion of which could be controlled by an increase or decrease of the radiations emitted in the chain reaction.~~ The time t within which such change would have to be brought about is of the order of magnitude.

Szilard 2

which such controlling action would have to take place is ~~exceedingly~~ short it is perhaps advisable to point out the following:

Of the neutrons which are emitted in the chain reaction by Uranium only a fraction $\varphi$ is absorbed within the system and $1-\varphi$ escapes across the boundary of the system without reacting with Uranium a stationary state can be maintained as long as

$$\mu \nu \varphi \leq 1$$

We write

$$\nu \varphi = \Gamma(T, x(t))$$

In order to indicate that this product is a function of the temperature T and also ~~temperature~~ on a parameter such as the position of some absorbing or scattering bodies near or within the system which can be shifted by the control and which can thus be made the functions of time t.

In order to have a large neutron production we must maintain a chain reaction near the point

$$\mu \Gamma_0 = 1$$

If this product becomes larger than 1, as it may well happen there is an exponential rise in the neutron production and accordingly also in the temperature. In case of a small deviation from 1

$$\Gamma = \Gamma_0 (1+\xi)$$

the time $t_2$ in which the number of neutrons is given by

$$t_2 \cong \xi t_1$$

Laboratory 3

Where $t_1$ is the time which a fast neutron which is emitted by a Uranium atom in the mixture and which is slowed down within and absorbed by the mixture would require to produce two fast neutrons

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In arriving at these conclusions we did not take into consideration the fact that the fraction is emitted by Uranium with a time delay of about 10 seconds. Thus this small fraction has an effect in considerations of this type and leads to longer times than those we have estimated. Since, however, the time which we have given is already long enough for all practical purposes we refrain for the present from including the delayed neutron emission in the treatment of this subject.