CORNELL UNIVERSITY
ITHACA, NEW YORK

DEPARTMENT OF PHYSICS
ROCKEFELLER HALL

30/11 34.

Lieber Szilard,

Mein Geschmier ist auf den Umstand zurückzuführen, dass meine Füllfeder verschwunden ist, u. zw., wie ich vermute, im Bette des Zimmers 315. Vielleicht können Sie so freundlich sein, diesbezügliche Recherchen anzustellen und im positiven Falle das gefundene Objekt mir nach Indiana zusenden zu lassen. Ich werde Ihnen dann einen leserlichen Dankbrief schreiben können.

Vorliegende Gelegenheit kann ich gleich benützen, um Ihnen eine Formel für das $\overline{r^2}$ mitzuteilen, die für jede Masse gültig bleibt. Es ist

LM

$$\overline{r^2} = \frac{2Nl^2}{1-\cos\vartheta} \qquad (1)$$

Nun ist $N = \dfrac{\ln \frac{E_0}{E}}{\overline{\ln \frac{E_0}{E_1}}} = \dfrac{\ln \frac{E_0}{E}}{1 + \frac{1-\alpha}{\alpha}\ln(1-\alpha)}$ (2) ; $\alpha = \dfrac{4M}{(M+1)^2}$

(Mittelwert des $\ln \frac{E_0}{E}$ nach einem Stoss)

Maximaler Energieverlust.

Weiters ist der mittlere cosinus des Ablenkungswinkels

$$\overline{\cos\vartheta} = \frac{2}{3M} \qquad (3)$$

Somit hat man:

$$\overline{r^2} = \frac{6M}{(3M-2)\left\{1+\frac{1-\alpha}{\alpha}\ln(1-\alpha)\right\}} \cdot l^2 \ln\frac{E_0}{E} = f \cdot l^2 \ln\frac{E_0}{E} \qquad (4)$$

Für Wasserstoff $(M=1)$ wird der Faktor f

$$f = 1,$$

für $M \gg 1$ wiederum

$$f = M,$$

wie es sein muss.

Für die Zwischenwerte folgende Tabelle:

	M	f
H	1	6
D	2	4,15
He	4	5,65
C	12	13,4
	$\gg 1$	M

Für C ist also der Grenzwert M schon eine gute Näherung. (13,4 gegen 12)

Je mehr ich mir übrigens das Uran überlege, desto wichtiger scheint mir es, die kombinierte Bor-Selbstabsorptionskurve zu machen, bevor man die Verlangsamung mit C oder D probiert, denn erst auf Grund dieser Kurve wird man imstande sein, die Versuchsbedingungen vernünftig zu wählen und so letzten Endes Zeit sparen.

Beste Grüße Ihr G. Placzek

420 West 116th Street
New York City

April 19, 1940

Dear Placzek:

Many thanks for your letter. It seems to me the best compromise is the following: Do send me your memorandum ~~in the stage in which~~ as it is. I shall have it copied and send you one copy. If you ~~will then be~~ are able to give me a longer memorandum within the next ten days, I shall withdraw your short memorandum and replace it by the longer one. A meeting is scheduled in Washington for Saturday next, and I want to send your short memorandum to Washington a few days before this meeting takes place.

Yours sincerely

L. S.

CORNELL UNIVERSITY
ITHACA, NEW YORK

DEPARTMENT OF PHYSICS
ROCKEFELLER HALL

9. Mai 1940.

Lieber Szilard,

Besten Dank für das Manuskript. Sie kriegen das Zusatzprodukt, sobald es getippt ist. Das Resultat ist sehr einfach: Die Anzahl der von einem schwarzen Detektor pro sec. ~~u~~ und Flächeneinheit absorbierten Neutronen*) ist:

$$\underline{I} = \frac{1}{8\pi}\,\frac{Q\, l_f}{\rho^3}\left\{1-\varphi\left(\frac{\rho}{L}\right)\right\}$$

Die Funktion $\varphi\left(\frac{\rho}{L}\right)$ für die Sie eine Kurve bekommen, ist Null für $\frac{\rho}{L} = 0$ und z. B. 0,6 für $\rho = L$, und geht dann schnell gegen 1. L ist etwa 1 km, l_f bekanntlich ca 140 m. Für Ihre Genauigkeit setzen Sie also einfach für mäßige Abstände ρ

mit $\underline{I} \sim \frac{1}{8\pi}\,\frac{Q\, l_f}{\rho^3}$

*) erzeugt von Quelle über fressendem Boden.

Das ist Ihnen hoffentlich einfach genug. Viel scheint es ja nicht zu sein. Ich weiss nicht wie Ihre Mordpläne sind, aber Sie sehen dass dies um einen Faktor $\frac{l_t}{\rho}$ weniger ist als der Effekt einer Quelle im <u>Vakuum</u> ohne jede Diffusion.

Beste Grüsse

Ihr

G. Placzek

CORNELL UNIVERSITY
ITHACA, NEW YORK

DEPARTMENT OF PHYSICS
ROCKEFELLER HALL

July 25, 1940.

Dear Szilard,

Enclosed with my thanks your undamaged MS. I do not need it at present, but it may be practical if I could refer to it at several passages of one of my future papers. {You know how bad my memory is. Therefore I would be grateful if I could dispose of your paper again when I come to the writing up of the passages in question. In order to avoid extralegal procedure, we may consider it as this time not as paper any longer, but as „aide-memoire".}

Please let me know about Teller.

Best regards Yours G.P.

Brandy

CORNELL UNIVERSITY
ITHACA, NEW YORK

DEPARTMENT OF PHYSICS
ROCKEFELLER HALL

Monday evening.

Dear Szilard,

Many thanks for your letter, which I found here today after my return from Rochester. Enclosed please find the product in two copies. You may like to send one to Washington and to keep the other. I am sorry the last page is only in the first copy, because I did not find ~~at~~ any carbon paper. But since this page looks not very clean anyhow, you ~~m~~ may perhaps find it possible to have it retyped. I am sorry I cant do it myself because the train is leaving.

As to the matter itself, I was not quite clear about the style to be adopted: how much discussion of the ~~mathe~~ mathematical methods, and how much interpretation of the results. So I have endeavoured to give above all ~~fro~~ some formulae, which are simple and easily understandable, and postpone the rest for the more complete version, which we can discuss in Washington.

Best regards

Yours

G. Placzek

CORNELL UNIVERSITY
ITHACA, NEW YORK

DEPARTMENT OF PHYSICS
ROCKEFELLER HALL

Thursday.

Dear Szilard,

Many thanks for your letter. I am sorry you have not yet got the memorandum, I have been working on it and it took me longer than I had thought. However I can send it away this Saturday in a tolerable form, so that you get it Sunday. If you allow me some more days for it, I could make it longer and more complete. In this case please send me a wire.

As to the matter itself ~~there is no doubt that the influence of the water has~~ influence of the water is not very exciting, just as I had always hoped. Best regards Yours

G. Placzek

Confidential

Memorandum

sent by Dr. G. Placzek to Dr. L. Szilard

April 1940.

On the Diffusion of Neutrons in Air

The diffusion of neutrons in air has been recently discussed by Bethe, Korff and Placzek[1], the average energy distribution has been derived by Placzek[2], and the ~~local~~ spatial distribution of neutrons in air over a water surface has been derived by Bethe[3] for the case of production of neutrons in air homogeneous in a horizontal plane.

§ 1. Diffusion of neutrons in free air. We first discuss the diffusion in free air. The results will hold if the distance of the neutron source from the ground is larger than the diffusion path (see below) of the neutrons produced by the source.

The average energy distribution is approximately given by[1,2]

$$N(v)\,dv = MQ\,l_s(v)\frac{dv}{v^2}\,e^{-M\int_v^{v_1}\frac{l_s(v')}{l(v')}\frac{dv'}{v'}} \tag{1}$$

Here $N(v)dv$ is the number of neutrons of velocities between v and $v + dv$ for initial velocity v_1, $M = 14.4$ the mean atomic weight of air, Q the number of neutrons produced per second (with velocity v_1) and $l_s(v)$ and $l_c(v)$ the mean free path of neutrons in air for scattering and capture respectively, as functions of the velocity.

---

[1] Bethe, Korff and Placzek, Phys. Rev. 57, 573, 1940

[2] See " " , ref. 30

[3] See " " , ref. 31

From a discussion of (1), together with the available experimental material about scattering and capture cross sections in nitrogen and oxygen it can be concluded (cf. ref. 1), that capture effects will probably not change the neutron density by orders of magnitude for all energies down to 1 ev. In the following, we shall neglect these effects, they can be roughly accounted for by multiplying all neutron densities with the reduction factor $\exp\left\{-M\int_{v}^{v_1}\frac{l_s(v')}{l_c(v')}\frac{dv'}{v'}\right\}$ as soon as the respective cross sections have been measured.

At energies considerably below 1 ev the capture due to the n-p reaction in nitrogen will come into play and remove most of the neutrons before thermal energies are reached. In the following table, we give approximate numerical values for some of the characteristic lengths, important for the study of the diffusion process in air.

These are: The mean free path for scattering l_s for fast and slow neutrons, the mean free path l_{ct} for capture of thermal neutrons, the diffusion path for thermal neutrons λ defined by:

$$\lambda = \left(\frac{l_s\, l_{ct}}{3}\right)^{1/2} \tag{2}$$

and the root mean square distance of diffusion $\sqrt{(r^2)_{AV}}$, for neutrons produced at energy E_1 and captured at energy E_c.

$$(r^2)_{Av}^{(E_c)} = M_{eff} \int_{E_c}^{E_1} l_s^2(E) \frac{dE}{E} \qquad (3)$$

$$M_{eff} = 15,6$$

E_c lies slightly above thermal energies.

		cm water equivalent	meters air at normal pressure
l_s	for fast neutrons	18	140
	" slow "	2.6	20
l_{ct}		24	190
λ		4.6	35
$\sqrt{(r^2)_{Av}}$	for 2Mev neutrons	150	1200

The most inaccurate of these figures is $(r^2)_{Av}$, because of insufficient data about the energy variation of the mean free path. (see ref. 1) The spatial distribution of the neutrons can be found approximately by assuming that the time spent in the slowing down process from the original energy E_1 to any energy E_2 is the same for all neutrons, so that the energy is a function of the "age" of the neutron. Then the problem can be treated by the standard methods of diffusion theory. Actually, the assumption is not quite correct, since to every energy belongs an age distribution of neutrons, but it can be shown that the use of this idealization is a good approximation as long as capture is unimportant.

If we define a symbolic time ϑ by

$$\vartheta(v) = \frac{1}{6} (r^2)_{Av}^{(v)} = 5,2 \int_{v}^{v_1} l^2(v') \frac{dv'}{v'} \qquad (4)$$

the diffusion equation reads

$$\frac{\partial F}{\partial \vartheta} = \nabla^2 F \tag{5}$$

where F is the neutron density. If Q neutrons of velocity v_1 are produced per second at r = 0, the solution of (5) is

$$F(v, r)\,dv = M\,Q\,\frac{l_s(v)\,dv}{v^2}\,\frac{e^{-\frac{r^2}{4\vartheta(v)}}}{(4\pi\vartheta(v))^{3/2}} \tag{6}$$

with (11) This is correct ✓

Or, if we introduce the diffusion path L(v) by

$$L(v) = 2\sqrt{\vartheta} = \sqrt{\tfrac{2}{3}\,(r^2)_{Av}} \tag{7}$$

:

$$F(v, r) = M\,Q\,l_s(v)\,\frac{dv}{v^2}\,\frac{e^{-\left(\frac{r}{L(v)}\right)^2}}{\pi^{3/2}L^3(v)} \tag{6a}$$

If we have a source of thermal neutrons, their density F_t is determined by the equation

$$\nabla^2 F_t - \frac{F_t}{\lambda^2} = \frac{Q_t\,\tau\,\delta(r)}{4\pi r^2} \tag{8}$$

where τ is the lifetime of thermal neutrons in air (about 1/100 sec), f and λ is given by (2). (7) is solved by

$$F_t = Q_t\,\tau\,\frac{e^{-r/\lambda}}{4\pi\lambda^2 r} \tag{9}$$

f Q_t the number of thermal neutrons produced per sec.

the number of thermal neutrons produced per sec.

§ 2. Diffusion of neutrons above ground. We now consider the following problem: The space of air is limited by a plane infinite boundary z = 0 (ground). At an altitude z_0 we have a point source of neutrons of velocity v_1. We ask for the neutron density in air as a function of velocity and coordinates. Obviously, the neutron density will depend on the capture and scattering properties, and hence on the chemical composition of the material forming the ground. However, we will get lower limit for the neutron density by assuming that every neutron reaching the ground is captured. This leads to the boundary condition F = 0 for $z = -\frac{l_s(v)}{\sqrt{3}}$.

The solution of the diffusion equation with this boundary condition is;

$$F(z,\rho,v)\,dv = MQl_s\frac{dv}{v^2}\cdot 2\pi^{-3/2}L^{-3}e^{-\rho^2/L^2+(z+\frac{l_s}{\sqrt{3}})^2/L^2+(z_0+\frac{l_s}{\sqrt{3}})^2/L^2}\sinh\frac{2(z+\frac{l_s}{\sqrt{3}})(z_0+\frac{l_s}{\sqrt{3}})}{L^2} \qquad (10)$$

z is the altitude and ρ the distance of the projections of source point and field point on the plane z = 0. If the source is at $z_0 = 0$ and we are interested in the spatial dependence of the neutron density on the ground, i.e. $z = 0$, (10) becomes approximately, for $L(v) \gg l(v)$:

$$F(0,\rho,v) = MQl\frac{dv}{v^2}\,\frac{e^{-(\rho/L)^2}}{\pi^{3/2}L^3}\,\frac{4}{3}\left(\frac{l(v)}{L(v)}\right)^2 \qquad (10a)$$

Comparing (10a) with (6a), we see that the neutron density at the ground is reduced by the factor $\frac{4}{3}\left(\frac{l}{L}\right)^2$ with respect

$\frac{1}{3}v$

to the case of free air. This reduction factor does not depend on the distance ρ .

If the ground is formed by a substance the neutron constants of which are sufficiently well known, as for instance water, the above treatment can be refined by calculating the probability $\varphi(v, v_0)$ that a neutron hitting the surface with velocity v_0, re-emerges from it with velocity v, and then using the product of the current of the density (6a) as function of v_0 times the function $\varphi(v,v_0)$ as a new source in the diffusion equation (5). This procedure can be applied again to the new result, and in this way a well converging series of positive terms is obtained. The physical meaning of this procedure is that one considers successively the neutrons which have entered and emerged from the boundary once, twice, three times, and so on. The most important difference of this solution compared to the first approximation (6a) in the case of water is the presence in air of a certain amount of thermal neutrons, which have been slowed down in the water and emerged from it. The further diffusion of these thermal neutrons can then be treated in a single step by solving a differential equation of the type (8), with a simple boundary condition which is obtained from albedo considerations.

§3. Total effect on small black detector.

We now calculate the total number of neutrons above velocity v absorbed by a black detector of dimensions small compared to the mean free path.

The number of neutrons above velocity v absorbed per second and unit surface of the detector can be shown to be (for a convex detector)

$$I(v) = \frac{1}{4} \int_{v}^{v_0} v\,F(v)\,dv \tag{11}$$

For constant mean free path, the integration gives in the case of the free atmosphere (6a):

$$I(v) = \frac{3}{16\pi} \frac{Q}{r\ell} \left\{ 1 - \Phi\left(\frac{r}{L}\right) \right\} \tag{12}$$

and in the case of source near capturing ground (10a):

$$I(v) = \frac{1}{8\pi} \frac{Q\ell}{\rho^3} \left\{ 1 - \varphi\left(\frac{\rho}{L}\right) \right\} \tag{13}$$

where Φ is the error function and φ is related to the incomplete Γ-function of order 3/2 by

$$\varphi(x) = \frac{2}{\sqrt{\pi}} \Gamma_{3/2}(x^2) .$$

If the distance from the source is small compared to L, Φ and φ can be neglected. Fig. 1 gives the dependence of $1 - \Phi$ and $1 - \varphi$ on r and ρ.

Actually, in air, the mean free path is not constant, but varies from about 140m for fast neutrons to about 20m for slow neutrons. The transition probably takes place in the region between 150.000 and 10.000 ev.(cf. ref. 1). We may roughly describe this behavior by putting

$$\begin{aligned} \ell &= \ell_f = 120\,m \quad \text{for} \quad E > E_1 \\ \ell &= \ell_s = 20\,m \quad \text{for} \quad E < E_1 \end{aligned} \tag{14}$$

where E_1 lies between 150.000 and 10.000 ev. With (14) we get instead of (12) and (13)

$$I(v) = \frac{3}{16\pi}\,\frac{Q}{r\ell_f}\left\{1-\Phi\left(\frac{r}{L_f}\right)+\frac{\ell_f}{\ell_s}\left(\Phi\left(\frac{r}{L_f}\right)-\Phi\left(\frac{r}{L}\right)\right)\right\} \tag{12a}$$

$$I(v) = \frac{1}{8\pi}\,\frac{Q\ell_f}{\rho^3}\left\{1-\varphi\left(\frac{\rho}{L_f}\right)+\frac{\ell_s}{\ell_f}\left(\varphi\left(\frac{\rho}{L_f}\right)-\varphi\left(\frac{\rho}{L}\right)\right)\right\} \tag{13a}$$

where L_f is the diffusion path from the initial energy to energy E_1. With the values (14) for ℓ, L_f has, for an initial energy of 2 Mev, a value between about 0.75 km and 1km depending on the value of E_1. Because of the small mean free path for slow neutrons, the total diffusion path L is not very different from L_f. For energies in the neighborhood of E_c (at which capture becomes important), $\frac{L-L_f}{L_f}$ is of the order of 5 per cent.

In spite of this, the slow neutron contribution to I is still important in the case of the free atmosphere and can be derived from fig. 1 (for r = L_f it is of the same order of magnitude as the fast neutron contribution). For capturing ground, however, it is in general negligible because of the small factor ℓ_s/ℓ_f in (13a), so that (13a) may be replaced by (13) with $\ell = \ell_f$.

TELEPHONE
UNIVERSITY 4-2700

KING'S CROWN HOTEL

OPPOSITE
COLUMBIA UNIVERSITY

UNDER KNOTT MANAGEMENT
420 WEST 116TH STREET, NEW YORK N.Y.

July 14, 1941

Professor G. Placzek
Cornell University
Ithaca, New York

Dear Placzek:

Many thanks for your letter and the enclosure. I will try to find your first expense account and then settle both together. It will perhaps interest you that it now seems that within a month or two we shall have no difficulty in discussing freely the work in which we are both interested. This change is coming about without any direct action on my part and even against my wishes insofar as the particular form is concerned which will apparently be chosen. However, this point is perhaps of small importance and you might find the form satisfactory from your own point of view.

I hope these somewhat dark hints will cheer you up even more than the prospect of ewentually getting your expense account settled.

Yours,

(Leo Szilard)

1947

Wood Ducks

SABRA

Christmas Greetings

AND BEST WISHES FOR A HAPPY NEW YEAR

George and

C. B. Placzek

10 W 305

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