The Graphite Uranium System

The purpose of the present paper is to show that a chain reaction can be achieved by using an element like carbon for slowing down the neutrons in certain particular systems composed of uranium and carbon. The theory which is given in the present paper can be applied to elements other than carbon but it does not give any useful information for systems composed of uranium and hydrogen.

Though one might think that carbon should be much less efficient for slowing down neutrons than hydrogen from several points of view it would be preferable to use carbon in the form of graphite rather than hydrogen in the form of water. The capture cross section of carbon for thermal neutrons $\sigma_c(C)$ is small. An upper limit of $\sigma_c(C) < 0.01 \times 10^{-24}$ cm^2 has been reported by Frisch, Halban and Koch, but this upper limit is not sufficiently low to allow us at present to conclude that a chain reaction could be maintained in homogeneous mixtures of uranium and carbon. For neutrons it takes about 6.5 collisions with carbon atoms to reduce their energy by a factor of e. Thus a neutron which is being slowed down by carbon stays for a long time within the resonance absorption region of uranium. Consequently, very low uranium concentrations would have to be used in order to avoid that a large fraction of the fast neutrons emitted by uranium is absorbed at resonance by uranium. At such very low uranium concentrations, on the other hand, the fraction of the thermal neutrons which is absorbed by carbon might perhaps be too large to permit a chain reaction. (16)

It will be shown, however, in the present paper that if instead of using a homogeneous mixture of uranium and carbon a large number of spheres of uranium which may form, for instance, a close-packed hexagonal or cubic lattice are embedded in carbon, the ratio of the number of thermal neutrons and the number of resonance neutrons absorbed by the uranium can be so much increased that a chain reaction will become possible.* It will be seen that this ratio strongly depends on the radius of the uranium spheres and that a rather small radius must be chosen in order to obtain most favorable conditions.

* 21

130
930

In order to get a simplified picture of the various factors which have a bearing on the balance of neutron production and neutron absorption let us consider a sphere of uranium ~~of infinite density~~ immersed into an infinite space filled with carbon, and let us assume that fast neutrons are produced uniformly everywhere in the carbon, ~~which~~ and are slowed down by collision with carbon atoms; ~~and~~ each neutron will then eventually reach the thermal region unless it is absorbed at resonance by the uranium sphere. We assume that the uranium sphere is by virtue of its resonance absorption ~~opaque~~ "black" to all neutrons in an energy interval δE, and ~~also opaque to thermal~~ has some abs. for thermal neutrons, and that everywhere within the carbon the same number of neutrons Q enter the energy region δE per cc and sec.

We are interested to find out the ratio Σ of the number of neutrons which are absorbed by the uranium sphere in the thermal region and the number of neutrons which are absorbed by the uranium sphere at resonance. Both, the resonance neutrons, which have an energy in the interval δE, and the thermal neutrons have a finite range in carbon. The former, because they leave the energy interval δE by being slowed down below that interval through elastic collisions with carbon atoms, and the latter because thermal neutrons are captured by carbon. We can use the same formalism for both the resonance neutrons and the thermal neutrons if we postulate that on the average the neutron which has an energy within the interval δE will after R collisions with Carbon have a probability of $1 - e^{-\frac{R}{R_0}}$ of having been slowed down below that interval δE ~~by Q collisions in carbon~~. Since a thermal neutron has a probability of ~~being absorbed in~~ $1 - e^{-\frac{p_c}{\sigma_c}}$ disappearing by p_c collisions by virtue of absorption in carbon, the law of "absorption" for the resonance neutrons and thermal neutrons in carbon is ~~compared to the range of resonance neut-~~ similar.

We define the range A of the thermal neutrons and the range B of the resonance neutrons by

$$A = \lambda_{sc}\sqrt{\frac{\sigma_{sc}}{3\sigma_c}} \quad ; \quad B = \lambda_{sc}\sqrt{\frac{k_0}{3}}$$ (≅ 7 cm for k = 25)

We are first of all interested in determining the ratio ε of the number of thermal neutrons absorbed by the uranium and the number of the resonance neutrons absorbed by uranium in the case of a single sphere of uranium in an infinite space filled with carbon.

We are next interested in the lattice of uranium spheres and want to find out what fraction q of the neutrons which are slowed down by carbon is absorbed by the uranium spheres in the thermal region, the rest being absorbed by the uranium spheres at resonance and the carbon. This fraction q will depend on the spacing of the uranium spheres and will be small both for a very small and a very large spacing. We are interested here in finding out the maximal value of q which can be obtained by choosing the optimal value for the spacing of the uranium spheres. If the distance between the uranium spheres is very large all the neutrons which are slowed down by carbon are captured by carbon. The number of thermal neutrons captured in unit time per unit volume by the carbon is proportionate to the average thermal neutron density. If the thermal neutron density for very wide spacing of the uranium spheres is ρ_0 the average neutron density $\bar{\rho}$ for a finite spacing of the uranium spheres will be

$$\alpha\rho_0 \quad \text{where } \alpha < 1$$

The fraction of the neutrons absorbed by carbon is proportionate to the average neutron density, and since for very wide spacing of the uranium spheres all neutrons are captured by carbon therefore for any given spacing of the uranium spheres the fraction of the neutrons captured by carbon will equal α and the fraction absorbed by U either in the thermal region or at resonance is $(1-\alpha)$

In the lattice the ratio of the number of thermal neutrons absorbed by the uranium spheres and the number of resonance neutrons absorbed by the uranium spheres is $\alpha\varepsilon$ if the spacing of the lattice is smaller than the range A of the thermal neutrons but large compared with the range B of the resonance neutrons. This ~~simply~~ follows from the fact that the number of thermal neutrons absorbed by the uranium spheres per sphere is reduced by a factor α due to the competition of the other uranium spheres, whereas the number of the resonance neutrons absorbed by the uranium spheres per sphere is not affected by ~~such~~ similar competition ~~due to the large spacing of the spheres as~~ as long as the distance of the U is large compared with the range B of the resonance neutrons.

For the ratio of the therm. neutrons abs. by U and the total number of n absorbed either by H or by C we have

$$q = \frac{u_U}{u_H + u_{res} + c_H} = \frac{u_U}{u_H + u_{res}} \times \frac{u_H + u_{res}}{u_H + u_{res} + c_H}$$

$$\frac{u_U}{u_H + u_{res}} = \frac{u_U/u_{res}}{1 + u_H/u_{res}} = \frac{\alpha\varepsilon}{1+\alpha\varepsilon}$$

$$\frac{u_H + u_{res}}{u_H + u_{res} + c_H} = 1 - \alpha$$

accordingly we have

~~of neutrons, both thermal and resonance~~, absorbed by the uranium spheres is given by

Since is the fraction of neutrons absorbed by uranium, the fraction of the neutrons absorbed as thermal neutrons by the uranium spheres is given by

$$q = \frac{\alpha \varepsilon}{1+\alpha\varepsilon}(1-\alpha)$$

This expression becomes a maximum for $\alpha = \alpha_m$

$$\alpha_m = \frac{-1+\sqrt{1+\varepsilon}}{\varepsilon}$$

and its maximum value is $q = q_m$

$$q_m = 1 - 2\alpha_m = 1 - 2\,\frac{-1+\sqrt{1+\varepsilon}}{\varepsilon}$$

or expressing ε by q_m it is

$$\varepsilon_m = \frac{4q_m}{(1-q_m)^2}$$

$$\alpha_m = \frac{1-q_m}{2}$$

~~For extremely high values of ε the~~

If the carbon capture cross sect. were much lower than expected the maximum value of q would correspond to such a wide spacing of the U spheres ~~that the neutrons emitted from one U sphere would be slowed down by the carbon~~ ...

energy of which lies in the energy interval ΔE) vanishes for the surface of the uranium sphere at $r = r_1$, i.e. $\rho(r_1) = 0$; $s(r_1) = 0$.

If r_1 is large compared to ~~xxxxxx~~ the density ρ of the thermal neutrons can be treated as a diffusion problem, and in the absence of a resonance absorption in uranium we would have

$$D \frac{d^2(r\rho)}{dr^2} = \sigma_a (r\rho) + Q r = 0$$

and

$$\rho = \frac{Q}{A}\left(1 - \frac{r_1}{r} e^{-a(r-r_1)}\right)$$

where $D = \frac{w \lambda_{sc}}{3}$ $\quad A = \frac{w}{\lambda_{sc}} \sigma_{sc}$ $\quad a = \frac{\sqrt{3}}{\lambda_{sc}} \sqrt{\frac{\sigma_c}{\sigma_{sc}}}$

Accordingly the number of thermal neutrons ~~xxxxxx~~ which would reach the uranium sphere by diffusion /in the absence of a resonance absorption by uranium would be

$$F_{th} = 4\pi r_1^2 D \left\{\frac{d\rho}{dr}\right\}_{r=r_1} = 4\pi Q \left\{\frac{r_1^2}{a} + \frac{r_1}{a^2}\right\}$$

This value is also correct if there is a resonance absorption by uranium provided that we have

$$r_1 \ll \frac{\lambda^*_{sc}}{\sqrt{3}} \sqrt{K_0} = B$$

Quite similarly we have for the density s of the neutrons, having an energy which falls into the interval ΔE

$$s = \frac{Q}{A^*}\left(1 - \frac{r_1}{r} e^{-b(r-r_1)}\right)$$

and

$$F^* = 4\pi Q \left(\frac{r_1^2}{b} + \frac{r_1}{b^2}\right) \qquad b = \frac{\sqrt{3}}{\lambda^*_{sc}} \sqrt{\frac{1}{K_0}}$$

where

The ratio of the neutrons absorbed by the uranium sphere in the thermal region and at resonance is therefore

$$\varepsilon = \frac{F}{F^*} \cong \frac{b^2}{a^2} \frac{a r_1^2 + r_1}{b r_1^2 + r_1}$$

and if we have $\frac{1}{a} \gg \frac{1}{b} \gg r_1$ we obtain

$$\varepsilon = \frac{F}{F^*} = \frac{b^2}{a^2} = \left(\frac{\lambda_{sc}}{\lambda^*_{sc}}\right)^2 \frac{\sigma_{sc}}{\sigma_c} \frac{1}{K_0}$$

the problem can no longer be treated as a diff. problem if $r_1 \ll \lambda_{sc}$

We now wish to calculate the value ε, for a uranium sphere which is black for resonance neutrons but not black for thermal neutrons. The density of the thermal neutrons within the uranium sphere is given by the equation

$$\frac{\lambda_{sc}(u)}{3}\frac{d^2(r\rho)}{dr^2} - (r\rho) = 0$$

and we have

$$\rho_u = \frac{c}{r}\left(e^{ur} - e^{-ur}\right)$$

where u stands for

$$u = \frac{1}{\lambda_{sc}(u)}\sqrt{\frac{3\sigma_c(u)}{\sigma_{sc}(u)}}$$

The ratio of the number of thermal neutrons absorbed by such a uranium sphere to the number of thermal neutrons absorbed by the black uranium sphere of the same radius is equal to the ratio of $\left\{\frac{d\rho}{dr}\right\}_{r=r_1}$ in carbon for the two cases and is calculated to be

$$= \frac{g^*}{\frac{1}{r_1} + a + g^*}$$

where G^* stands for

$$g^* = \left\{\sqrt{\frac{3\sigma_c(u)}{\sigma_{sc}(u)}}\,\frac{e^{ur_1} + e^{-ur_1}}{e^{ur_1} - e^{ur_1}} - \frac{\lambda_{sc}(u)}{r_1}\right\}\frac{1}{\lambda_{sc}(C)}$$

For $a \ll \frac{1}{r_1}$ we have

$$= \frac{1}{\frac{1}{r_1 g^*} + 1} \quad \text{and} \quad \varepsilon_1 = \varepsilon_0(r_1) = \frac{6^2}{a^2}\frac{1}{\frac{1}{r_1 g^*} + 1}$$

and for very large r_1, we have $\frac{1}{r} \ll a$

and $$= \frac{1}{\frac{a}{6^*} + 1} = \frac{1}{a\lambda_{sc}(C)\sqrt{\frac{3\sigma_c(u)}{\sigma_{sc}(u)}} + 1}$$

$$\simeq 1$$

E. 1 February 17, 1940

A method has been found to set up and to maintain a divergent chain reaction in a system which contains uranium. About ten tons of uranium will be needed to reach the point of divergence at which atomic transmutation will proceed at the rate limited only by the necessity of avoiding over heating. If the chain reaction is to be used for purposes of supplying power a system has to be used which permits a rise of temperature to 500° to 900 ° C. so as to have sufficient temperature gradient available for leading away the heat produced. A system has been found which is able to withstand such temperatures and a method has been found for leading away and utilizing the heat without interfering with the nuclear processes in the chain reaction. A method has also been found to stabilize and control the chain reaction so as to avoid accidental over heating.

A number of physical constants have to be measured rather accurately to be able to find the most favorable conditions for operating an atomic engine and three experiments have been devised for this purpose. Two of these are quite inexpensive and are now being started. The third requires larger quantities of material which have been ordered [at my request] by a department of the Administration and will be supplied to Columbia University where the experiments will be started in about two months time. We thus should be in a position within a few months to begin with the construction of an atomic engine and various steps ought to be taken right now in order to avoid unnecessary delay. ~~A fourth experiment which would be~~ A fourth measurement which would be very useful could perhaps be carried out at Columbia or elsewhere if additional facilities could be obtained.

As far as the production of power is concerned the field of application is limited by the necessity of having the engines surrounded by water tanks to protect the personnel from the irradiations emanating from the atomic engine. This would not prevent, though, the use of the atomic engine for driving boats which could, no doubt, out run every other boat and stay on the high seas for a long period of time, as they would not be dependent on a fuel base. Still it is not possible ~~at~~ present to ~~some~~ form a final opinion on the scope of this application since it is not known which of the two uranium isotopes is responsible for the liberation of energy in the chain reaction. If it is the rare isotope then ten tons of uranium would supply only as much power as about fifty thousand tons of coal before the active agent in the uranium is used up

E. 1

to a point where the machine would stop operating. If it is the abundant isotope then ten tons of uranium could supply more power than five million tons of coal. Though N. Bohr has put forward interesting arguments in favor of the view that it is the rare isotope which is the active agent it is now generally agreed that this question has to be decided by a direct observation performed on a small sample of the separated isotope. I have found a new method by which such samples weighing a few milligrams could be prepared without much difficulty and this could be done at any time if the facilities were obtainable. Expenses in this connection would essentially consist in the salary of some person who would have to be entrusted with this work. It is of course possible that in the meantime other physicists will succeed in preparing samples of the separated isotopes weighing at least a few micro-grams by using a mass spectograph. One way or another this point ought to be cleared up as quickly as possible since it will determine the scope of applications as far as power production by an atomic engine is concerned.

embedded in graphite appeared even more favorable from the point of view of a chain reaction than the system of plane uranium layers which was initially considered. The efficiency of such a system was calculated and it was found that if small spheres of uranium metal were embedded in graphite there would be quite a good chance of obtaining a chain reaction in an experiment performed on a large but entirely practicable scale. There appeared to be an appreciable chance for success even if uranium oxide had to be used in place of uranium metal. In the circumstances, the conclusion was reached that it would be better to perform an experiment on a large scale rather than to wait for measurements to be performed for the purpose of determining all the nuclear constants involved.

(July 9, 1939)

SINGLE SPACE

Note for Memorandum:

Tentative steps were thereupon taken in this direction and among others, E. P. Wigner and E. Teller were informed of these considerations. They shared ~~my~~ the opinion that no time must be lost in following up this line of development and in the discussion that followed the opinion crystallized that an attempt ought to be made to enlist the support of the Government rather than that of private industry. Dr. Wigner, in particular, urged very strongly that the Government of the United States be advised of certain possible consequences of this as well as ~~certain~~ some other lines of work connected with uranium. With this in mind, we approached Professor Albert Einstein and Dr. Alexander Sachs, and after a number of consultations, Dr. Einstein wrote a letter to the President of the United States recommending that a personal committee be appointed to act as a permanent liason between the Government and the physicists who are working on uranium. (August 2, 1939)

Einstein's letter and a memorandum which I was asked to write were submitted by Dr. Sachs to the President and Dr. Lyman J. Briggs was appointed as chairman of a Government committee. Dr. Sachs, Dr. Wigner, Dr. Teller and I were given an opportunity to explain to this committee why we believed that the work which is being done on uranium deserved the attention and the support of the Government. (October 21, 1939)

On this occasion a plea was made for the Government's support either financial or moral both for the work on uranium in general and for the work along the lines indicated in the paper in particular. It was stated that a lattice of uranium metal spheres embedd in graphite appeared to offer the greatest chance for immediate success; that about 100 tons of graphite and 10 to 20 tons

of uranium would have to be used in a large scale experiment in order to produce a divergent chain reaction. A recommendation was made that steps be taken to prepare for the performance of such a large scale experiment and that methods of producing uranium metal from uranium oxide be explored. It was emphasized that before starting a large scale experiment the capture cross-section of carbon would be measured and that a few tons of graphite are required for this purpose. A memorandum summarizing these statements and recommendations was submitted to Dr. Briggs (October, 1939).

The value of the capture cross-section of carbon for thermal neutrons was not known at that time; only an upper limit for this magnitude of was published by Frisch, v. Halban, and Koch. To measure this value appeared, therefore, to be an urgent task, indeed. The usual methods for measuring small absorption cross-sections did not seem to be adequate for this purpose and so a method was devised which called for the study of the spatial distribution of the thermal neutron density in a large mass of graphite. The thermal neutron density in graphite obeys a diffusion equation which contains the mean free path for scattering and the ratio of the capture cross-section and the scattering cross-section. Since the thermal neutrons are produced in the graphite through the slowing down of the fast neutrons emitted from the source, this diffusion equation is not homogeneous. By introducing screens which are black for thermal neutrons one can obtain, however, experimental values which obey the homogeneous diffusion equation. The first experiment of this type which was originally planned (July 5, 1939) is described in the paper.

Due to various circumstances, experimentation along this line was halted between July, 1939, and March, 1940. While no information on the capture cross-section of carbon was as yet forth-coming, an increasingly optimistic view on the chances of a chain reaction in a uranium - graphite system appeared to be justified when it was realized that the neutron absorption in carbon could be materially reduced by allowing the bulk of the graphite to heat up to high temperatures. Thus having finally reached the conclusion that we may expect a divergent chain reaction in such a system, the paper was submitted in February, 1941, to the Physical Review for publication. After its acceptance, and after consultations with a number of my colleagues and Dr. Lyman J. Briggs, its publication was deferred at my request.

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Oct 40

These experiments demonstrate beyond doubt that on the average more than one fast neutron is emitted for one thermal neutron absorbed by uranium, but this does not necessarily mean that a chain reaction can be maintained in a mixture of water and uranium oxide.

A fraction of the fast neutrons emitted from uranium is absorbed by uranium at resonance before reaching thermal energies, and does not cause fission. As it was first pointed out by G. Placzek, this fraction becomes so large at high uranium con centrations that it may very well prevent the possibility of a chain reaction at such concentrations. At low concentrations the fraction of thermal neutrons, which is absorbed by hydrogen, rather than uranium, becomes considerable, and so there may not be any concentration at which a nuclear chain reaction may take place in a homgenous mixture of uranium oxide and water. For this reason, Placzek advocated the use of helium instead of hydrogen for slowing down the neutrons. Helium may be expected to have no absorption for thermal neutrons and would therefore permit the use of low uranium concentrations at which only a small fraction of the neutrons would be absorbed at resonance. A general theory of the process of slowing down fast neutrons in media heavier than hydrogen had been given by Placzek in 1939.

Fermi was the first to realize that the fraction of neutrons absorbed at resonance may be reduced in mixtures of uranium and water by keeping the uranium oxide and water in separate layers. He found that by using thin layers of uranium oxide and water the conditions for a chain reaction can be improved. Whether this improvement is sufficient for making a chain reaction actually possible in such a system is not known.

It is easy to understand why uranium layers of finer thickness embedded in paraffine are preferable to layers of an infinitely small thickness, i.e. to a homogenous mixture. If the thickness of a very thin layer of uranium is increased the thermal neutron absorption of the layer increases proportionally to the thickness. The absorption for resonance neutrons of uranium, however, increases more slowly than proportional to the thicknesses at which the thermal neutron absorption is still increasing proportionally to the thickness. This is due to the fact that for not too thick uranium layers the resonance absorption is mainly due to the first sharp resonance line of uranium. At larger thickness the absorption of thermal neutrons ~~fax~~ flattens out and there is a thickness which is optimal from the point of view of ratio of the thermal absorption and the resonance absorption of uranium. This optimum may be even more marked and more favorable for lumps of uranium than for flat layers of uranium. Since the range of the thermal neutrons in paraffine ~~wax~~ is of the same order of magnitude as the range of resonance neutrons, and since the thickness of the paraffine layers which may be sandwiched between uranium layers must not be made large compared to the range of the thermal neutrons, systems of this type may be considered as quasi homogenous since the velocity distribution of the neutrons will not vary very much within the system.

While the question whether ~~xhx~~ a chain reaction can be maintained in such a system remained open it appeared of interest primarily from the point of view of possible practical applications to raise the question whether a chain reaction could be maintained in a system composed of uranium and carbon. Even if it were possible to maintain a chain reaction in a system in which the neutrons are slowed down by hydrogen the rate at which the chain reaction could be maintained would necessarily be limited by the fact that hydrogen containing substances

decompose or evaporate at moderately elevated temperatures. If carbon can be used in the place of hydrogen for slowing down the neutrons for the purpose of the chain reaction, there would be no such limitation of the chain reaction rate, and it would be possible to have a sufficiently high temperature gradient available for dissipating the heat which would be generated.

The purpose of the present paper is to show how a chain reaction can be achieved by using an element like carbon for slowing down the neutrons in certain particular systems composed of carbon and uranium. The calculations in the present paper can be applied equally well to an element other than carbon provided that the range of thermal neutrons in a medium composed of this element is large compared to the range of the resonance neutrons.

For a number of practical reasons it would be preferable to use carbon in the form of graphite rather than hydrogen in the form of water for the purposes of a chain reaction. The capture cross-section of carbon for thermal neutrons is small. An upper limit of has been reported by Frisch, Halban, and Koch, but this upper limit is not sufficiently low to allow us at present to conclude that a chain reaction could be maintained in homogeneous mixtures of uranium and carbon. For neutrons it takes about 6.5 collisions with carbon atoms to reduce their energy by a factor of e. Thus a neutron which is being slowed down by carbon stays for a long time within the resonance absorption region of uranium. Consequently, very low uranium concentrations would have to be used in order to avoid that a large fraction of the fast neutrons emitted by uranium be absorbed by uranium at resonance. At such very low uranium concentrations, on the other hand, the fraction of the thermal neutrons which is absorbed by carbon might perhaps be too large to permit a chain reaction.

In the present paper we consider a system in which a large number of fairly large spheres of uranium which are practically black for resonance neutrons of uranium in an energy interval stretching from perhaps 2 to 20 volts are embedded in carbon. It will be shown that the ratio of the number of thermal neutrons and the number of resonance neutrons absorbed by the uranium can be made so small in such a system that we may expect a chain reaction to become a reality.

is practically independent of the carbon cross-section since we have
Its value is determined by the density of graphite and uranium and the nuclear values ; . The value of R may be chosen as to make this factor a maximum.

All expressions for were so far obtained from diffusion equations involving the assumptions:

For small values of R the problem can no longer be treated as a diffusion phenomenon and we shall, therefore, refrain from using expression No. 20 or 20a for values of R of less than 5 cm.

Oct 40

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CORRECTION OF VALUES

By calculating the value of from we have neglected the effect of the absorption of resonance neutrons (by the uranium spheres in the lattice) on the production on thermal neutrons in the carbon in the neighborhood of the uranium spheres. The absorption of resonance neutrons reduces in reality the value of near the spheres below the average value of and accordingly the correct number of thermal neutrons absorbed by the uranium spheres per sphere will be smaller than . Corresponding to this, the correct value of q, q will be smaller than q .

If the uranium sphere in the lattice did not absorb resonance neutrons then a neutron which reaches a given sphere at least once while its energy is in the resonance region would have a certain probability of reaching the sphere at least once after it had been slowed down to the thermal region. It can be shown that we have

For values of

we find

In order to take into account this and other corrections and in order to deviate on the conservative side we shall in discussing the chances of a chain reaction use a value of q which is ten per cent below the calculated value .

SPACING OF THE LATTICE

Formula No. 26 giving the value of q was derived under the assumption that there is a uniform production of resonance neutrons throughout the whole mass of carbon into which the lattice of uranium spheres is embedded. We have to verify that this assumption is correct. For this reason we have to compare the distance to which a fast neutron "diffuses" away from a uranium sphere from which it is emitted before it is slowed down to thermal energies with the distance L between two neighboring uranium spheres in a close-packed hexagonal or cubic lattice. If the value of L which corresponds to the maximum value of q were large compared to then obviously equation No. 26 giving the value of could not be used.

In order to estimate L, as well as for other reasons, we shall calculate the volume V of carbon per uranium sphere in the lattice.

In the lattice of uranium spheres from the Q V neutrons which are slowed down per second within the volume V to resonance energies, the carbon absorbs neutrons and the uranium spheres in the lattice absorb neutrons. Accordingly we have

embedded in graphite appeared even more favorable from the point of view of a chain reaction than the system of plane uranium layers which was initially considered. The efficiency of such a system was calculated and it was found that if small spheres of uranium metal were embedded in graphite there would be quite a good chance of obtaining a chain reaction in an experiment performed on a large but entirely practicable scale. There appeared to be an appreciable chance for success even if uranium oxide had to be used in place of uranium metal. In the circumstances, the conclusion was reached that it would be better to perform an experiment on a large scale rather than to wait for measurements to be performed for the purpose of determining all the nuclear constants involved. (July 9, 1939)

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the upper limit which has so far been established and consequently there is hope that conditions will be much more favorable for a chain reaction than would seem from the values so far quoted.

The amount of carbon and uranium required to reach the point of divergence at which nuclear transmutation will proceed at a rate limited only by the necessity of avoiding over-heating is essentially determined by the value of

In the following we shall calculate how the value of this expression depends on the value of the carbon capture cross-section at room temperature.

We shall take the density of graphite to 1.7; the density of uranium metal to be 15 and choose R = 8 cm. We then obtain for a capture cross-section of carbon at room temperature of 0.005 the following set of values at 900° C:

$A = 64\ cm$; $\frac{A^2}{B^2} = 90$; $\varphi = 0.666$; $\varepsilon = 27$; $q_m = 0.68$

and for $\mu = 2$ $(\mu q\, 0.9 - 1) = 0.32$

Critical Dimensions

For a large sphere of graphite which contains a large number of small spheres of uranium the critical value, l, for the radius of the graphite sphere for which the chain reaction becomes divergent may be calculated for various distributions of uranium within the graphite sphere. The treatment of this question may as well be postponed until the value of the carbon capture cross-section is known. In the meantime, a very rough approximation may give an idea of the order of magnitudes which are involved. In graphite of 1.7 density the meandistance $\tilde{\tau}$ to which a fast neutron emitted by uranium diffuses away from its point of origin until it becomes a thermal neutron and reacts with uranium or carbon is about 50cm

For $(\mu q - 1) = \frac{1}{8}$ we find for the critical radius $l \sim 250\ cm$.

$$l \sim \sqrt{\frac{3\tilde{\tau}^2}{\mu q - 1}} \quad \text{or} \quad l \sim \sqrt{\frac{7500}{\mu q - 1}}$$

The amount of uranium required can be calculated from equation No. 33a

$$\frac{4\pi}{3} R^3 / \gamma = \frac{1 - q_m}{6} \frac{R^2}{B^2} \frac{1}{1 + R/B}$$

It may be kept down by choosing a smaller value for R than the value corresponding to the

Note for Memorandum

The approximation used here corresponds to a treatment of the problem which was first given in a patent application that was filed in England in 1934, and was assigned to the British Admiralty. Appendix No. is a copy of pages of the corresponding American patent application which was filed in March, 1935. The mean distance $\tilde{\tau}$ in this paper has to be identified with the mean free path "a" in the appendix. The differential equation quoted in the appendix leads, if written in the notation of the present paper, to a critical radius of

$$l = \pi \tilde{\tau} \sqrt{\frac{1}{3(\mu q - 1)}}$$

It may be kept down by choosing a smaller value for R than the value corresponding to the maximum value of ε. For R = 5 cm. and $q_m = 0.6$ 33a gives

$\frac{4\pi}{3} R^3/V = 0.022$ corresponding to 20 tons of uranium for 100 tons of graphite.

MEASUREMENTS

In order to determine the critical dimensions and the most favorable distribution of the small uranium spheres within a large graphite sphere it is essential to have an accurate value for μ and for the capture cross-section of carbon for thermal neutrons.

In order to have an accurate value for μ it will be necessary to have a direct measurement of p for small hydrogen concentrations preferably for n = 3.

The upper limit which has been reported for the carbon cross-section by Halban, Frisch, and Koch is already so low that it would be difficult to improve upon it unless a method is used which is specifically designed to measure extremely small capture cross-sections. Such a method will be described in the following:

Let us consider a sphere of carbon and a neutron source in the center of the sphere. The thermal neutron density inside the carbon will then obey equation No. 3 only in this case Q is a function of r of which we must not assume $\frac{dQ}{dr} \equiv 0$. Let the carbon sphere be immersed in a water tank or surrounded by paraffin wax. The thermal neutron density will then have a certain value at the surface of the sphere and inside the sphere it will be some function of r: $\rho_1(r)$. If in a second experiment the surface of the sphere in the water is covered by cadmium the thermal neutron density at the surface of the sphere is then reduced to practically zero and inside the sphere it will be another function of r, $\rho_2(r)$. The difference

$\rho(r) = \rho_1 - \rho_2$ obeys the homogeneous equation

$$D \frac{d^2(r\rho)}{dr^2} - \lambda r\rho = 0$$

which has the solution

$$\rho(r) = C \frac{e^{r/A} - e^{-r/A}}{r}$$

We can thus find A by determining the value of ρ for two values of r, for instance: r = 0 and r = r. It is

$$\frac{\rho(r)}{\rho(0)} = \frac{e^{r/A} - e^{-r/A}}{2\, r/A}$$

Or for small values of r/A

$$\frac{\rho(r)}{\rho(0)} \cong 1 + \frac{1}{6}\left(\frac{r}{A}\right)^2$$

Using a sphere of graphite of fifty to seventy centimeters of radius it should be possible to measure the range A with sufficient accuracy. There is a limit to using very large spheres which arises out of the fact that for a very large sphere most of the neutrons emitted in the center of the sphere will be slowed down to the thermal region within the carbon and the thermal neutron density near the surface of the carbon may become very low. If that happens, then the difference $\rho = \rho_1 - \rho_2$ will become small and will therefore set a limit to the accuracy of the measurement.

Stabilizing the chain reaction

Soon after the discovery of an abundant neutron emission from uranium the question of stabilizing such a reaction was a subject of discussion (14,15) but the situation as we see it appears to be rather different.

If a chain reaction could be maintained in a homogeneous mixture of water and uranium or carbon and uranium it would have a certain natural stability in the sense that with rising temperature there would be a decrease in the neutron production. The reason for this is the fact that the absorption of both uranium and hydrogen obey the 1/v law in the thermal region and thus at higher temperatures the range of thermal neutrons in the mixture is larger. Correspondingly, at higher temperatures a larger fraction of the thermal neutrons will escape across the boundary of the mixture without having reacted with the uranium in the mixture. This natural stability could even be enhanced by having bodies of strong thermal neutron absorbers inserted in the mixture. Fairly thin sheets of such absorbers as boron, for instance, are practically "black" for thermal neutrons and any strong thermal neutron absorber would stabilize equally well.

A system, on the other hand, in which uranium bodies which are almost "black" for thermal neutrons are embedded in carbon, like the system which we have considered in great detail in the present paper, has no such stability. This is due to the fact that with rising temperature the capture cross-section of the carbon decreases whereas the absorption by the uranium spheres remains almost unchanged. Accordingly, at higher temperatures, a larger fraction of the thermal neutrons is absorbed by uranium and a smaller fraction is absorbed by carbon and this leads to an increase in q and thermal instability.

The rate at which a nuclear chain reaction can be maintained is determined by the rate at which the heat liberated in the chain reaction can be dissipated. A system can not be efficient in this respect unless considerable temperature differences can be maintained, i.e.; unless the system can withstand high temperatures. Carbon is an excellent material from this point of view and it was this point of view which led me to estimate the chance of a chain reaction in carbon. Finding that carbon can be obtained in considerable purity the high density and the moderate in the form of graphite bricks which can be easily used to be the structure composed of alternate layers of uranium oxide and graphite I tried to estimate the chance of a chain reaction in such a system. The behaviour of such a system is very different from systems in which water is used for slowing down the neutrons and in the case of the carbon I found that the result is independent of the intensity and shape of the lowest resonance line of uranium in such a system. Therefore in spite of inadequate knowledge of this line it was possible to arrive at a favorable forecast.

In order to understand the difference between a hydrogen system and a carbon system one has to realize that the range of resonance neutrons in carbon is small compared to the range of thermal neutrons. This makes it possible to summarily take into account the lowest resonance of uranium, but assuming that the uranium is black for neutrons which have energies within a certain energy interval, say between 10 and 100 volts. Such a simple assumption has been made in the enclosed paper and leads to very simple formulas. Naturally, uranium will have some absorption at xxx higher energies which can not be taken into account in such a way by the general experience which I had from some previous work on the capture by heavy elements of even atomic number led me to believe that this neglected absorption will not be very significant. While neglecting the higher resonance tends to overestimate the chances of a chain reaction the approximation used in the paper which is made possible by considering small dense spheres of uranium metal only exaggerates the value of the loss of neutrons due to absorption in carbon and might to some extent counter-balance the neglect of the higher resonance.

range of the resonance neutrons.

For a number of practical reasons it would be preferable to use carbon in the form of graphite rather than hydrogen in the form of water for the purposes of a chain reaction. The capture cross-section of carbon for thermal neutrons is small. An upper limit of has been reported by Frisch, Halban, and Koch, but this upper limit is not sufficiently low to allow us at present to conclude that a chain reaction could be maintained in homogeneous mixtures of uranium and carbon. For neutrons it takes about 6.5 collisions with carbon atoms to reduce their energy by a factor of e. Thus a neutron which is being slowed down by carbon stays for a long time within the resonance absorption region of uranium. Consequently, very low uranium concentrations would have to be used in order to avoid that a large fraction of the fast neutrons emitted by uranium be absorbed at resonance by uranium. At such very lw uranium concentrations, on the other hand, the fraction of the thermal neutrons which is absorbed by carbon might perhaps be too large to permit a chain reaction.

In the present paper we consider a system in which a large number of fairly large spheres of uranium which are practically black for resonance neutrons of uranium in an energy interval stretching from perhaps ~~two~~ 2 to ~~twenty~~ 20 volts are embedded in carbon. It will be shown that the ratio of the number of thermal neutrons and the number of resonance neutrons absorbed by the uranium can be made so small in such a system that we may expect a chain reaction to become a reality.

In order to get a better picture of this limitation we may assume for the sake of ~~argument~~ argument that one half of the fission neutrons has an initial energy above the dissociation energy of beryllium, and that the cross-section for the disintegration of ~~beryllium~~ is one third of ~~the~~ of total cross-section ~~of beryllium~~ (and one half of its elastic collision cross-section). A fission neutron would then in its first collision with a beryllium nucleus on the average knock out .166 neutrons. If we further assume, rather arbitrarily, that the fission neutrons withstand two elastic collisions with beryllium with undiminished capacity for the disintegration of beryllium, but that after the third elastic collision their energy ~~has become so low that they are no longer capable of splitting beryllium~~ is below the threshold we find that a fission neutron moving entirely in beryllium would liberate about $0.5\left(\frac{1}{3}+\frac{2}{9}+\frac{4}{27}\right)=0.35$ neutrons and not more. [[In our arrangement collisions will take place with carbon atoms as well as beryllium atoms, and accordingly the total number of neutrons liberated from beryllium by one fission neutron would be smaller. It should be emphasized though that a value of 0.2 would ~~be~~ already be very significant since it would raise, μ, the value of the neutrons generated in the system per thermal neutron absorbed in uranium from a value between 1.5 and 2 to a value between 1.8 and 2.4. The data available at present do not permit to estimate the increase in μ which we may expect from the introduction of beryllium into a system composed of uranium and carbon. ~~But~~ Experiments using 75 to 150 lbs of beryllium are in preparation for the purpose of clearing up this point.

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Following the discovery[1,2,3] of an abundant emission of fast neutrons from uranium the balance of neutron absorption and production has been studied by means of different types of experiments.[4,5,6] Carrying out one type of experiment Anderson, Fermi, and Szilard reported[5] that on the average about $\mu = 1.5$ fast neutrons are emitted by uranium for every thermal neutron absorbed by uranium. A more accurate value for μ can be obtained in principle from a second type of experiment such as the one performed by Halban, Joliot, Kowarski and Perrin for which we have

$$(1) \qquad \mu = \frac{1}{1-p} + \frac{n\sigma_H}{\sigma_U}\left(\frac{1}{1-p} - J\right)$$

p is the fraction of neutrons absorbed by uranium at resonance in a uranium oxide water mixture containing n atoms of hydrogen for one atom of uranium, σ_H/σ_U is the ratio of the capture cross-section of hydrogen and the absorption cross-section of uranium comprising both radiative capture and fission. The value of J is determined by the values directly measured in an experiment of this second type. For the experiment which has been actually carried out by Halban, Joliot, Kowarski, and Perrin[6] I find for J from the published data the value of J = 1.8 for n = 3. ~~to have p = 0.5 it is~~ Taking into account that for $n = 3$ p has a value of 0.5 we have

$$\mu = 2 + 0.2\,\frac{3\sigma_H}{\sigma_U}$$

The value obtained for μ if calculated from this equation is scarcely affected by the wide limits of error of the present experimental values of σ_U. A very accurate value for μ could thus be obtained by measuring the value of p accurately and an attempt is being made in this direction. In the following we shall use as the best value available at present $\mu = 2$.

As stated before in a homogeneous mixture of uranium and water a considerable fraction of the neutrons is absorbed by uranium at resonance and it remains quite uncertain whether in any such mixture the number of neutrons produced can exceed the number of neutrons absorbed, i.e. whether a chain reaction in the true sense of the word can be maintained in such mixtures.[5]

From several points of view it would be preferable to use carbon in the form of graphite rather than hydrogen in the form of water for slowing down the neutrons. The capture cross-section of carbon for thermal neutrons σ_C is small; an upper limit of $\sigma_C > 0.01 \times 10^{-24}$ cm^2

has been reported for room temperatures by Frisch, Halban, and Koch. It is not possible though to state at present whether or not a chain reaction can be maintained in homogeneous mixtures of uranium or uranium oxide and carbon.

The purpose of this communication is to report that a method for obtaining a chain reaction in a system composed of uranium and carbon has been devised and worked out in detail. This method is based on the fact that in a lattice of uranium spheres embedded in graphite the ratio of the number of thermal neutrons and the number of resonance neutrons which are absorbed by the uranium spheres can be made very small if only the radius R of the uranium spheres is chosen rather small. In spite of our present insufficient knowledge of the value of the capture cross-section of carbon it can be predicted that by using such a lattice of uranium spheres instead of a homogeneous mixture a chain reaction can be maintained which will be divergent for a lattice of small uranium spheres embedded in a large sphere of graphite.

We find for a lattice of uranium spheres in carbon that a chain reaction can be maintained provided we have

$$\frac{4.4\mu}{(\mu-1.1)^2} < \varepsilon \qquad (2)$$

ε is defined in the following way: a single uranium sphere is embedded in an infinite mass of graphite. Neutrons are generated in the graphite and the numbers of thermal neutrons and resonance neutrons produced per c.c. and sec. are equal and have the same value throughout the whole infinite mass of graphite. ε is the ratio of the number of thermal neutrons and resonance neutrons absorbed under these conditions by the uranium sphere.

In order to obtain a conservative estimate for ε we calculate its value under the assumption that every neutron which reaches the uranium sphere by diffusion while its energy is between 0.2 E and 2E is absorbed by the ~~sphere~~ uranium at resonance, E being the energy at which the resonance absorption of uranium has its maximum. Using only such values of nuclear cross-sections in our computations which appear to be well established we find thus for a sphere of uranium metal of less than 8 cm. radius at 900 Centigrade, $\varepsilon > 14$. This value is higher than the value given by Formula (2) which is 11 for $\mu = 2$.

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In these circumstances we can expect a chain reaction to take place in a sufficiently large mass of graphite which contains for instance a close-packed hexagonal or cubic lattice of uranium spheres. The capture cross-section of carbon may of course be much smaller than the upper limit quoted above and consequently there is hope that only moderately large masses of graphite and uranium or uranium oxide will be needed to reach the point of divergence at which nuclear transmutation can be maintained at an intensity which is limited only by the necessity of avoiding overheating. An experiment requiring several tons of graphite has been devised for the purpose of measuring the exact value of σ_C ,

Large quantities of radio-active elements will be produced directly from the splitting uranium atoms and indirectly by the intense neutron emission. The necessity of protecting human beings from deadly irradiation emanating from the chain reaction will undoubtedly limit the scope of practical applications and perhaps will slow down the industrial development of this field but it is difficult to imagine that practical applications should not follow in due course of time the present turn of events in physics.

It can be shown that a chain reaction maintained in a lattice of uranium spheres embedded in graphite increases in intensity if the temperature of the carbon rises; this represents a type of thermal instability which can not easily be remedied. A dynamic method for stabilizing the chain reaction has therefore been devised and encounters less difficulties than might be supposed. The problem of leading away the heat produced (most of which is liberated within the uranium spheres) without blocking the chain reaction has also been investigated and is capable of a satisfactory solution.

In so far as the production of power for practical purposes is concerned the crucial question which will determine the scope of applications is now whether the rare isotope of uranium 235 or the abundant isotope 238 is the active agent in the thermal neutron reaction. If the rare isotope is the active agent ten tons of uranium may become exhauseted by the chain reaction after having supplied as much power as can be obtained from about fifty thousand tons of coal. In case of the other

alternative, ten tons of uranium could supply more power than five million tons of coal without being used up. Though N. Bohr put forward interesting arguments[7] in support of the view that it is the rare isotope which is split by thermal neutrons this question will have to be decided by direct observations performed on small samples of the separated isotopes. Only after some such observation shall we be able to express a well balanced opinion upon the immediate future of "atomic engineering."

Leo Szilard

Pupin Physics Laboratories
Columbia University
New York, New York
February 6, 1940

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$$(1)\qquad \mu = \frac{1}{1-p} + \frac{n\,\sigma_c(H)}{\sigma_a(U)}\left\{\frac{1}{1-p} - J\right\}$$

p is the fraction of neutrons absorbed by uranium at resonance in a uranium-oxide-water mixture containing n atoms of hydrogen for one atom of uranium, $\sigma_c(H)/\sigma_a(U)$ is the ratio of the capture cross-section of hydrogen and the absorption cross-section of uranium comprising both radiative capture and fission. The value of J is determined by the values directly measured in an experiment of this second type. For the experiment which has been actually carried out by Halban, Joliot, Kovarski, and Perrin I find[6] from the published data for J the value of $J = 1.8$ for $n = 3$. Taking into account that for $n = 3$ we have $p = 0.5$ ~~we have~~ it is

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The value obtained for μ if calculated from this equation is scarcely affected by the wide limits of error which are still attached to the experimental value of $\sigma_a(U)$. A very accurate value for μ could thus be obtained by measuring the value of p accurately and an attempt is being made in this direction. In the following we shall use as the best value available at present; $\mu \cong 2$.

As stated before[5] in a homogeneous mixture of uranium and water a considerable fraction of the neutrons is absorbed by uranium at resonance and it remains quite uncertain whether in any such mixture the number of neutrons produced can exceed the number of neutrons absorbed, i.e. whether a chain reaction in the true sense of the word can be maintained in such mixtures.

From several points of view it would be preferable to use carbon in the form of graphite for slowing down the neutrons rather than water. The capture cross-section of carbon for thermal neutrons $\sigma_c(C)$ is small; an upper limit of $\sigma_c(C) < 0.01\times10^{-24}$[7] has been reported for room temperatures by Frisch, Halban, and Koch. It is not possible though to state at present whether or not a chain reaction can be maintained in homogeneous mixtures of uranium and carbon.

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In order to obtain a conservative estimate for ε we calculate its value under the assumption that every neutron which reaches the uranium sphere by diffusion while its energy is between 0.2 E and 2 E is absorbed by the sphere at resonance, E being the energy at which the resonance absorption of uranium has its maximum. Using only such values of nuclear cross-sections in our computationx which appear to be well established we find thus for a uranium sphere of less than 8 cm. radius at 900 Centigrade, $\varepsilon > 14$.

In these circumstances we can expect a chain reaction to take place in a sufficiently large mass of graphite which contains for instance a close-packed hexagonal or cubic lattice of uranium spheres. The capture cross-section of carbon may of course be much smaller than the upper limit quoted above and consequently there is hope that only moderately large masses of graphite will be needed to reach the point of divergence at which nuclear transmutation can be maintained at an intensity which is limited only by the necessity of avoiding over-heating. An experiment requiring several tons of graphite is being prepared in order to measure the exact value of $\sigma_d(C)$.

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Large quantities of radio-active elements will be produced directly from the splitting uranium atoms and indirectly by the intense neutron emission. The necessity of protecting human beings from deadly irradiations emanating from the chain reaction will undoubtedly limit the scope of practical applications, and it will, perhaps, slow down the industrial development of this field but it is difficult to imagine that practical applications should not follow in due course of time the present turn of events in physics.

It can be shown that a chain reaction maintained in a lattice of uranium spheres embedded in graphite increases in intensity if the temperature of the carbon rises; ~~and correspondingly has an essential~~ (It represents) thermal instability which can not easily be remedied. A dynamic method for stabilizing the chain reaction has therefor been devised and encounters less difficulties than might be supposed. The problem of leading away the heat produced (most of which is liberated within the uranium spheres) without blocking the chain reaction has also been investigated and is capable of a satisfactory solution .

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In these circumstances we can expect a chain reaction to take place in a sufficiently large mass of graphite which contains for instance a close-packed hexagonal or cubic lattice of uranium spheres. The capture cross-section of carbon may of course be much smaller than the upper limit quoted above and consequently there is hope that only moderately large masses of graphite and uranium or uranium oxide will be needed to reach the point of divergence at which nuclear transmutation can be maintained at an intensity which is limited only by the necessity of avoiding over-heating. An experiment requiring several tons of graphite has been devised for the purpose of measuring the exact value of σ_c.

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Following the discovery[1][2][3] of an abundant emission of fast neutrons from uranium the balance of neutron absorption and production has been studied by means of different types of experiments.[4][5][6] Carrying out one type of experiment[3] Anderson, Fermi, and Szilard reported that on the average about $\mu = 1.5$ fast neutrons are emitted by uranium for every thermal neutron absorbed by uranium. A more accurate value for μ can be obtained in principle from a second type of experiment such as the one performed by Halban, Joliot, Kovarski and Perrin for which I find

$$\mu = \frac{1}{1-p} + \frac{n\sigma_H}{\sigma_u}\left\{\frac{1}{1-p} - J\right\} \qquad (1)$$

p is the fraction of neutrons absorbed by uranium at resonance in a uranium oxide water mixture containing n atoms of hydrogen for one atom of uranium, σ_H/σ_u is the ratio of the capture cross-section of hydrogen and the absorption cross section of uranium comprising both radiative capture and fission. The value of J is determined by the values directly measured in an experiment of this second type. For the experiment which has been actually carried out by Halban, Joliot, Kovarski, and Perrin[6] I find for J from the published data the value of $J = 1.8$ for $n = 3$. Taking into account that for $n = 3$ we have $p = 0.5$ it is

$$\mu = 2 + 0.2\,\frac{3\sigma_H}{\sigma_u}$$

The value obtained for μ if calculated from this equation is scarcely affected by the wide limits of error which are still attached to the experimental value of σ_u. A very accurate value for μ could thus be obtained by measuring the value of p accurately and an attempt is being made in this direction. In the following we shall use as the best value available at present: $\mu = 2$.

As stated before in a homogeneous mixture of uranium and water a considerable fraction of the neutrons is absorbed by uranium at resonance and it remains quite uncertain whether in any such mixture the number of neutrons produced can exceed the number of neutrons absorbed, i.e. whether a chain reaction in the true sense of the word can be maintained in such mixtures.[5]

From several points of view it would be preferable to use carbon in the form of graphite rather than water for slowing down the neutrons. The capture cross-section of carbon for thermal neutrons σ_C is small; an upper limit of $\sigma_C > 0.01 \times 10^{-24}\ cm^2$ has been reported for room temperatures by Frisch, Halban, and Koch. It is not possible though to state at present whether or not a chain reaction can be maintained in homogeneous mixtures of uranium and carbon.

The purpose of this communication is to report that a method for obtaining a chain reaction in a system composed of uranium and carbon has been devised and worked out in detail. This method is based on the fact that in a lattice of uranium spheres embedded in graphite the ratio of the number of thermal neutrons and the number of resonance neutrons which are absorbed by the uranium spheres can be made very small if only the radius R of the uranium spheres is chosen rather small. In spite of our present insufficient knowledge of the value of the capture cross-section of carbon it can be predicted that by using such a lattice of uranium spheres instead of a homogeneous mixture a chain reaction can be maintained which will be divergent for a lattice of small uranium spheres embedded in a large sphere of graphite.

I find for a lattice of uranium spheres in carbon that a chain reaction can be maintained provided we have

$$\frac{4.4\,\mu}{(\mu - 1.1)^2} < \varepsilon \qquad (2)$$

ε is defined as the ratio of the number of thermal neutrons and the number of resonance neutrons which would be absorbed per second by a single uranium sphere which is embedded in an infinite mass of graphite in the special case in which the numbers of resonance neutrons and thermal neutrons produced per c.c. and second are equal and have the same value throughout the whole infinite mass of graphite.

In order to obtain a conservative estimate for ε we calculate its value under the assumption that every neutron which reaches the uranium sphere by diffusion while its energy is between 0 and 2 E and 2 E is absorbed by the sphere at resonance, E being the energy at which the resonance absorption of uranium has its maximum. Using only such values of nuclear cross-sections in our computations which appear to be well established we find thus for a uranium sphere of less than 8 cm. radius at 900 Centigrade, $\varepsilon > 14.$ — This value has to be compared with the value given by Formula (2) which is 11 for $\mu = 2$.

In these circumstances we can expect a chain reaction to take place in a sufficiently large mass of graphite which contains for instance a close-packed hexagonal or cubic lattice of uranium spheres. The capture cross-section of carbon may of course be much smaller than the upper limit quoted above and consequently there is hope that only moderately large masses of graphite will be needed to reach the point of divergence at which nuclear transmutation can be maintained at an intensity which is limited only by the necessity of avoiding over-heating. An experiment requiring several tons of graphite is being prepared in order to measure the exact value of σ_C.

Large quantities of radio-active elements will be produced directly from the splitting uranium atoms and indirectly by the intense neutron emission. The necessity of protecting human beings from deadly irradiations emanating from the chain reaction will undoubtedly limit the scope of practical applications and will, perhaps, slow down the industrial development of this field but it is difficult to imagine that practical applications should not follow in due course of time the present turn of events in physics.

It can be shown that a chain reaction maintained in a lattice of uranium spheres embedded in graphite increases in intensity if the temperature of the carbon rises; this represents a type of thermal instability which can not easily be remedied. A dynamic method for stabilizing the chain reaction has therefore been devised and encounters less difficulties than might be supposed. The problem of leading away the heat produced (most of which is liberated within the uranium spheres) without blocking the chain reaction has also been investigated and is capable of a satisfactory solution.

In so far as the production of power for practical purposes is concerned the crucial question which will determine the scope of applications is now whether the rare isotope of uranium 235 or the abundant isotope 238 is the active agent in the thermal neutron reaction. If the rare isotope is the active agent ten tons of uranium would become exhausted by the chain reaction after having supplied as much power as can be obtained from about fifty thousand tons of coal. In case of the other alternative, ten tons of uranium could supply more power than five million tons of coal without being used up. Though N. Bohr put forward interesting arguments (7) in support of the view that it is the rare isotope which is split by thermal neutrons this question will have to be decided by direct observations performed on small samples of the separated isotopes. Only after some such observation shall we be able to express a well balanced opinion upon the immediate future of "atomic engineering."

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7. N. Bohr Phys Rev. (1939(