Approximate Theory

A very simple but only approximately correct theory of the most important quantities which enter into the construction of a potentially chain reacting system had been given for a lattice, for instance, a close-packed lattice, the elements of which are spheres of uranium metal, in an unpublished paper submitted to Phys. Rev. in February, 1940. The outlines of this theory are the following:

The picture is simplified by assuming that the fast neutron generated in a chain reacting lattice can be considered as produced uniformly throughout the graphite within the cell of the lattice (Q fast neutrons per cc and sec). The average thermal neutron density within the lattice is smaller by a factor $\mathcal{L}$ than it would be in graphite in the absence of uranium for equal fast neutron production per cc in graphite. The approximation used in the theory assumes that the flow of thermal neutrons $\mathcal{J}^{th}$ into a uranium sphere within the lattice is equal to $\mathcal{L}\,\mathcal{J}^{th}$ where $\mathcal{J}^{th}$ is the flow of thermal neutrons into a single uranium sphere embedded in an infinite amount of graphite for equal production of fast neutrons per cc in graphite. The theory further assumes that the flow of resonance neutrons, $\mathcal{J}^{res}$, into a sphere in the lattice is equal to $\mathcal{J}^{res}$ the flow of resonance neutrons into a single sphere embedded into an infinite amount of graphite for equal fast neutron production per cc in the graphite.

With these assumptions it is easy to determine the fraction q of the neutrons which are slowed down to the thermal region and are absorbed in the thermal region by the uranium spheres in the lattice. For a uranium sphere of given radius R, the most favorable ratio of uranium to carbon (the most favorable lattice spacing) is the ratio for which q becomes a maximum. For the corresponding value of $\mathcal{L} = \mathcal{L}_m$, the theory gives the equation

(8) $$\mathcal{L}_m = \frac{1 - q_m}{2}$$

Since $\mathcal{L}$ is equal to the fraction of the neutrons which are slowed down to the thermal region and are absorbed in the thermal region by carbon, and since q is the fraction of the usefully absorbed neutrons, the above equation expresses the fact that for the most favorable ratio of uranium to carbon, half of the neutrons which are not usefully absorbed are absorbed as thermal neutrons by carbon. It follows that the other half of the 'not-usefully' absorbed neutrons must be absorbed at resonance by uranium. This fact can be expressed by writing:

(9) $$\frac{1 - q_m}{2} \quad \frac{1}{V} \quad \frac{J_{res}}{Q}$$

which we can write also in the form

(9a) $$\frac{1 - q_m}{6} = \frac{\frac{4 \pi R^3}{3}}{V} \left(\frac{J_{res}}{Q\ 4\ \ R^3}\right)$$

The expression J_{res}/Q signifies the number of resonance neutrons absorbed per sec by a single sphere of uranium which is embedded in an infinite mass of graphite if one fast neutron is produced in the graphite per cc and sec; V stands for the volume of graphite per uranium sphere in the lattice. (9A) may be used to calculate $\frac{4\pi R^3}{3} / V$, the ratio of uranium volume to carbon volume in the lattice. From this, the ratio of weights is obtained by multiplying with the ratio of the densities.

q can be calculated within the validity of this simple theory for any given radius R of the uranium spheres which form the lattice element from ε, the ratio of the thermal neutron absorption to the resonance neutron absorption which holds for a single sphere of the radius R that is embedded into an infinite mass of graphite in which there is a uniform production of fast neutrons per cc of graphite.

According to the above definitions we may write for

(10) $$\varepsilon = \frac{J^{th}}{J^{res}}$$

According to the simple theory, we may calculate q from ε from the equation

(11) $$\frac{4\, q_m}{(1 - q_m)^2}$$

In order to obtain an approximate value for $\frac{J^{res}}{Q}$ the resonance absorption of a uranium sphere may be represented for a given size sphere by assuming that the uranium is black for resonance neutrons with a certain energy interval and does not absorb resonance neutrons outside that energy interval. The resonance absorption of the uranium sphere is then determined by a certain range B of the resonance neutrons in graphite, for which we may write

(12) $$B = \frac{\lambda^*}{\sqrt{3}} \sqrt{\frac{\ln \quad E_1/E_2}{\ln \left(1 - \frac{2m}{(1+m)^2}\right.}}$$

where E_1 and E_2 are the limite of the energy interval within which the uranium is considered as black for resonance neutrons. Or putting in general $E_1/E_2 = 1/10$ and for graphite in particular, m = 12. We obtain for graphite of density 1.7

(12b) $$B \sim \frac{\lambda^*}{\sqrt{3}} \sqrt{6.5 \ln 10}$$

If the resonance absorption of the uranium sphere is thus represented, we may write

(13) $$\frac{J^{res}}{Q} = 4 \pi R B^2 \;(1 + R/B)$$

so that we have from 9a

(14) $$\frac{1 - q m}{2} = \frac{4 \pi R B^2 \;(1 + R/B)}{V}$$

or

(15) $$\frac{4 \pi R^3}{3} \Big/ V - \frac{1 - qm}{6} \frac{R^2}{B^2} \frac{1}{1 + R/B}$$

Within the limits of the approximation for the resonance neutron absorption, we may write for

(16) $$\varepsilon = \frac{A^2}{B^2} \times \left\{ \frac{1}{1 + R/B} \times \frac{1}{\dfrac{\lambda(C)}{RG \dfrac{3\sigma_a(U)}{\sigma_{sc}(U)}} + \dfrac{1}{1 + R/A}} \right\}$$

where

(17) $$G = \frac{e^{R/U} - e^{-R/U}}{e^{R/U} - e^{-R/U}}$$

and

$$R/U \quad \frac{R}{\lambda(U)} \sqrt{\frac{3\sigma_a(U)}{\sigma_{sc}(U)}}$$

According to the simplified theory, we obtain the highest value of q for a value of R for which ε becomes maximum. In choosing the most suitable size for the lattice element, there are, however, other points of view which have to be considered and the value of $R = 5$ cm for a uranium density of 15 gm/cm^2 was recommended on the basis of this simplified theory. A ratio of about 30 tons of uranium per 100 tons of graphite was recommended for $R = 5$ cm.

It should be noted that while this simplified theory may be applied to small uranium spheres, and while the formulae give correctly the trend which characterized the variations of q, ε, and V, the formulae given are not strictly speaking correct and in order to improve them, one would have to consider η and B as functions of R. The variation of η with R may then take into account the fact that the neutron contribution of fission caused by fission neutrons in the uranium sphere increases with increasing R and by writing B as a function of R, we may account for the fact that the uranium sphere is not black for higher energy resonance neutrons so that we have a term in the resonance absorption which is proportionate to the mass of the sphere. The corrections which one would have to apply, cancel out, however, rather well for spheres of about 5cm radium. By neglecting the fission by fission neutrons, we have shifted the optimum towards smaller spheres than would correspond to reality, while by neglecting the absorption of higher energy resonance neutrons, we have shifted the optimum towards larger spheres than would correspond to reality. Finally, by writing we have tended to favor smaller spheres than correspond to reality.

A 5 cm sphere of 15 gms/cc density weighs about 7.5 kg, and it may be that from the strict point of view of having the highest value for q, a somewhat smaller mass of the lattice element which has a spheroid shape would be preferable, if no gap is left free between the lattice element and the graphite mass which contains the lattice.

The formula 9A or 15 permits a quick survey of the most favorable ratio of uranium to carbon if various factors are varied, since the value of q does not change much, with the radius or the density of the uranium in the sphere as long as we do not go to extreme values. With q having a value between .7 and .8, one has only to insert into 9A or 15 the value for g^{res} or B respectively, to obtain the approximate ratio of the volume of uranium to carbon. By multiplying this with the ratio of the densities, one obtains the ratio of the weights of uranium to carbon.

As it has been stated before, a lattice of uranium spheres is capable of maintaining a chain reaction even in impure graphite which corresponds to a capture cross section of carbon of nearly 0.01×10^{-24} cm^2, particularly if the uranium is cooled while part of the graphite is allowed to heat up. If the uranium and enough of the surrounding graphite is kept cool, the thermal neutrons which diffuse to the uranium spheres will be cold even though the rest of the graphite has been allowed to heat up. In these conditions of the neutrons which are slowed down to thermal energies, a smaller fraction is absorbed by carbon and a larger fraction is absorbed by uranium, than would be the case for a uniform temperature throughout the lattice. In order to maintain a large temperature difference with the heat flow that can be taken care of by the cooling system, the uranium and its surrounding graphite layer may be thermally insulated from the rest of the graphite by a gap (which may be filled with lamp black or some other fluffy material that does not appreciably absorb neutrons). Structurally, these conditions are easier to realize for a lattice of rods than for a lattice of spheres.

Approximate Theory

A very simple but only approximately correct theory of the most important quantities which enter into the construction of a potentially chain reacting system had been ~~given~~ described for a lattice, for instance, a close-packed lattice, the elements of which are spheres of uranium metal, in an unpublished paper submitted to Phys. Rev. in February, 1940. The outlines of this theory are the following:

The picture is simplified by assuming that the fast neutron generated in a chain reacting lattice can be considered as produced uniformly throughout the graphite within the cell of the lattice (Q fast neutrons per cc and sec). The average thermal neutron density within the lattice is smaller by a facter than it would be in graphite in the absence of uranium for equal fast neutron production per cc in graphite. The approximation used in the theory assumes that the flow of thermal neutrons into a uranium sphere within the lattice is equal to , where is the flow of thermal neutrons into a single uranium sphere embedded in an infinite amount of graphite for equal production of fast neutrons per cc in graphite. The theory further assumes that the flow of resonance neutrons, , into a sphere in the lattice is equal to the flow of resonance neutrons into a single sphere embedded into an infinite amount of graphite for equal fast neutron production per cc in the graphite.

With these assumptions it is easy to determine the fraction q of the neutrons which are slowed down to the thermal region and are absorbed in the thermal region by the uranium spheres in the lattice. For a uranium sphere of given radius R, the most favorable ratio of uranium to carbon (the most favorable lattice spacing) is the ratio for which q becomes a maximum. For the corresponding value of , the theory gives the equation

(8)

Since is equal to the fraction of the neutrons which are slowed down to the thermal region and are absorbed in the thermal region by carbon, and since q is the fraction of the usefully absorbed neutrons, the above equation expresses the fact that for the most favorable ratio of uranium to carbon, half of the neutrons which are not usefully absorbed are absorbed as thermal neutrons by carbon. It follows that the other half of the 'not-usefully" absorbed neutrons must be absorbed at resonance by uranium. This fact can be expressed by writing:

(9)

which we can write also in this form

(9A)

The expression signifies the number of resonance neutrons absorbed per sec by a single sphere of uranium which is embedded in an infinite mass of graphite if one fast neutron is produced in the graphite per cc and sec; V stands for the volume of graphite per uranium sphere in the lattice. (9A) may be used to calculate the ratio of uranium volume to carbon volume in the lattice. From this, the ratio of weights is obtained by multiplying with the ratio of the densities.

q can be calculated within the validity of this simple theory for any given radius R of the uranium spheres which form the lattice element from the ratio of the thermal neutron absorption to the resonance neutron absorption which holds for a single sphere of the radius R that is embedded into an infinite mass of graphite in which there is a uniform production of fast neutrons per cc of graphite.

According to the above definition we may write for

(10)

According to the simple theory, we may calculated q from from the equation

(11)

In order to obtain an approximate value for the resonance absorption of a uranium sphere may be represented for a given size sphere by assuming that the uranium is black for resonance neutrons with a certain energy interval and does not absorb resonance neutrons outside that energy interval. The resonance absorption of the uranium sphere is then determined by a certain range B of the resonance neutrons in graphite, for which we may write

(12)

where E_1 and E_2 are the limits of the energy interval within which the uranium is considered as black for resonance neutrons. If the resonance absorption of the uranium sphere is thus represented, we may write

(13)

so that we have

(14)

or

(15)

Within the limits of the approximation for the resonance neutron absorption, we may write for

(16)

According to the simplified theory, we obtain the highest value of q for a value of R for which becomes maximum. In choosing the most suitable size for the lattice element, there are, however, other points of view which have to be considered and the value of R=5 cm for a uranium density of 15 gm/cm^2 was recommended on the basis of this simplified theory. A ratio of about 30 tons of uranium per 100 tons of graphite was recommended for R=5 cm. On the basis of equation (1) it was stated that a graphite sphere containing a uranium lattice having a radius of more than 250 cm with the graphite weighing more than 100 tons may be expected to be capable of sustaining a divergent chain reaction. These estimates were based on the following estimate of the constants which are involved.

It should be noted that while this simplified theory may be applied to small uranium spheres, and while the formulae give correctly the trend which characterizes the variations of q, , and V, the formulaes given are not strictly speaking correct and in order to improve them, one would have to consider and B as functions of R. The variation of with R may then take into account the fact that the neutron contribution of fission caused by fission neutrons in the uranium sphere increases with increasing R and by writing B as a function of R, we may account for the fact that the uranium sphere is not black for higher energy resonance neutrons so that we have a term in the resonance absorption which is proportionate to the mass of the sphere. The corrections which one would have to apply, cancel out, however, rather well for spheres of about 5 cm radius. By neglecting the fission by fission neutrons, we have shifted the optimum towards smaller spheres than would correspond to reality, while by neglecting the absorption of higher energy resonance neutrons, we have shifted the optimum towards larger spheres than would correspond to reality. Finally, by writing

we have tended to favor smaller spheres than correspond to reality.

A 5 cm sphere of 15 gms/cc density weighs about 7.5 kg, and it may be that from the strict point of view of having the highest value for q, a somewhat small mass of the lattice element which has a spheroid shape would be preferable, if no gap if left free between the lattice element and the graphite mass which contains the lattice.

The formula 9A or 15 permits a quick survey of the most favorable ratio of uranium to carbon if various factors are varied, since the value of q does not change much, with the radius or the density of the uranium in the sphere as long as we do not go to extreme values. With q having a value between .7 and .8, one has only to insert into 9A or 15 the value for or B respectively, and obtain the approximate ratio of the volume of uranium to carbon. By multiplying this with the ratio of the densities, one obtains the ratio of the weights of uranium to carbon.

As it has been stated before, a lattice of uranium spheres is capable of maintaining a chain reaction even in impure graphite which corresponds to a capture cross section of carbon of , particularly if the uranium is cooled while part of the graphite is allowed to heat up. If the uranium and enough of the surrounding graphite is kept cool, the thermal neutrons which diffuse to the uranium spheres will be cold even though the rest of the graphite has been allowed to heat up. In these conditions of the neutrons which are slowed down to thermal energies, a smaller fraction is absorbed by carbon and a larger fraction is absorbed by uranium, than would be the case for a uniform temperature throughout the lattice. In order to maintain a large temperature difference with the heat flow that can be taken care of by the cooling system, the uranium and its surrounding graphite layer may be thermally insulated from the rest of the graphite by a gap which may be filled with lamp black or some other fluffy material that does not appreciably absorb neutrons. Structurally, these conditions are easier to realize for a lattice of rods than for a lattice of spheres.

Approximate theory (2)

A very simple but only approximately correct theory of the most important quantities which enter into the construction of a potentially chain reacting system had been given for a lattice, for instance a close-packed lattice, the elements of which are spheres of uranium metal, in an unpublished paper. submitted to Phys. Rev. in Febr. 1940: The outlines of this theory are the following:

The picture is simplified by assuming that the fast neutron generated in a chain reacting lattice can be considered as produced uniformly throughout the graphite within the cell of the lattice (Q fast neutrons per cc and sec). The average thermal neutron density within the lattice is smaller by a factor α than it would be in graphite in the absence of uranium for equal fast neutron production per cc in graphite. The approximation used in the theory assumes that the flow of thermal neutrons J^{th} into a uranium sphere within the lattice is equal to $\alpha\, \mathcal{J}^{th}$, where $\mathcal{J}^{th}$ is the flow of thermal neutrons into a single uranium sphere embedded in an infinite amount of graphite for equal production of fast neutrons per cc in graphite. The theory further assumes that the flow of resonance neutrons, J^{res}, into a sphere in the lattice is equal to $\mathcal{J}^{res}$ the flow of resonance neutrons into a single sphere embedded into an infinite amount of graphite for equal fast neutron production per cc in the graphite.

With these assumptions it is easy to determine the fraction q of the ~~Q~~ neutrons which are slowed down to the thermal region and are absorbed in the thermal region by the spheres in the lattice. ~~The most favorable arrangement~~ For a uranium sphere of given radius R, the most favorable ratio of uranium to carbon (the most favorable lattice spacing) is the ratio ~~from~~ for which q becomes a maximum.

For the corresponding value of α, the theory gives the equation

$$\alpha_{max} = \frac{1 - q_{max}}{2} \tag{8}$$

Since α is equal to the fraction of the neutrons which are slowed down to the thermal region and are absorbed in the thermal region by carbon, and since q is the fraction of the usefully absorbed neutron, the above equation expresses the fact that for the most favorable ratio of uranium to carbon, half of the neutrons which are not usefully absorbed are absorbed as thermal neutrons by carbon. It follows that the other half of the 'not usefully' absorbed neutrons must be absorbed at resonance by uranium. This fact can be expressed by writing:

$$\frac{1 - q_{max}}{2} = \frac{1}{V} \frac{\sigma^{res}}{Q} \tag{9}$$

which we can write also in this form ~~In this formula~~

$$\frac{1 - q_m}{6} = \frac{\frac{4\pi R^3}{3}}{V} \left(\frac{\sigma^{res}}{Q\, 4\pi R^3} \right) \tag{9a}$$

The expression $\frac{\sigma^{res}}{Q}$ signifies the number of resonance neutrons absorbed per sec by a single sphere of uranium which is embedded in a infinite mass of graphite if one fast neutron is produced in the graphite per cc and sec.; V stands for the volume of graphite per uranium sphere in the lattice. (9e) may be used to calculate $\frac{4\pi R^3}{3}$ / V the ratio of Uranium volume to Carbon volume in the lattice. From this the ratio of weights is obtained by multiplying with the ratio of the densities.

q can be calculated within the validity of this simple theory for any given radius R of the uranium spheres which form the lattice element from (ε) the ratio of the thermal neutron absorption to the resonance neutron absorption which holds for a single sphere of the radius R that is embedded into an infinite mass of graphite in which there is a uniform production of fast neutrons per cc of graphite.

According to the above definition we may writer for ε

$$\varepsilon = \frac{\sigma^{th}}{\sigma^{res}} \tag{10}$$

According to the simple theory, we may calculated q from ε from the equation

$$\varepsilon = \frac{4 q_m}{(1 - q_m)^2} \tag{11}$$

The same fact can be expressed by writing

$$\frac{1-q_m}{2} = \frac{\mathcal{I}^{res}/Q}{V}$$

In this formula the expression $\frac{I^{res}}{Q}$ signifies the number of resonance neutrons absorbed by a single sphere of uranium imbedded into an infinite mass of graphite if one fast neutron is produced in the graphite per cc. and second, and V stands for the volume of graphite per uranium sphere in the lattice. q can be calculated from this simple theory which gives the following relation between Σ and q for the most favorable ratio of uranium to carbon (for which q becomes a maximum):

$$\Sigma = \frac{4 q_m}{(1-q_m)^2}$$

In this way by ~~determining~~ calculating q^m and the resonance absorption ~~of~~ $\mathcal{I}^{res}/Q$ of a single uranium sphere which forms the lattice element ~~and from q we~~ may calculate V and thereby determine the lattice spacing.

In order to obtain an approximate value for $\frac{\mathcal{I}^{res}}{Q}$ the resonance absorption of a uranium sphere may be represented for a given size sphere by assuming that the uranium is black for resonance neutrons with a certain energy interval and does not absorb resonance neutrons outside the energy interval. The resonance absorption of the uranium sphere is then determined by a certain range B of the resonance neutrons in graphite, for which we may write

(12) $$B = \lambda^* \ln E_2/E_1$$

where E_1 and E_2 are the limits of the energy interval within which the uranium is considered as black for resonance neutrons. If the resonance absorption of the uranium sphere is thus represented we may write

(13) $$\mathcal{I}^{res}/Q = 4\pi R B^2 (1 + R/B)$$

so that we have

(14) $$\frac{1-q_m}{2} = \frac{4\pi R B^2 (1+R/B)}{V}$$

or

(15) $$\frac{\frac{4\pi R^3}{3}}{V} = \frac{1-q_m}{6}\,\frac{R^2}{B^2(1+R/B)}$$

Within the limits of the approximation for the resonance neutron absorption we may write ~~for~~ for ε

~~These formulae were used to determine for a uranium sphere of 5 cm. radius and a density of 15 gms. per cc. from formula No. the most favorable ratio of uranium to carbon. The range B was assumed for graphite of 1.7 gms. per cc. density to be 6.5 cm. and on this basis the use of about 30 tons of uranium per 100 tons of graphite was recommended.~~ Total Quantity

estimate of constants;

$\varepsilon =$

(16) $G =$

According to the simplified theory, we obtain the highest value of q for a value of R for which ξ becomes maximum. In choosing the most suitable size for the lattice element, there are, however, other points of view which have to be considered and the value of R=5 cm. for a uranium density of 15 gm./cm^2 was recommended on the basis of this simplified theory. A ratio of about 30 toms of uranium per 100 tons of graphite was recommended for R=5 cm. ~~and~~ On the basis of equation (1) it was stated that a graphite sphere containing a uranium lattice having a radius of more than 250 cm. ~~and~~ with the graphite weighing more than 100 tons may be expected to be capable of sustaining a divergent chain reaction. These estimates were based on the following estimate of the constants which are involved.

/ space 2½ inches free /

It should be noted that while this simplified theory may be applied to small uranium spheres, and while the formulae give correctly the trend which characterizes the variations of q, ξ, and V, the formulas given are not strictly speaking correct and in order to ~~have them correct,~~ improve them one would have to consider ζ and B as functions of R. The variation of ζ with R ~~and would~~ may then take into account the ~~effect~~ fact that the contribution of neutron fission caused by fission neutrons in ~~a~~ the uranium sphere increases with increasing R and by writing B as a function of R, we ~~could~~ may account for the fact that the uranium sphere is not black for higher energy resonance neutrons so that we have a ~~turn~~ term in the resonance absorption which is proportionate to the mass of the sphere. ~~Some of~~ The corrections which one would have to apply cancel out, however, ~~very~~ rather well for spheres of about 5 cm. radius. By neglecting the fission by fission neutrons, we have shifted the optimum towards smaller spheres that would correspond to reality, while by neglecting the absorption of higher energy resonance neutrons, we have shifted the optimum towards larger spheres than would correspond to reality. Finally, by writing $J^{th} = \alpha\, \mathcal{J}^{th}$ we have tended to favor smaller spheres than correspond to reality.

A 5 cm. sphere of 15 gms./cc density weighs about 7.5 kg, and it may be that from the strict point of view of having the highest value for q, a somewhat smaller mass of the lattice element which has a spheroid shape would be preferable, if no gap is left free between the lattice elements and the graphite mass ~~that~~ which ~~the~~ the lattice contains).

Formulae (15) permits a quick orientation since $q \cong 0.7$ and 0.8

Doppl heating

if no gap is left free between the lattice element and the graphite mass which contains the lattice.

The formula 9A or 15 permits a quick survey of the most favorable ratio of uranium to carbon if various factors are varied, since the value of q does not change much, with the radius or the density of the uranium in the sphere as long as we do not go to extreme values. With q having a value between .7 and .8, one has only to insert into 9A or 15 the value for or B respectively, and obtain the approximate ratio of the volume of uranium to carbon. By multiplying this with the ratio of the densities, one obtains the ratio of the weights of uranium to carbon.

As it has been stated before, a lattice or uranium spheres is capable of maintaining a chain reaction even in impure graphite which corresponds to a capture cross section of carbon of $\sigma_c(C) = 0.01$, particularly if the uranium is cooled while part of the graphite is allowed to heat up. If the uranium and enough of the surrounding graphite is kept cool, the thermal neutrons which diffuse to the uranium spheres will be ~~cooled~~ cold even though the rest of the graphite has been allowed to heat up. In these conditions of the neutrons which are slowed down to thermal energies, a smaller fraction is absorbed by carbon and a larger fraction is absorbed by uranium ~~Then~~ than would be the case for a uniform temperature throughout the lattice. In order to maintain a large temperature difference with the heat flow that can be taken care of by the cooling ~~of uranium;~~ system ~~of the power~~ the uranium and its surrounding graphite layer may be thermally insulated from the rest of the graphite by a gap which may be filled with lamp black or some other fluffy material that does not appreciably absorb neutrons. Structurally, these conditions are easier to realize for a lattice of rods than for a lattice of spheres.

VIII

(2)

Approximate Theory

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With these assumptions it is easy to determine the fraction q of the neutrons which are slowed down to the thermal region and are absorbed in the thermal region by the uranium spheres in the lattice. For a uranium sphere of given radius R, the most favorable ratio of uranium to carbon (the most favorable lattice spacing) is the ratio for which q becomes a maximum. For the corresponding value of $\alpha = \alpha_m$, the theory gives the equation

$$\alpha_m = \frac{1-q_m}{2} \tag{8}$$

Since α is equal to the fraction of the neutrons which are slowed down to the thermal region and are absorbed in the thermal region by carbon, and since q is the fraction of the usefully absorbed neutrons, the above equation expresses the fact that for the most favorable ratio of uranium to carbon, half of the neutrons which are not usefully absorbed are absorbed as thermal neutrons by carbon. It follows that the other half of the 'not-usefully" absorbed neutrons must be absorbed at resonance by uranium. This fact can be expressed by writing:

$$\frac{1-q_m}{2} = \frac{1}{V}\,\frac{g^{res}}{Q} \tag{9}$$

which we can write also in this form

$$\frac{1-q_m}{6} = \frac{\frac{4\pi R^3}{3}}{V}\left(\frac{g^{res}}{Q\,4\pi R^3}\right) \tag{9A}$$

The expression g^{res}/Q signifies the number of resonance neutrons absorbed per sec by a single sphere of uranium which is embedded in an infinite mass of graphite if one fast neutron is produced in the graphite per cc and sec; V stands for the volume of graphite per uranium sphere in the lattice. (9A) may be used to calculate $\frac{4\pi R^3}{3}/V$, the ratio of uranium volume to carbon volume in the lattice. From this, the ratio of weights is obtained by multiplying with the ratio of the densities.

q can be calculated within the validity of this simple theory for any given radius R of the uranium spheres which form the lattice element from E, the ratio of the thermal neutron absorption to the resonance neutron absorption which holds for a single sphere of the radius R that is embedded into an infinite mass of graphite in which there is a uniform production of fast neutrons per cc of graphite.

According to the above definitions we may write for ε

(10) $$\varepsilon = \frac{g^{th}}{g^{res}}$$

According to the simple theory, we may calculate q from ε from the equation

(11) $$\varepsilon = \frac{4qm}{(1-qm)^2}$$

In order to obtain an approximate value for $\frac{g^{res}}{Q}$ the resonance absorption of a uranium sphere may be represented for a given size sphere by assuming that the uranium is black for resonance neutrons with a certain energy interval and does not absorb resonance neutrons outside that energy interval. The resonance absorption of the uranium sphere is then determined by a certain range B of the resonance neutrons in graphite, for which we may write

(12) $$B = \frac{\lambda^*}{\sqrt{3}} \sqrt{\frac{\ln E_1/E_2}{3 \ln \left(1 - \frac{2m}{(1+m)^2}\right)}}$$

where E_1 and E_2 are the limits of the energy interval within which the uranium is considered as black for resonance neutrons. Or putting in general $E_1/E_2=1/10$ and for graphite in particular, m = 12. We obtain for graphite of density 1.7

(12b) $$B \approx \cancel{\frac{\lambda^*}{\sqrt{3}} \sqrt{\frac{\ln 1/10}{3 \ln \left(1 - \frac{24}{(13)^2}\right)}}} \frac{\lambda^*}{\sqrt{3}} \sqrt{6.5 \ln 10}$$

If the resonance absorption of the uranium sphere is thus represented, we may write

(13) $$\frac{g^{res}}{Q} = 4\pi R B^2 (1 + R/B)$$

q can be calculated within the validity of this simple theory for any given radius R of the uranium spheres which form the lattice element from $\mathcal{E}$, the ratio of the thermal neutron absorption to the resonance neutron absorption, which holds for a single sphere of the radius R that is embedded into an infinite mass of graphite in which there is a uniform production of fast neutrons per cc of graphite.

According to the above definition we may write for $\mathcal{E}$

(10) $$\mathcal{E} = \frac{g^{th}}{g^{res}}$$

According to the simple theory, we may calculated q_m from $\mathcal{E}$ from the equation

(11) $$\mathcal{E} = \frac{4q_m}{(1-q_m)^2}$$

In order to obtain an approximate value for $\frac{g^{res}}{a}$ the resonance absorption of a uranium sphere may be represented for a given size sphere by assuming that the uranium is black for resonance neutrons with a certain energy interval and does not absorb resonance neutrons outside that energy interval. The resonance absorption of the uranium sphere is then determined by a certain range B of the resonance neutrons in graphite, for which we may write

(12) $$B = \lambda^* \sqrt{\frac{\ln E_1/E_2}{3\ln\left(1-\frac{2\times m}{(2+m)^2}\right)}}$$ or putting $\frac{E_1}{E_2} = \frac{1}{10}$ and putting $m = 12$ for graphite

where E_1 and E_2 are the limits of the energy interval within which the uranium is considered as black for resonance neutrons. If the resonance absorption of the uranium sphere is thus represented, we may write

(13)

so that we have

$$(14) \qquad \frac{1-\eta q_m}{2} = \frac{4\pi R B^2 (1 + R/B)}{V}$$

or

$$(15) \qquad \frac{4\pi R^3}{3}\Big/ V = \frac{1-\eta q_m}{6} \frac{R^2}{B^2} \frac{1}{(1+R/B)}$$

Within the limits of the approximation for the resonance neutron absorption, we may write for

$$(16) \qquad \xi = \frac{A^2}{B^2} \times \left\{ \frac{1}{1+R/B} \times \frac{1}{\dfrac{\lambda(C)}{R\, G \sqrt{\dfrac{3\sigma_a(U)}{\sigma_{sc}(U)}}} + \dfrac{1}{1+R/A}} \right.$$

where

$$(17) \qquad G = \frac{e^{R/u} + e^{-R/u}}{e^{R/u} - e^{-R/u}}$$

and

$$\frac{R}{u} = \frac{R}{\lambda(U)} \sqrt{\frac{3\sigma_a(U)}{\sigma_{sc}(U)}}$$

According to the simplified theory, we obtain the highest value of q for a value of R for which ξ becomes maximum. In choosing the most suitable size for the lattice element, there are, however, other points of view which have to be considered and the value of R=5 cm for a uranium density of 15 gm/cm^2 was recommended on the basis of this simplified theory. A ratio of about 30 tons of uranium per 100 tons of graphite was recommended for R=5 cm. ~~On the basis of equation (1) it was stated that a graphite sphere containing a uranium lattice having a radius of more than 250 cm with the graphite weighing more than 100 tons may be expected to be~~

~~On the basis of the best available values in Febr 1940 the corresponding value of ηq was estimated to be about~~ $\cancel{\eta q = 2(1+4/800) \times 0.6}$

~~capable of sustaining a divergent chain reaction. These estimates were based on the following estimate of the constants which are involved.~~

It should be noted that while this simplified theory may be applied to small uranium spheres, and while the formulae give correctly the trend which characterizes the variations of q, ε , and V, the formulaes given are not strictly speaking correct and in order to improve them, one would have to consider η and B as functions of R. The variation of η with R may then take into account the fact that the neutron contribution of fission caused by fission neutrons in the uranium sphere increases with increasing R and by writing B as a function of R, we may account for the fact that the uranium sphere is not black for higher energy resonance neutrons so that we have a term in the resonance absorption which is proportionate to the mass of the sphere. The corrections which one would have to apply, cancel out, however, rather well for spheres of about 5 cm radius. By neglecting the fission by fission neutrons, we have shifted the optimum towards smaller spheres than would correspond to reality, while by neglecting the absorption of higher energy resonance neutrons, we have shifted the optimum towards larger spheres than would correspond to reality. Finally, by writing $J^{th} = \alpha\, \mathcal{J}^{th}$ we have tended to favor smaller spheres than correspond to reality.

A 5 cm sphere of 15 gms/cc density weighs about 7.5 kg, and it may be that from the strict point of view of having the highest value for q, a somewhat small mass of the lattice element which has a spheroid shape would be preferable, if no gap if left free between the lattice element and the graphite mass which contains the lattice.

The formula 9A or 15 permits a quick survey of the most favorable ratio of uranium to carbon if various factors are varied, since the value of q does not change much, with the radius or the density of the uranium in the sphere as long as we do not go to extreme values. With q having a value between .7 and .8, one has only to insert into 9A or 15 the value for g_{res} or B respectively, and obtain the approximate ratio of the volume of uranium to carbon. By multiplying this with the ratio of the densities, one obtains the ratio of the weights of uranium to carbon.

As it has been stated before, a lattice of uranium spheres is capable of maintaining a chain reaction even in impure graphite which corresponds to a capture cross section of carbon of (nearly $\sigma_c(C) = 0.01 \times 10^{-24}\ \mathrm{cm}^2$, particularly if the uranium is cooled while part of the graphite is allowed to heat up. If the uranium and enough of the surrounding graphite is kept cool, the thermal neutrons which diffuse to the uranium spheres will be cold even though the rest of the graphite has been allowed to heat up. In these conditions of the neutrons which are slowed down to thermal energies, a smaller fraction is absorbed by carbon and a larger fraction is absorbed by uranium, than would be the case for a uniform temperature throughout the lattice. In order to maintain a large temperature difference with the heat flow that can be taken care of by the cooling system, the uranium and its surrounding graphite layer may be thermally insulated from the rest of the graphite by a gap (which may be filled with lamp black or some other fluffy material that does not appreciably absorb neutrons). Structurally, these conditions are easier to realize for a lattice of rods than for a lattice of spheres.

The approximate theory which treats all the resonance absorption as surface absorption and which leads to formula 15 gives a somewhat too rapid rise of the ratio of carbon weight to uranium weight with the increasing radius R, and one may, therefore, not extrapolate to very small values of R from formula 15.

For instance, for a density of 18 and a radius of ~~330 meters~~ 3 cm the most favorable ratio of carbon weight to uranium weight is close to 4.5, whereas formula 15 would give a larger value. If we have to deal with such small spheres we may replace the lattice of spheres by a lattice of cylindrical rods by choosing the radius of the cylindrical rod to about two-thirds of the radius of the sphere and by having in both cases about the same ratio of weights of carbon to uranium. Accordingly for a lattice of cylindrical rods of a radius of 2 cm. a ratio of carbon weight to uranium weight of about 4.5 is close to the most favorable ratio. A sphere of 2 cm. radius and density 18 would require a carbon to uranium weight ratio of about 5.5 and about the same weight ratio would correspond to a lattice of cylindrical rods if the rods have a radius of 2 x 2/3, or about 1.3 cm. If we have to deal with such spheres or cylindrical rods of such small radius and if we change the bulk density of the uranium then we ought to increase the radius of the sphere or the radius of the cylindrical rod in the following manner: In the case of a sphere the radius should be changed so that the weight should lie somewhere between the product of the old weight multiplied with the ratio of the densities and the old weight multiplied with the square of the ratio of the densities. The corresponding size for the cylindrical rods is again obtained by multiplying the radius of the sphere with 2/3 in order to obtain the radius of the cylindrical rod. In practice the greatest change of density will be a reduction of the density by a factor 2. For a density of 9 and the radius of 3 cm. the ratio of weight of carbon to the weight of uranium for a lattice of spheres would be about 6.2 and the same would hold for lattices of cylinders of density 9 and radius of 2 cm. Such low bulk densities may occur if either uranium carbide is used or if an aggregate of uranium metal is used in a helium cooled lattice.

Fig

While for a simple body like a sphere, the surface of the sphere determines the value of the surface absorption, for more complicated shapes it is not always the total surface of the uranium which counts. For instance, if we have a rod of uranium which has a cross section as illustrated in Fig. 2, and if this rod is placed in a cylindrical hole in the graphite, it is the surface of the cylinder and not the surface of the rod which determines the surface absorption. Moreover, internal surfaces which are not exposed to neutrons coming from the graphite but are shielded from such neutrons by uranium do not contribute to the surface absorption. Such internal surfaces fall into two categories: We may have a lattice element consisting of a single body which has slits or holes going through it so that internal surfaces arise; or we may have an aggregate of a number of uranium bodies assembled together and have the aggregates play the role of one single lattice element. An example for this has been described in connection with Fig. 1.

The absence of surface absorption on internal surfaces makes it possible to have heat transfer from the lattice element to the cooling agent across internal surfaces without appreciable loss of efficiency. This is certainly true if we use as a cooling agent a gas-like helium at a density which corresponds to about 10 times atmospheric pressure, since, at this density, the gas does not appreciably slow down neutrons in their passage from a point of internal surface to another point of internal surface. It is also true if we use as a cooling agent a heavy liquid metal like bismuth in thin layers since bismuth, on account of its great atomic weight, does not appreciably slow down energy resonance neutrons.

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General Statements about Cooling Agents

Most of the heat generated in the chain reaction is generated by the fission process in the lattice element and the cooling agent must, therefore, be in good thermal contact with the lattice element. Some heat, however, is generated in the graphite, where the neutrons and gamma radiations emitted in the fission process are scattered or absorbed. The uranium graphite lattice may be surrounded by a layer of graphite which does not contain uranium and a further layer of material which serves as a radiation shield. This radiation shield may be a heterogeneous system composed of iron and graphite. Heat is produced both in the scattering layer and the radiation shield while neutrons and gamma rays are scattered or absorbed. Provisions must be made for cooling both the scattering layer and the radiation shield.

The chain reacting pile should be enclosed into a steel tank and an inert atmosphere be maintained within the tank. Helium gas is a suitable inert gas for this purpose both because it does not absorb neutrons and because it facilitates the heat transfer across gaps between solid bodies.

Helium gas can also be used as a cooling agent and may be circulated either in parallel flow or in series flow through the graphite pile. See examples given in figures . Another, and the most obvious cooling agent is, of course, water. If water is used, it is advisable to use series flow, have a closed circulation, and have the water flow through pipes which are inserted into the graphite pile. It is advisable to protect the uranium against chemical action by water and a possible arrangement is shown in Fig. 3. In Fig. 3, (1) is a uranium rod which forms the lattice element. This uranium rod is covered with a thin aluminum tube (2). This Al-covered uranium rod is placed inside an aluminum tube (3) which goes through the graphite pile. The gap between the Al tube and the graphite is kept as small as possible. Water is flown through the anular gap between the Al tubes (3) and the Al-covered uranium rod (1). In order to have a potentially chain reacting system, it is necessary to have the gap which is filled with water rather small, but if the uranium rod has a diameter of about 3 cm. and if the gap is about 1 mm. and if water is flowing through the gap, we still have a potentially chain reacting lattice.

If liquid bismuth is used as a cooling agent, we may have either parallel or series flow. The liquid bismuth may be in direct touch with the graphite and we may have either closed circulation or a gravity flow through the pile. Uranium carbide may be used in direct contact with bismuth, but uranium metal may be used coated with iron. If uranium rods are used, a thin steel coating consisting, for instance, in a thin steel tube covering the uranium rod will permit the construction of a potentially chain reacting unit if the wall thickness of the steel tube is about 1% of the diameter of the uranium rod or less. In the example given in figures , such steel coated uranium rods are drawn. If the same arrangement is used for He cooling rather than bismuth cooling, the uranium may remain uncoated.

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