Indlæg ①

The neutron emission of Uranium.

It should be possible to ~~measure~~ obtain the value of μ ~~rather~~ with great accuracy by the following method. Let us consider a sphere containing a homogeneous mixture of U and H of n atoms of H per U atom. The hydrogen be present in the form of water and δ gr/cc be the density of water inside the sphere. This sphere is immersed in an infinitely large water tank and a photoneutron source such as used in previous experiments ~~is placed inside~~ be placed ~~inside~~ in the center of the sphere. The density ρ of the thermal neutrons be measured along a radius and

Joliot (2)

and the integrals of $\int_0^l r^2 \rho \, dr$ is I_1 calculated inside the sphere $(r<l)$ and outside the sphere $r>l$ $\left(I_2 = \int_l^\infty\right)$. Then ~~the same exp~~ in an other experiment the integral I_0 of $\int_0^\infty r \rho \, dr$ is determined for the same neutron source in pure water.

~~If that an~~ experiment has to be ~~is~~ performed for values of l sufficiently large to neglect transition phenomena near the surface of the sphere.

~~Under this cond~~

~~I find that the value of μ can be derived from~~

I find that one can derive from I_0, I_1, I_2 the value of μ and it is

$$\mu = \frac{1}{1-p} + \frac{n\,\sigma_c(H)}{\sigma_c(U)+\sigma_f(U)} \left\{ \frac{1}{1-p} - \frac{1}{5}\,\frac{1 - I_2/I_0}{I_1/I_0} \right.$$

This expression hold provided the ~~value of~~ radius of the U + Water sphere etc.

Szilard ob (3)

~~It so happens that ... performed an experiment of this type. from which they they arrived at the conclusion that a primary photo neutron causing a~~

An experiment of this type was ~~see~~ recently performed by ~~from~~ who used the activity ~~such~~ in a dysprosium as measure of the slow neutron density. They concluded from these experiment that about 8 neutrons are generated in their arrange- ~~ment~~ by every primary n which causes a fission process in a U atom. in the case of n=3

~~Which~~ We can make use of the values of I/I_0 0.72 I/I_0 0.045 which they reported to calculate the number μ of neutrons emitted by U per thermal neutron absorbed in U. I find that μ can be derived from these two values and that it is

μ =

which they interpret by stating that a considerable number of re. and destroying neutrons appears in this experiment x.

~~It appears~~ Page 16

As it takes about 6 collisions to reduce the energy of a neutron by a factor 1/e, a neutron which is being ~~slowed~~ slowed down ~~and enters~~ has entered the ~~resonance Region of~~ energy interval of the ~~Uranium in which U absorbs~~ of the res absorption of U will stay in this energy interval for a comparatively long time. It has therefore a large probability to be captured by U unless very low U concentrations are used:— ~~The carbon~~ Though the capture + of C is very small at such very low U concentrations the Carbon might capture enough to make a chain reaction impossible ~~Since only an upper limit of~~ the C cross section is known at present ~~If~~ we are not able to state with certainty ~~that~~ whether in ~~a homogeneous mixture of U and C~~

The carbon + section for which we have at present only ~~a~~ the above given upper limit will have to be measured ~~by before~~ before this question can be answered. ~~with certainty~~

Ins to Page 1c

~~We shall however show~~

~~Since only a small fraction of the C weight need to be U and also because the scattering of C is much more spherically in contrast to the sc. of is practically sph. sym.~~

(The ~~distance~~ to which ~~within~~ a fast neutron diffuses away from the point of its origin before it ~~reaches~~ ~~energy~~ is slowed down to en of a few volts is only of the order of   if "mixtures" of U uranium in graphite are used.

~~This is though 6 collisions in which~~

~~This~~ That this distance is so small is due to various circumstances. 1) ~~a to~~ In graphite we have x uranium atoms per liter against .. of hydrogen in water

2) The scattering of neutrons by carbon is practically sph sym the sc by H is not

3) The mixture is almost 90% and contains only little U)

Case 1 A

~~$J_{th} = 4\pi Q R A^2 \frac{\lambda_{sc}(U)}{U} R \left\{ \quad - \frac{u}{R} \right\} (1 + R/A)$~~

$$J_{th} = 4\pi Q R A^2 (1 + R/A) \frac{\frac{\lambda_{sc}(U)}{U} \left\{ \frac{e^{R/u} + e^{-R/u}}{e^{R/u} - e^{-R/u}} - \frac{U}{R} \right\}}{\frac{\lambda_{sc}(C)}{R} (1 + R/A) + \frac{\lambda_{sc} u}{U} \left\{ \quad - \frac{u}{R} \right\}}$$

~~At~~ since $J^{res} = J_0^{res}$

$$\varphi = \frac{J^{u}_{th}}{J_0^{th}} =$$

ad (f

$$\cancel{\varphi = \frac{1}{R/A+1} \quad \frac{\sqrt{R}\{3\,(\ldots)\}}{\lambda_{sc}(C)\,(R/A+1)+\sqrt{R}\{\ldots\}}}$$

$$\cancel{\frac{1}{U} = \sqrt{\frac{3\sigma_a(U)\,\sigma_t(U)}{\sigma_{sc}(U)}} = \frac{\lambda_{sc}}{U}}$$

$$\varphi = \frac{\dfrac{\lambda_{sc}(U)}{U}\left\{\dfrac{e^{R/U}+e^{-R/U}}{e^{R/U}-e^{-R/U}} - \dfrac{U}{R}\right\}}{\dfrac{\lambda_{sc}(C)}{R}\left(R/A+1\right) + \dfrac{\lambda_{sc}(U)}{U}\left\{\dfrac{e^{R/U}+e^{-R/U}}{e^{+R/U}-e^{-R/U}} - \dfrac{U}{R}\right\}}$$

the same expression also hold for Uranium oxide

U is the range in Uranium oxide

$$\frac{\lambda_{sc}(U)}{U} \cong 1 \text{ in pure U}$$

Sol 1 f

Numerical expl

$R = 5 \quad \frac{\Delta x^4}{U} = 1 \qquad \frac{R}{A} << 1$

$$\varphi = \frac{5\left\{1 - \frac{1.5}{5}\right\}}{2.44 + 5\left\{1 - \frac{1.5}{5}\right\}} =$$

$$= \frac{5 \times 0.7}{2.44 + 5 \times 0.7} = \frac{1}{\frac{2.44}{3.5} + 1}$$

$$\frac{1}{1.7} \qquad C$$

Add 1f page 2

(1)

Our formulas ~~appear~~ ~~to~~ give ~~to~~ ~~the~~ a too low value for $\varepsilon = \varepsilon_0$ if we have no longer $R \gg l$ $R \gg \lambda_{sc}$ (C)

~~For small values of R we have~~ ~~then the~~ However
For $R << \lambda_{sc}$ C
we found for a ~~th~~ U sphere which is „black" for thermal neutrons" again

$$\varepsilon = \frac{A^2}{B^2}$$

As a matter of fact it would not be safe (for our purpose) to use the ~~f~~ value of f given by eqn for R < 5 cm.

(2) Add 1 p. paper

It is ~~always~~ in principle possible to make ~~Rsc~~ $\lambda_{sc}(C)$ as large as we wish by choosing the density small. — ~~The density of the U can then be made~~ U-spheres however are not black for thermal neutrons, ~~For instance~~ even if the U density is made infinite. ~~The ratio of the~~ If $\lambda_{sc}(C) >> R$ it is then the ratio which determines ε, This ratio is not very well known but it appears likely that

$$\varepsilon \approx \frac{1}{2}\frac{R^2}{\beta^2}$$

may be the limiting value

Lattice (1)

If we have loose ~~a~~ lattice of U spheres embedded in Carbon ~~and lower~~ we will ~~assume~~ again assume ~~the~~ for the time being that everywhere in the Carbon ~~of~~ the same number of neutrons ~~force~~ enter the res region and the thermal region per cc. ~~There If we know the distance between~~ Formel No 1 shows that a U sphere does not affect the ~~neutron density~~ thermal neutron density appreciably ~~at~~ at a distance which is large compared to R; at a distance from the center of 2 R ~~half of the cylinder~~ the ρ is almost equal to $\frac{\rho_0}{2}$ if ρ_0 is the value at infinity. In these circumstances the number of thermal neutrons absorbed by a single sphere is only in so far affected by the other spheres as the presence of these other spheres reduces from ρ_0 the average value of ρ by ~~a~~ some factor α k which is smaller than one

$$\bar{\rho} = \alpha \rho_0$$ [illegible]

The num of res. neutrons abs. by a U is not affected at all lattice (2)

If the distance between spheres is large compared to the range of the res. n.

U_{res} (lattice) = U_{res} (~~single~~ isolated)

we have therefore $\frac{U_{th}\text{(lattice)}}{U_{res}\text{(lattice)}} = \varepsilon$

The fraction of the neutrons absorbed by the carbon is proportionate to the average neutron density $\bar{\rho}$. — Since ~~for~~ in the absence of U $\bar{\rho} = \rho_0$ and since in this case all the neutrons which are produced are absorbed by the Carbon the fraction absorbed by the Carbon in the presence of U is

$$\alpha$$

Accordingly the fraction of the neutrons ~~which~~ ~~absorb~~ which are absorbed by the U spheres either as res neutrons or as thermal neutrons is

$$1 - \alpha$$

Accordingly the fraction of all the neutrons which absorbed by the U spheres in the thermal region alone is given by

$$q = \frac{U_{th}\text{(lattice)}}{(U_{th} + U_{res}\text{(lattice)})}(1 - \alpha)$$

or

$$q = \frac{\varepsilon\alpha}{1 + \varepsilon\alpha}(1 - \alpha)$$

L. text (1)

small

The value hold only under the assumption that q is the same everywhere in the C. This would no longer be true ~~it~~ for such ~~very small values of $\sigma c(f)$~~ ~~for very small ... if very~~ ~~the distances~~ for which our formula ~~would~~ imply that the distance between U spheres is large compared with the distance to which a fast neutron diffuses away (in C) from ~~the~~ its point of origin before it is slowed down to a few volts for very small σ_c q is therefore in reality smaller than the value given by ... x ~~This happen~~ ~~this however does not become~~

The value of $\sigma c(C)$ for which this happens is however much smaller than the values which are required ~~for it~~ for having $\mu y > 1$ and therefore ~~in the~~ formula given can be used ~~for all practical pur~~ for purposes of studying the canalies of a ch. r.

~~The~~ In order to see this we estimate the volume L^3 of material

L. text (3)

$$\alpha\varphi g_0 + g_0^*$$

so that we have

$$\alpha\varphi g_0 + S_0^* = (1-\alpha) Q L^3$$

~~$\alpha\varphi g_0 + g_0^*$~~

St is $\alpha\varphi g_0 + S_0^* = \alpha\varphi g_0 \frac{\varphi\varepsilon_0\alpha + 1}{\varphi\varepsilon_0\alpha}$

$$\text{and } \frac{\varphi\varepsilon_0\alpha + 1}{\varphi\varepsilon_0\alpha} = \frac{1-\alpha}{q}$$

So that we have

$$\varphi g_0 \frac{\alpha}{q} = Q L^3$$

and further

$$L^3 = 4\pi\varphi\frac{\alpha}{q}\left\{A^2 R\left(1 + R/A\right)\right\}$$

in max $\frac{\alpha}{q} = \frac{1-\varphi m}{2\varphi m}$

$$L^3 = 4\pi\varphi\frac{1-\varphi m}{2\varphi m} A^2 R\left(1 + R/A\right)$$

for large values of ~~Σ~~

for large Σ

$$\frac{1-\varphi m}{2\varphi m} \cong \frac{1}{\sqrt{\varphi\varepsilon_0}} =$$

~~th~~ and $L^3 = 4\pi\sqrt{\varphi}\sqrt{\frac{1+R/B}{1+R/A}}\, ABR$

z. B.

for $\sigma_c(C) = 0.003$

L text (2)

per U sphere ×

In the lattice a single U sphere absorbs/sec

~~[illegible]~~ $\alpha\varphi g_0^{th} + g_0^{res} =$ ~~$\alpha\varphi g_0^{th} + \varepsilon_0$~~

~~$\alpha\varphi\varepsilon_0+1$~~

~~[illegible]~~

~~$\alpha\varphi g_0^{th}+$~~

$$g_0^{th}\,\frac{\alpha\varphi\varepsilon_0+1}{\dfrac{g_0^{res}}{g_0^{th}}} = 4\pi g_0^{th}\,\frac{\alpha\varphi\varepsilon_0+1}{4\pi\varepsilon_0}$$

In the lattice of U spheres [illegible]

from the neutrons (QL^3) ~~produced within the volume~~ which are slowed down within the volume L^3 ~~the~~ to res. energies the carbon absorbs $\alpha Q, L^3$ and the U sphere absorbs $(1-\alpha)QL^3$, ~~From eqn. on the other hand we found for the number of~~

On the other hand a single U sphere within a lattice obviously absorbs

~~It is~~ The methods appear ~~...~~ promising in so far as ~~...~~ the J activities need not be measured in the same units as the In activities and ~~...~~ ~~...~~ β ray ranges and the number efficiency can be left out of consideration. An exp. of this type is now being performed for n = 3

~~Cross sections~~

~~We consider published~~ ~~The fission cross section of the $\sigma_f(u)$ are too unreliable is not very well known to be used of much use for our consideration. The total abs of the Cross sections and notations~~

$\sigma_f(U)$

$(\sigma_c + \sigma_f)$ is not

$\sigma_c(H)$ ~~has been ...~~ has ~~the nature~~

been

be

~~$\sigma_c(C)$~~

Jaldat

Solmoss (3)

The value of p has been studied ~~from~~ for values $n > 30$ by ... and by from which one obtains for instance for $n = 3$

$$p = 0.5$$

~~The knowledge~~ An more accurate knowledge of the value of p might perhaps be obtained for ~~[illegible]~~ such low H. conc. by the following method

Let ~~[illegible] be the activity of~~ $J_1(U)$ ~~[illegible]~~ be the ~~intensity~~ activity with which a Cd covered foil of Indium is activated by neutrons in the U water mixture and $J_1(H_2O)$ the same in H_2O. Let $J_2[U]$ and $J_2[H_2O]$ be the ~~same~~ corresponding β activities for a Iod indicator covered by both Cd an Indium absorbers. Let further $In_1[U]$, ~~[illegible]~~ $In_1 H_2O$ $In_2[U]$ and $In_2[H_2O]$ be the ~~same~~ corresponding activities for an Indium detector covered with Cd or Cd + In in U-mixture or in Water. We have then

$$\frac{J}{\qquad} = p$$

this holds under the well founded assumption that the U resonance is between the In resonance and the Iodine resonance. ~~And the [illegible]~~

Schumis (2)

We shall call the energy interval $E_2 - E_1 = \Delta E$ the resonance region and have $\frac{E_2}{E_1} = 10$

~~In the following we s~~

Throughout this paper we shall assume that every neutron which comes into contact with highly concentrated U while within ΔE is ~~always~~ absorbed by radiative capture by U without causing fission. To the energy region below $0.2 E_0$ we shall refer as the "thermal region".

In a homogeneous mixture of U and water the ~~number of~~ fraction of the neutrons which are captured at resonance by U and therefore never reach thermal energies depends on the number H atoms n per U atom in the mixture. ~~For thermal The value of p for 2 different values of n are interrelated~~ Dancoff may hold for ~~for~~ theoretical reasons which are generally known that if p is known for a given H concentration $n = n_1$ the value ~~p~~ for another H conc can be calculated from the equation

11

Insert

Page 1 a

Turn round

Szilard (1)

~~Constant~~

Cross sections and ~~other~~ Notations,

~~The fraction P of the neutrons which reaches thermal energies in a homogeneous mixture of U and Water ~~depents~~ depends on the number of H atoms per U atom in the mixture~~

Uranium has a strong resonance absorption line at energies of the order of magnitude of 10 volts. ~~Such For such an absorbtion line~~ ~~If which the absorption If the position For such an abs. whole diagram~~ If we have to deal with an abs. line having its max. at E_0 in this en. region and if this line obeys the B-W Formula then the following will hold. At thermal energies the absorption will follow the 1/v law will then go through a minimum at $E_1 = 0.2\,E_0$ ~~and fall to a very small value~~ go through a maximum at E_0 then ~~and~~ fall again and ~~fall to a very small value will~~ reach a very small value for $E_2 = 2E_0$

Stability

~~Of the neutrons~~

Of ~~the~~ a ~~neutrons~~ neutrons ~~a~~ produced by the U only a fraction q ~~reacts within within~~ is absorbed within the system and ~~the~~ $(1-q)$ escapes across the boundary of the system. A stationary state can be maintained as long as

$$\mu q \Psi < 1$$

~~or by writing~~
~~both q and Ψ may be functions of the Temp and also~~
~~or by writing~~

we write $q\Psi = \Gamma(T, \cancel{x}t)$
to indicate that this product is a function of the Temp of this system and also depends ~~on the~~ parameter such as the position of scattering ~~or~~ bodies ~~near the system or~~ absorbing bodies near or within the system.

In order to have a large neutron production we must ~~have operate~~ maintain a chain reaction near the point

$$\mu \cancel{q} \Gamma_0 = 1$$

I Stabilizing

By decreasing or increasing ~~the average~~ the average ~~value~~ number of neutrons per neutron absorbed in the system with the increasing or decreasing intensity of the neutron or other radiations emanating from the chain reaction. This can be achieved simply by ~~slowly~~ ~~inserting~~ ~~into~~ or removing from absorbing bodies into or from the system. ~~Contrary to what has been said on this point they seem to think these movements of the this shifting of absorbing in the time element involved is contrary to what has been thought about such a procedure a fairly the times involved can take place quite slowly what he finds in the literature in gener~~

[illegible]

II

~~If the neutron produces a large number of neutrons~~

~~[illegible]~~ ~~Restr.~~ It can be ~~shown easily seen that~~ shown ~~that~~ by decreasing or ~~increasing~~ ~~and thereby changing~~ μ the average ~~number~~ number of neutrons produced ~~[illegible]~~ per neutron absorbed in the system & with increasing or decreasing intensity of the neutron rad. If the chain reaction becomes divergent for a certain ~~critical~~ value of $\mu = \mu_0$ and of ~~then the time if which the number~~ $\mu > \mu_0$ ~~of If $\mu > \mu_0$ the number of neutrons would double in a time of the order of~~ then the time T which it takes for the number of neutrons to double is of the ~~avg~~ order of magn of

$$T = \cancel{[illegible]}\ \tau \frac{\mu_0}{\mu - \mu_0}$$

where τ is the time which a thermal neutrons need to ~~deposit~~ produce 2 thermal neutrons & ~~[illegible]~~ for if accidentally μ should suddenly ~~exceed~~ μ_0 by as it may one per mille ~~[illegible]~~ ~~we have~~ and if ~~τ~~ we have $\tau \cong 1/100$ sec ~~[illegible]~~ we have

$$T \cong 10 \text{ sec.}$$

~~Contrary to what one finds elsewhere it is not the value of τ~~ This is much longer than the time ~~[illegible]~~

Stabilisation

Since there is an ~~intens~~ exponential rise in the number of neutrons produced if ~~Since there is an ex~~

$$\mu \Gamma > 1$$

the question of stability ~~arises~~ has to be discussed. —

$$\cancel{\lambda = t_0 \ldots}$$

$$\Gamma = \Gamma_0 (1 + \delta)$$

$$t_2 = t_1 \delta$$

In arriving at ~~this far we have left out of~~ we did not take into consideration the fraction of the neutrons emitted ~~is not~~ with a time delay of about 10 seconds. ~~This the the frac~~ Though this fraction is small it ~~has an~~ ~~works on~~ ~~not be~~ ~~on the value of t_2 but since the time involved towards is~~ leads to a larger value of t_2 than given by eqn. Since however ~~the t_2~~ the time obtained is very long ~~it~~ we need not discuss this ~~point~~ point at this juncture. —

which 5

moving ~~the~~ accurately.
This is now being done by
a new method which has
been devised for the purpose.

Joliot (4)

In the case of Joliot's experiment we have $n = 3$ $s = 0.42$ gm/cc. The value of p has so far not been measured for $n = 3$ and has to be ~~exp~~ extrapolated from ~~exp~~ results obtained for larger values of n. ~~[illegible]~~ If This is done for theoretical reasons according to the ~~f~~ equation

$$\frac{\log(1-p_1)}{\log(1-p_2)} = \frac{\sqrt{n_2}}{\sqrt{n_1}}$$

which yield in ~~appr~~ agreement with . . .

$$p = 0.5 \quad \text{for } n = 3$$

Using this value of p ~~for~~ we find

$$\mu = 2.05 \text{ or } \mu = 2.02$$

~~if we assume~~

according to whether we attribute to $\dfrac{3\sigma_{cH}}{\sigma_c(H) + \sigma_f(H)}$ the value 4 or 8

The fact that μ is so very insensitive to an error in the determination of these absorption + sections gives ~~the method~~ makes it possible to determine μ ~~acc~~ very accurately by mea

Valont O 2

The number of neutron emitted by Uranium (per ~~primary~~ neutron emitted from the photo source)

$$\left(Q^{*}_{int} + Q_{ext} - 1 \right) \equiv \text{Prod.} \quad (1)$$

$$= \frac{Q_{int}}{1-p} + Q_{ext} - 1$$

And R_1, the number ~~emitted per neutron~~ per neutron which reacts "within the mixture is

$$R_1 = \frac{\text{Prod}}{Q_{int}/1-p} = 1 - \frac{(1-p)(1-Q_{ext})}{Q_{int}}$$

~~the number of neutrons which reach the thermal region within the sphere~~

$$\cancel{R_2 = R_1(1-}$$

And R_2 the number produced per per neutron which reaches the thermal region within the sphere

$$R_2 = R_1 / 1-p = \frac{1}{1-p} - \frac{1-Q_{ext}}{Q_{int}}$$

Finally the

From this the R_2 the number produced per neutron which reaches the thermal region within the sphere

$$R_2 = \frac{p}{Q_{int}} = \frac{1}{1-p} + \frac{Q_{ext} - 1}{Q_{int}}$$

Jnldat O a

The ~~number of neutron~~ which all reach the res. region of U produced by the photoprocesse $Q_0^* = I_0/\tau_0$

The number of neutrons which reach the thermal region inside the sphere in the mixture Q_{int}

$$Q_{int} = \frac{I_{int}\,\tau_0}{I_0\,\tau_{int}}$$

whereas the number of ~~fast~~ neutrons which reach ~~Q^*_{int}~~ the resonance region in the sphere is

$$Q^*_{int} = (1-p)\,Q_{int}$$

The number of thermal neutrons reaching the thermal region outside the sphere

$$Q_{ext} = \frac{I_{ext}}{I_0}$$

and

$$Q^*_{ext} = Q_{ext}$$

and

$$Q_0^* = Q_0$$

„reaching" with thrill i.e.

~~The number of neutrons produced by the Q^* in the mixture is therefore~~

$$\cancel{Q^*(a) = Q^*_{int} + Q^*_{ext} - Q_0^*}$$

$$\cancel{\frac{Q^*(a)}{Q_0} = \frac{Q_{int}/Q_0}{1-p} + Q_{ext}/Q_0 - 1}$$

Joldot O c

And finally since of the thermal neutrons within the sphere the fraction $\dfrac{\sigma_c(U)+\sigma_f(U)+n\,\sigma(H)}{\sigma_c(U)+\sigma_f(U)}$

is absorbed by uranium μ ~~the R_3~~ number of neutrons produced per thermal n ~~[illegible]~~ abs. by uranium is

$$\mu\, \cancel{R_3} \frac{\sigma_c(U)+\sigma_f(U)+n\,\sigma(H)}{\sigma_c(U)+\sigma_f(U)}\, R_2 =$$

$$= \underline{\qquad\qquad} \left\{ \frac{1}{1-p} + \frac{Q_{ext}-1}{Q_{int}} \right\}$$

~~If we write Q_{ext} $\frac{I_2}{I_0}$~~

~~and Q_{int}~~

For which we can write

$$\mu\, \cancel{R_3} = \underline{\qquad\qquad} \left\{ \frac{1}{1-p} + \frac{\frac{I_{ext}}{I_0}-1}{\frac{I_{int}}{I_0}} \frac{\tau_{i}}{\tau_0} \right.$$

Since we have

$$\frac{\tau_i}{\tau_0} = \underline{\qquad\qquad\qquad}$$

we obtain

$$\mu =$$

(1) Text instead of page "5"

~~If we have now to deal with a really sphere of U wh~~

A sphere of U is in reality not "black" for thermal neutrons though it can in our case be considered black for resonance neutrons. ~~Therefore the~~ Only a fraction φ of the thermal neutrons which reach the sphere will be ultimately absorbed by it and the ratio ε of ~~the~~ thermal neutrons to resonance neutrons abs. by the sphere is therefore

$$\varepsilon = \varphi \varepsilon_0$$

~~Since inside the U sphere~~

In order to calculate φ we take into account ~~then poly~~ inside the U sphere thermal neutron density ρ obeys the equation

We find from 1.) and 2.) ~~by applying the boundary condi~~ that ~~[illegible]~~ the number of thermal ~~n~~ neutrons diffusing into the sphere is given by

Since the sphere is ~~still~~ black for res. we have φ ="

All this holds of course strictly
~~All this hold~~
~~If we trav~~
only if ~~R~~ $R >> \lambda_{sc}$
Otherwise the problem can no longer be treated as a diffusion problem but may of course be calculated by other methods ⊕

~~If $R << \lambda_{sc}$ I found for a~~ ~~small black sphere~~ sphere which

~~$\Sigma = R_0$~~

~~and sphere with only~~ in which ~~scattering an~~
is black for ~~res. = than~~ res n and also for thermal neutrons

$\Sigma =$

and for a sphere which is black for res. n. but only faintly scattering and absorbing for thermal neutrons

$\Sigma =$

~~If one knew for instance the~~
~~[illegible]~~

Text Instead page 5

(2)

Insert 4

~~We see that~~ for small values of r Σ for a sphere is larger by ~~the~~ a factor ... than for a plane sheet i.e. for a set of values ...

~~by a set~~ by a factor of ...

~~Some of this advantage~~ (but by no means all) ~~is lost by going over~~

In practice we cannot have ~~r~~ r very much smaller than perhaps $\frac{1}{2}$ b and therefore part of this advantage (is left) but by no means all of it. —

~~...~~

~~Lattice of spheres~~

~~Correction for res on~~

~~$q^* = (\quad) q$~~

~~that~~ every where the same number Q of res. neutrons is produced per cc and sec in the carbon and does not yet take into account the ~~fact~~ fact that the number of thermal neutrons produced in the carbon will be less than Q ~~in the~~ near the "black" uranium ~~Alter due to this latter fact~~ ~~we have ε c~~ ~~but the correction is not important if B << A. In our case B is about 6 cm and A ≥ 40 cm (?)~~

Let us now consider in detail ~~one of a~~ having a radius R ~~A~~ spherical sphere of "black" uranium imbedded in carbon × Again we assume that the same number of thermal neutrons Q is produced every where per cc and sec. ~~and will calcul~~

~~and~~ ~~The necessary the results obtain~~

the ~~~~ In reality this is not so and accordingly a correction factor will be calculated later and applied to the final result. Solve the diff eqn:

second half of
Insert 3

The diffusion of thermal neutrons and resonance neutrons towards the U layer can be treated with good approximation in an alike manner if the following is born in mind:

has the dimension of a length and we shall refer to it for the sake of brevity as the "range" of thermal neutrons.

Similarly, faster neutrons in pure Carbon disappear out of a given energy interval $\Delta E = E_2 - E_1$

because they are slowed down by elastic collisions with Carbon atoms. ~~The~~ For the probability that a neutron which enters the energy region ΔE

will still be within the region after k elastic collisions with Carbon atoms is given (with sufficient good approximation) by the expression

1 —

~~With an accuracy which is sufficient for our purpose the value of k is given by~~ Where $k_0 =$

~~Again the expression ~~is a~~ has the dimension of a length and we shall refer to it for the sake of brevity as the range of resonance neutrons.~~

Again the expression

B

will be called the "range" of res. n.

By treating the problem as a diffusion problem ~~that~~ we find the following ~~If the number of Res. neutrons produced per cc and sec is the same everywhere in the carbon and if the same ... we neglect~~ value for Σ in case of a black plane sheet of U this value holds under the ass.

A slight excess of unity means that very large masses of Uranium are required to approach a divergent chain reaction if a chain reaction is possible at all.

Introductory argument

Whether conditions are more favorable if instead of Hydrogen Carbon is used for slowing down the neutrons it can not be said with certainty since only an upper limit of the Carbon cross-sention is known. But we shall show if a non-homogeneous mixture is used Carbon is certainly much more favorable than Hydrogen ~~and that~~ because there seems to be little doubt in the case of Carbon it is possible to make use of tricks which considerably increase the value of ξ. Indeed ξ can be increased to a point where a chain reaction becomes possible. This difference in the behaviour in the Carbon and Hydrogen is connected with the fact that the scattering cross-section of Carbon is about the same for thermal neutrons as for resonance neutrons of Uranium whereas the scattering cross-section of Hydrogen varies by a factor of about 3. Insert "3"

~~Let us consider an infinitely large mass of pure Carbon.~~ A thermal neutron produced ~~in such~~ a mass of Carbon will disappear after a certain number of elastic collisions by being captured by a Carbon atom. The probability that ~~a thermal neutron in the Carbon~~ survives h elastic collisions with Carbon atoms before being captured is given ξ ~~in the expression~~

$(1- \quad)$

the expression

A

Let us first ~~consider~~ a plane sheet of U embedded in an infinite space filled with C and assume that the U is black both for thermal neutrons and for resonance ~~neu~~-neutrons i.e. ~~every neutron which~~ every th. n and every neutron which has ~~an~~ an energy within an energy interval $\Delta E = E_2 - E_1$ around the resonance energy ~~E~~ E_0 ~~is absorbed by U, an which~~ is absorbed by U if it reaches the U layer. ~~If a neutrons~~ ~~In order to find ε What we want~~ ~~In order to determine ε we~~ ~~treat the problem as a diffusion problem under the ass. that in the C everywhere the ~~tot~~ same number of neutrons Q enter the res. region per cc and~~ ~~We can then easily ... easily~~ ~~the ratio of~~

We are interested in finding ε for this arrangement and ~~comparing~~ ~~by~~ to find other arrangement for which its value is increased

Insert 3^4 11

introducing C ~~HH~~
will perhaps give a ~~x~~
reliable value for $\sigma_c(C)$
~~Meanwhile~~
~~It can be shown~~
Here I wish to show
that if nor have mixtures
of C is certainly more
favourable than H because
in the case of C it is possible
to make use of certain tricks ~~to~~ which
reduce the prob. of the neutrons
abs. at resonance x ~~The main~~
~~difference~~ One reason for this
diff. in favour of C is .....

Introducing Carbon III

Since only an upper limit for the C + section is known at present it is not possible to state yet whether a chain reaction is possible in homog. mixtures of C and U. [known method] Though the carbon + section is very small T.C. $\cdot 01 \cdot 10^{-24}$ ~~a neutron such as at thermal energy of U~~ it may not be small enough since very low conc. of U has to be used in order to avoid a capture ~~of the ... at~~ a large fraction of the neutrons at res. by U. — Since a single collision with Carbon slows the neutrons only very little a neutron which has ~~been slowed down~~ entered the resonance region stays ~~there for a long time~~ within that region for a long time and has therefore a large probability to be captured by the neutrons

~~[illegible] the chief [illegible]~~

~~[illegible]~~ A new method for measuring very small cr. + sections has now been devised which is now being ~~will be~~ applied to graphite ~~and~~

Introducion Carbon II

~~Yet besides in the form of~~
~~graphite~~
accordingly ~~the path~~ a fast neutron produced in the C will diffuse further away than it would in water before it is slowed down to energies of the order of volts. Yet this distance in graphite is only about cm× [illegible] ~~[illegible]~~ Since the capture cross section of carbon is small mixtures could be used which contain only a small amount of U ~~and~~ this has the double ~~[illegible] and~~ advantage ~~that from the point of view~~ the distances required for slowing down are not materially increased and that the number of U atoms ~~total~~ volume taken ~~[illegible]~~ by U is a comparatively small fraction of the total volume ~~In a homogeneous mixture of C and U~~

~~U there is however a considerable~~
~~[illegible] abs. of res. [illegible] neutrons by U~~
~~and since only an approx. estimate~~
~~of the C × section of carbon is~~
~~known at present it is not possible~~
~~for the present to state [illegible] whether~~
~~a chain reaction would be maintaining~~

Introducing Carbon I

~~It appears that carbon is~~
~~the~~ an ~~element which has very~~
~~affinity~~

It appears that in order to make a chain reaction possible Carbon is a very much better element ~~to be used~~ for slowing down the neutrons than H_2O. ~~It is true that~~ It takes ~~It takes about~~ 6.5 collisions in carbon ~~which~~ diminishes the neutron energy ~~on the average only by a factor~~ by a factor of $\frac{1}{e}$ ~~and has also a scattering cross section~~

0.862

~~is 3 to 4 times smaller than that of H for.~~

$$e^{-x} = \frac{1}{0.858}$$

.142

1.165 0.1527 2

~~but on the other hand carbon in the form of graphite has a density~~
~~very~~ containing

~~but nevertheless to be the large number of atoms which are contained per cc in~~

by uranium than

μ of neutrons are produced per neutron which is absorbed by the system and

$\mu q > 1$ difference between water and carbon

is necessary to have a chain reaction.

~~If Σ designates~~

~~In order to keep~~ water/carbon statement or some other similar element

We wish to consider a system which is composed of C and U; the ~~or some other~~ carbon slows down the neutrons and owing to its small capture cross section the range ~~for thermal neutrons the range is~~ of slow neutron in carbon ~~is very much larger than~~ is perhaps 40 cm ~~as~~. — ~~compared with water~~ [In contrast to water where this range is about 2 cm ~~For this reason and because of the~~ and other reasons ~~little or no useful information our treatment cannot be usefully applied to~~ few of the statements which we shall make will apply to the U + H_2O mixtures.—]

We are interested in finding Σ the ratio of the neutrons absorbed by thermal energies and those absorbed by U at resonance ~~when~~ in the core when

Liter 3

~~Since it is known that more than 1 neutron is emitted per neutron absorbed by uranium itself~~

About $p = 1.5$ neutrons have been found to be emitted per thermal neutrons which is absorbed by uranium and from certain ~~values which were~~ quantities which were measured by .... I calculate ~~this value a probably more exact value~~ as a better value ~~better~~

~~of~~ $\mu = 2$. ~~[illegible]~~

~~If p is the fraction of neutrons which is abs in the system by U at resonance before the neutrons reach the thermal region~~

~~If we have a homogeneous mixture of uranium and water ~~and have~~ having n molecules of water per ~~uranium at~~ Atom of Uranium the fraction of the neutrons q which are absorbed by U in the thermal region is given by where ...~~

$$q = \frac{\sigma_d(U) + \sigma_f(U)}{\sigma_d(H) + \sigma_{ph} + n\sigma(H)}(1 - p)$$

~~[illegible]~~

If q is ~~number of~~ the fraction of neutrons which ~~reach the thermal region~~ are absorbed in the thermal region

Later!

~~A nuclear chain reaction could be maintained in a system composed of U and a light element (which serves for the neutrons down to thermal ... purpose of slowing down the neutrons to thermal energies)~~

In the following we shall consider the balance of neutron emission and absorption in a system composed of uranium and a light element which serves the purpose of slowing down the neutrons. ~~[illegible]~~ The question in which we are ~~just~~ interested is whether and under what condition it is possible to maintain a nuclear chain reaction in such a system. We ~~shall~~ ~~agree~~ ~~use~~ ~~talk the~~ call by the name ~~word~~ of a chain reaction only ~~to designate~~ ~~in connection with~~ a process in which more than 1 neutron is produced for every neutron absorbed ~~in the~~ within system. If fewer than one neutron is ~~obviously even if~~ ~~the this chain~~ ~~Such a chain reaction may be called convergent if more neutrons escape across the boundary of the system per unit time than are emitted by the uranium in the process~~ ~~[illegible]~~

Later 2

~~and this will always be the~~
~~case of the amounts~~
~~or divergent~~ produced it will
obviously still be true that secondary
neutrons will be emitted which in turn
will lead to tertiary neutrons, but we
~~the term~~ shall ~~not call~~ speak ~~of chain reaction in this~~
not call this a chain reaction* x ~~The~~
~~two~~ ~~A difference of terminology exists~~ exist
~~in this respect between certain american~~
~~and french publications~~ ~~makes it necessary~~ and makes it
~~to emphasize~~ necessary to redefine our
usage of the word x A chain reaction
may be called convergent if more
neutrons escape across the boundary
of the system than are emitted ~~per~~
by the uranium per unit time and
otherwise it may be called ~~convergent~~ divergent
~~but the question of divergence or conver-~~
~~gence depends on the size dimensions~~
~~of the system extended the system~~
~~is whereas and not on the composi~~
Whether a chain reaction takes
place depends essentially on the natural constants
and the composition of the system;
whether it is convergent or divergent
depends ~~essentially~~ then on ~~the~~ ~~how~~ its
~~extended in space the system~~
extension in space. * ⑩

~~by Uranium~~

the Uranium ~~is not slowed~~ (metall or oxide) and Carbon (graphite) are kept in separate layers. We shall ~~still~~ show that the value of ε may be greatly increased by using small ~~[illegible]~~ spheres of U instead of plane ~~[illegible] of the Uranium~~ layers embedded in the Carbon. In order to demonstrate this let us assume that the Uranium has infinite density and ~~that it does not scatter thermal~~ neglect the scattering of thermal neutrons and res neutrons in that layer. In these circumstances ~~the uranium thermal neutron density~~ U ~~would~~ layer would absorb not only ~~every~~ resonance neutron but also every thermal neutron which reaches it. ~~If we [illegible]~~ If we have a plane layer of U ~~the thermal neutron density can be calculated a~~ embedded in an infinite space filled with graphite the density of the thermal neutrons as a function of the distance from the plane is given by a sort of diffusion equation

Löff

4

~~to~~ $\mu g/$ exceeds for any value so unity. The constants involved are not sufficiently exactly known to enable us to exclude with certainty that a chain reaction might be possible in homogeneous mixtures of Uranium and Water but if μg exceeds unity it can only do so by a very small amount + this means that very large masses of U ~~are so~~ would be needed to approach a divergent chain reaction —

~~If~~ Whether conditions are more favourable if ~~a homogeneous mixture of U and carbon is used~~ ~~instead~~ instead of H carbon is used for slowing down the neutrons can not be said with certainty since at present only an upper limit of the carbon capture cross section is known. But in another respect carbon is certainly much more favourable than Hydrogen and this makes it possible to make use of a trick in increasing the value of E to a point where a chain reaction becomes possible. ~~The difference or~~ This difference in the behaviour of C and H is ~~c~~connected with the fact that the scattering cross section of ~~of Carbon varies only little~~ C is ~~only little different for the~~ about the same ~~an~~ for the neutrons as for res. neutrons of H ~~whereas~~ whereas the sc + of H varies by a factor of ~~3~~ about 3

the probability that ~~A thermal neutron in pure carbon has a probability of disapp disappears in carbon by capture after~~ which suffers k elastic collisions ~~in Ca~~ pure Carbon ~~with carbon~~ ~~at~~ with C atoms in pure C has a probability of $e^{-k\frac{\sigma_c(C)}{\sigma_{sc}(C)}}$ of surviving and accordingly a probability of $(1-e^{-k\frac{\sigma_c(C)}{\sigma_{sc}}})$ of ~~being this~~ having disappeared by capture. ~~Accordingly we shall postulate that~~ Thermal neutrons have thus something like ~~a~~ somewhat dwelling range of ~~fairly dis~~

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in Carbon ~~pulleded~~ per we may
~~define by put by defdualdian~~
~~as~~ $A = N_{sc}(C)\sqrt{\frac{\sigma_{sc}(C)}{3\sigma_c}}$

Similarly ~~resonance neutrons~~ faster neutrons
in pure Carbon ~~disappear~~ out of ~~the~~ a given energy int.
ΔE ~~in which there is an appreciably resonance absorption of U in Uranium~~
because they are slowed down by elastic collisions with C. ~~It is~~
We can put with for us sufficiently good approximation for the probability that a neutron ~~w~~ which enters the ~~energy~~ energy ~~resonance~~ region ΔE will be out of the region after K collisions with C atoms

$$1 - e^{-\frac{k}{k_0}}$$

~~where k_0 depends on the form of~~
~~In the following the~~ the
value of k_0 is determined by the breadth of the interval ΔE. ~~Thus~~
Accordingly ~~these~~ ~~used~~ ~~as~~ such "resonance" neutrons have a range of

$$B = \lambda_{sc}^{x}\sqrt{\frac{k_0}{3}}$$

We shall ~~consider~~ only consider

(3)

K

$\frac{\sigma_c + \sigma_f}{\sigma_c + \sigma_f + n\sigma_c H}$

β

$$q = \frac{\varepsilon}{\varepsilon + \cancel{k}\beta} \cancel{k}\beta \left(= \varepsilon p\right)$$

$$p = \frac{\cancel{k}\beta}{\varepsilon + \cancel{k}\beta}$$

$$q = (1-p)K$$

We can calculate q from the ratio ε of the neutrons captured by uranium ~~at~~ in the thermal region and those captured at resonance and the ~~capture cross sections of absorbtio~~ absorbtion cross sections of U ($\sigma_c(U) + \sigma_f(U)$) and Hydrogen ($\sigma_c(H)$) ~~We have~~ and find

$$q = \frac{\varepsilon}{\varepsilon + \beta} \cancel{k}\beta$$

$$K = \frac{\sigma_c + \sigma_f}{\sigma_c + \sigma_f + n\sigma_c(H)}$$

$n =$

for low Hydrogen conc ε becomes low for high Hydrogen concentrations K becomes small and it appears doubtful whether there

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the case in which ~~the~~ ~~Uranium is~~ bodies of uranium are embedded in Carbon and shall then assume that every neutron which reaches the Uranium while in the interval ΔE will be absorbed at resonance i.e. we assume that uranium has an infinite absorption in the interval $\Delta E = E_{max} - E_{min}$

For any absorption line which obeys the B.W ~~Formula~~ and has a resonance at E_R we have for the energy interval which extends from the energy for which the abs becomes a minimum ~~for~~ and which we choose as E_{min} ~~the value of~~ ~~1.5 E_R~~ $E_{min} = 0.2 E_R$ ~~and for~~ and if we choose as our E_{max} ~~the value of~~ as appears reasonable the value of $1.6 E_R$ we have $\frac{E_{max}}{E_{min}} = 2^3$ and accordingly we shall put $k_0 \approx 12.5$

~~In view of our present knowledge concerning~~ The same value of k_0 appears to be justified ~~in the light of our present knowledge~~ for the resonance abs. of U. –

Putting ~~...~~ $E_{max} = 1.6 E_R$ and $E_{min} = 0.2 E_R$ we have

$\frac{E_{max}}{E_m} = 8$ $k_0 \cong 12.5$

Accordingly we have

$B =$ when $A > \ldots$

Let us now first consider a plane layer of U embedded in an infinite space filled with carbon and assume that the Uranium ~~is black for all reson~~ is [perfectly] black not only for every resonance neutron but also every thermal neutron which reaches its surface

for $E = 2, E_r$

gives $\frac{E_{max}}{E_{min}} = 10$

$$k_0 = \frac{\ln \frac{E_{max}}{E_{min}}}{\ln\left(1 - \frac{\alpha}{2}\right)}$$

$$\alpha = \frac{4M}{(1+M)^2} \cong \frac{M}{2} \ln \frac{E_{max}}{E_{min}} \quad \text{for } M \gg 1$$

$$k_0 \cong 6 \ln 10 = 2.3 \times 6 =$$

we shall use $k_0 = 12.5$

Correction

~~Correction~~ In reality Σ_E is somewhat smaller than given by express.

No - ~~We shall now~~ ~~give an upper~~ $\Sigma_1 = \varphi \Sigma_0$
~~limit for the correction factor $f < 1$~~
~~which has to~~ ~~Since not we have~~
~~so far not taken into account the~~
~~reduction of Σ due to the fact that~~
~~the production of thermal neutrons~~
~~is reduced~~ because the production
of thermal neutrons is somewhat
reduced in the neighbourhood of
the U spheres

In reality the ratio $\frac{U_{th}}{U_{res}}$ is

somewhat smaller than $\alpha\Sigma$, $\frac{U_{th}}{U_{res}} = f \Sigma \alpha = f \varphi \Sigma_0 \alpha$
$f < 1$, because in reality ~~the~~ ~~[illegible]~~ less than α thermal neutrons
are produced per cc and sec close to
the uranium spheres × It is easy though which is due to this effect
to give a lower limit for f. ~~Since the~~
~~mean free path is about~~ Since for carbon ~~[illegible]~~ $\lambda_{sc} \equiv \lambda^*_{sc}$
it follows for a lattice of black U
spheres that ~~at the most~~ the number
of thermal neutrons lost by the effect is at the most one ~~[illegible]~~
~~to~~ for average resonance neutron ~~the number of res~~ absorbed by
the spheres. since not every res neutron

Correction

$\cancel{N}\ \ \cancel{N \cdot \mathcal{A}^3}$ N spheres in A^3 cc

$u_{th} =$ $\alpha = \frac{1}{4}$

$\alpha \varepsilon$

$\frac{|\alpha\varepsilon_0 - 1|}{\alpha\varepsilon_0}\ \varepsilon$

$\frac{30}{4} - \frac{7}{6}$

$\gamma\varepsilon$ $\gamma > \frac{\alpha\varepsilon_0 - 1}{\alpha\varepsilon_0}$

$\gamma >= 1 - \frac{1}{\alpha\varepsilon_0}$

$\varphi\varepsilon_0 = \varepsilon$ $\gamma >= 1 - \frac{\varphi}{\alpha\varepsilon}$

$\alpha\varepsilon = \frac{4\varphi_m}{(1-\varphi_m)^2} \cdot \frac{(1-\varphi_m)}{2}$

$\frac{1}{117}$ $\gamma > 1 - \frac{\varphi}{2} \times \frac{1 - \varphi_m}{\varphi_m}$

$\varphi_m = \frac{1}{2}$

Greenfall $\gamma > 1 - \frac{\varphi}{2}(2 - 1)$

$\underline{L \text{ now very small } \sigma_c(\ell)}$ ①

blocks or not blocks

~~1' then above would always be dent~~

~~$\phi U_{\mu}(\text{single}) = U$~~

~~ϕ~~ $U_{\mu}(\text{single}) \propto \dfrac{\phi \varepsilon_0 \alpha + 1}{\phi \varepsilon_0 \alpha} = Q L^3 (1-\alpha)$

$$\frac{1-\alpha}{q}$$

$$\phi U_{\mu}(\text{single}) \frac{\alpha}{q} = Q L^3$$

in max $\dfrac{\alpha}{q} = \dfrac{1-q_m}{2q_m}$

~~if $q_m \to 1$ $L^3 \to 0$~~

~~all right~~

for $q_m = \frac{1}{2}$ $\alpha = 1/4$ $\dfrac{1-q_m}{2q_m}$

$\dfrac{\alpha}{q} = \dfrac{1}{8}$ | $L^3 = 4\pi\phi \left(\dfrac{\alpha}{q}\right)\left\{\dfrac{r_1^2}{a} + \dfrac{r_1}{a^2}\right\}$

$$L^3 = \frac{4\pi}{8}\phi\left\{\frac{r_1^2}{a} + \frac{r_1}{a^2}\right\} \cong \frac{\pi}{2}\phi A^2 r_1$$

$\frac{\pi}{2}\phi \cong 1$ $A \cong \dfrac{\lambda_{rc}}{\sqrt{3}}\sqrt{\dfrac{\sigma_{c_0}}{\sigma_c}}$

$= \dfrac{2.4}{17}\sqrt{500} = 3.1\ cm$

(2)

L

$L \cong \sqrt[3]{A^2 r_1} = \cancel{10}\,17\ \mathrm{cm}$

for large A

$\cancel{}\quad L^3 = 4\pi\varphi \frac{1-\varphi_m}{2\varphi_m}\left\{A^2 r_1 + \cancel{A r_1^2}\right\}$

~~for~~ ε large

$$\varphi \cong 1 - \frac{2}{\sqrt{\varepsilon}}$$

$$1-\varphi = \frac{2}{\sqrt{\varepsilon}}$$

$$L^3 \cong \frac{4\pi\varphi}{\sqrt{\varepsilon}} A^2 r_1$$

$$L^3 \cong \frac{4\pi\varphi}{\sqrt{4\varepsilon_0}} A^2 r_1 \cong \frac{4\pi\varphi}{A\sqrt{\frac{b^2}{6r_1+1}}}$$

$$\varepsilon_0 = A^2 \frac{b^2}{6r_1+1}$$

$\sqrt{2}$

$$L^3 \cancel{A} \cong \frac{4\pi\varphi\sqrt{6r_1+1}}{\cancel{b}} ABr_1$$

corresponding $\quad 4\pi\ 5 \times 5 \times 100$

$\sigma_c \cong 0.003 \quad L = 30\ \mathrm{cm}$

Corr ~~last page~~

$$\rho = \frac{1}{\varepsilon_0 \alpha \left(1+\varphi \varepsilon_0 \alpha\right)}$$

$$\alpha = \frac{(1-\varphi)}{2}$$

$$\varphi \varepsilon_0 = \frac{4\varphi}{(1-\varphi)^2}$$

$$\varepsilon_0 = \frac{8\varphi}{(1-\varphi_m)^3}$$

$$\cancel{\rho} = \varepsilon_0 \alpha = \frac{1}{\varphi}\frac{2\varphi_m}{1-\varphi_m}$$

Meissner

$$\frac{1}{\varphi}\frac{2q_m}{1-q_m}\left(1+\frac{2q_m}{1-q_m}\right)$$

$$\cancel{\rho = \frac{\varphi(1-q_m)}{2q_m\left(1+2q_m\right.}}$$

$$\rho = \frac{\varphi\left(1-\varphi_m\right)}{2\varphi_m\left(1+\frac{2q_m}{1-q_m}\right)}$$

$$= \frac{\varphi}{\frac{2\varphi_m}{1-\varphi_m}+\left(\frac{2\varphi_m}{1-q_m}\right)^2}$$

e.g. $\varphi_m = 1/2$, $\rho = \frac{4+2}{\varphi}$

for $\rho = \frac{1}{1.85}$ $\varphi <$ 0.09

Correction

We have ~~neglected~~ so far in calculating the value of g ~~the~~ the effect of the resonance absorption on the production of thermal neutrons near the U-spheres. ~~where~~ Due to this effect ~~the~~ ~~po~~ the production of ~~slow~~ thermal neutrons ~~will not be homogeneous~~ will not have the same value everywhere in the carbon but will fall of near the spheres. Accordingly the correct value for $\frac{U_{th}}{U_{res}}$ is ~~Eq~~ ~~instead of~~

$$\left(\frac{U_{th}}{U_{res}}\right)_{corr.} = f\varepsilon\alpha = f\varphi\varepsilon_0\alpha$$

where $f < 1$.

~~Since a resonance neutron (which the of number of them~~
~~it were not abs. resonances (one of the U spheres, U would~~
~~have~~ In order to find an upper limit for f let us assume that the resonance neutrons which reaches the U sphere ~~it is~~ they are not absorbed by the U but go on diffusing and

Corrections
finally became thermal
~~If the U sphere is black~~
~~at the most an equal num~~
A fraction of these neutron would then be absorbed as therm by the sphere but this fraction can certainly not exceed the value φ. ~~Since the Since the number of f~~ The number of the neutron which the U sphere fails to absorb as thermal neutrons due to the effect of res abs is accor ding φ per resonance neutron which reaches the sphere i.e per resonance neutron absorbed by the sphere. Corresponding we have

$$\frac{1}{\varepsilon_0}\delta$$

$$U_{th}^{(korr)} > U_{th} - \varphi\, U_{res}\, \frac{U_{th}}{U_{res}}$$

$$\frac{U_{th}(korr)}{U_{res}} > \frac{U_{th}}{U_{res}} - \varphi \;\; \cancel{= \alpha \varepsilon - \varphi}$$

$$\cancel{= \alpha\varepsilon\left(1 - \frac{\varphi}{\alpha\varepsilon}\right)}$$

$$= \frac{U_{th}}{U_{res}}\left(1 - \frac{\varphi}{\frac{U_{th}}{U_{res}}}\right)$$

Putting this value into eqn. we find

$$\varphi\,(korr) = \varphi\left(1 - \frac{1}{\varepsilon_0 \alpha}\;\frac{1}{1 + \varphi\varepsilon_0\alpha}\right)$$

Correction

would ~~be~~ came back to the sphere as a thermal neutron if there was no res abs. in U.

~~Similarly for~~ For a non black U sphere ~~if we ... to~~ the probability that a thermal neutron which reaches the sphere will be eventually absorbed by the sphere is given by φ and therefore the number of neutrons lost by this effect is on the average at the most φ ~~for every resonance for one resonance neutron~~ every resonance neutron which is absorbed

Accordingly

$$f_0 > 1 - \frac{1}{\varepsilon_0 \alpha} = \frac{\varepsilon_0 \alpha - 1}{\varepsilon_0 \alpha}$$

for black spheres and

$$f > 1 - \frac{\varphi}{\varepsilon \alpha} = 1 - \frac{\varphi}{\varepsilon_0 \alpha}$$

But $1 - \frac{1}{\varepsilon_0 \alpha} = 1 - \frac{\varphi}{2} \frac{1 - \varphi_m}{\varphi_m}$

$$f > 1 - \frac{\varphi}{2} \frac{1 - \varphi_m}{\varphi_m}$$

Therefore

$$q > \frac{8 \varphi \varepsilon_0 \alpha}{1 + 8 \varphi \varepsilon_0 \alpha} (1 - \alpha) =$$

$$= (1 - \alpha) \frac{\varphi \varepsilon_0 \alpha - \varphi}{1 + \varphi \varepsilon_0 \alpha - \varphi} =$$

$$= \frac{(\varepsilon_0 \alpha - 1) \varphi}{1 + (\varepsilon_0 \alpha - 1) \varphi} (1 - \alpha)$$

Correction

$$q \geq \frac{\alpha\varepsilon}{\frac{1}{f} + \alpha\varepsilon} = \frac{\alpha\varepsilon}{1+\frac{1}{\varepsilon_0\alpha}+\alpha\varepsilon}(1-\alpha)$$

$$\cancel{q} \geq \frac{\alpha\varepsilon / 1+\alpha\varepsilon}{1+\frac{1}{\varepsilon_0\alpha}\cdot\frac{1}{1+\alpha\varepsilon}}(1-\alpha)$$

$$q \geq q_{old}\left(1-\frac{1}{1+\alpha\varepsilon}\times\frac{1}{\varepsilon_0\alpha}\right) =$$

$$q > q_{old}\left(1-\frac{1}{1+q\alpha\varepsilon_0}\times\frac{1}{\varepsilon_0\alpha}\right)$$

$1/2$ $\quad$ $1/4$

Anzahl N_1 strikes block body

N at block N_1 strike once $N_1 x y$ twice $N_1 (xy)^2$

or n times escape return

$$R_y \varphi N_1 = N_1 \sum_{n=0}^{\infty} (xy)^n$$

Neu

steht für N_1

$R y \sum_0^{\infty} (xy)^n$ contribution of Res

neutrons

$$\Sigma = \varphi$$

Corr

$\boxed{R y \varphi}$ in contribution

$\boxed{y < 1}$

we have to add

$$\boxed{J^{th} < J^{th}(\text{korr}) < J^{th} + J^{Res} \varphi}$$

$$\cancel{\frac{J^{th}(\text{korr})}{J^{th}} \leq J^{th} + \frac{J^{th}}{J^{res}}}$$

$$q \cancel{\geq} q(\text{korr}) > \frac{\dfrac{J^{th} - J^{res} \varphi y}{J^{res}}}{1 + \dfrac{J^{th} - J^{res} \varphi y}{J^{res}}} (1 - \alpha) =$$

Let us say

$$q_{\text{korr}} = \frac{\varepsilon\alpha - \varphi}{1 + \varepsilon\alpha - \varphi} (1 - \alpha)$$

zu B

3/4

2

$$\frac{\cancel{8}\frac{1}{4} - \frac{1}{1.05}}{1 + \cancel{8}\frac{1}{4} - \frac{1}{1.05}} \quad 3/4$$

$$\varphi = \frac{1}{2}$$

$$q_{korr} = \frac{2 - \frac{1}{2}}{1 + 2 - \frac{1}{2}} \quad 3/4$$

$$\frac{\frac{3}{2}}{\frac{5}{2}}$$

$$\frac{3}{5} \times \frac{3}{4}$$

$\frac{9}{20}$ instead of $\frac{10}{20}$

$$q_{korr} = \frac{\frac{\Sigma\alpha}{1+\Sigma\alpha} - \frac{\varphi}{1+\Sigma\alpha}(1-\alpha)}{1 - \frac{\varphi}{1+\Sigma\alpha}}$$

$$q_{korr} = \frac{q - \frac{\varphi}{1+\Sigma\alpha}(1-\alpha)}{1 - \frac{\varphi}{1+\Sigma\alpha}} =$$

$$\frac{\varphi}{1+\Sigma\alpha} = q\frac{\left(1 - \frac{\varphi(1-\alpha)}{q(1+\Sigma\alpha)}\right)}{1 - \frac{\varphi}{1+\Sigma\alpha}}$$

$$\parallel$$

$$X = \frac{q\left(1 - \frac{X(1-\alpha)}{q}\right)}{1 - X}$$

$$q_{korr} = q\left(1 - \frac{X(1-\alpha)}{q}\right)\left(1 + X + X^2 + X^3\right)$$

$q\left(1 - \frac{1+q}{2q} x\right.$

3

$\frac{\varphi}{1+\varepsilon\alpha}(1-\alpha)$

$q(1+\varepsilon\alpha) = \varepsilon\alpha(1-\alpha)$

$$\frac{q\left(1-\frac{\varphi}{\varepsilon\alpha}\right)}{\frac{\cancel{1-q\varphi}}{q(1+\varepsilon\alpha)}}$$

$q\ 1 -$

$\alpha\varepsilon = \frac{2qm}{(1-q)}$

$$q\left(1 - \frac{\varphi(1-\alpha)}{q(1+\varepsilon\alpha)}\right)\left(1 + \frac{\varphi}{1+\alpha\varepsilon} + \left(\frac{\varphi}{1+\alpha\varepsilon}\right)^2 + \dots\right.$$

$$q\left(1 + \frac{\varphi}{1+\alpha\varepsilon} - \frac{\varphi(1-\alpha)}{q(1+\varepsilon\alpha)} - \frac{\varphi^2(1-\alpha)}{q(1+\alpha\varepsilon)^2} + \frac{\varphi^2}{(1+\alpha\varepsilon)^2}\right.$$

$$= q\left(1 + \frac{(q+\alpha-1)\varphi}{q(1+\varepsilon\alpha)} + \frac{(\alpha+q-1)\varphi^2}{q(1+\alpha\varepsilon)^2} + \text{l.s.}\right.$$

$$= q\left(1 + \frac{(q+\alpha-1)\varphi}{q(1+\varepsilon\alpha)}\left(1 + \frac{\varphi}{1+\varepsilon\alpha}\right)\right.$$

4

$$\alpha\Sigma = \frac{2q}{1-q}$$

$$\alpha\Sigma + 1 = \frac{1+q}{1-q}$$

$$q+\alpha = \frac{1-q}{2} + q = \frac{1+q}{2}$$

$$q+\alpha-1 = \frac{q-1}{2} = -\frac{1-q}{2}$$

$$q = \left| \cancel{1} - \frac{1}{2}\frac{(1-q)^2}{q(1+q)}\varphi\left(1 + \varphi\frac{(1-q)}{1+q}\right)\right.$$

$$\cancel{\frac{2}{3}\;\frac{3}{2}\;2\;\frac{1}{4}\times\frac{1}{4}\;\frac{1}{2}}$$

$$\cancel{\frac{3}{16}\left(1+\right.}$$

$$-\frac{1}{4}\;\cancel{\frac{1}{4}\;\frac{2}{3}}$$

$$-\frac{1}{12}\left(1 + \frac{1}{4}\,\frac{2}{3}\right)$$

$$\frac{1}{12} + \frac{1}{12}\times\frac{1}{6}$$

$$\frac{1}{12} + \frac{1}{72} = \frac{7}{72}$$

(5)

$$\varphi\left(\frac{1-\dfrac{\varphi(1-\alpha)}{\varphi(1+\varepsilon\alpha)}}{1-\dfrac{\varphi}{1+\varepsilon\alpha}}\right) =$$

$$= \frac{\varphi\left(1-\dfrac{\varphi}{\alpha\varepsilon}\right)}{1-\dfrac{\varphi(1-\varphi)}{1+\varphi}} = \varphi\,\frac{1-\dfrac{\varphi(1-\varphi)}{2\varphi}}{1-\dfrac{\varphi(1-\varphi)}{1+\varphi}}$$

$\varphi(1-\varphi)$ ist klein

neglecting 3rd power of it

$$\left(1-\frac{\varphi(1-\varphi)}{2\varphi}\right)\left(1+\frac{\varphi(1-\varphi)}{1+\varphi}+\frac{[\varphi(1-\varphi)]^2}{(1+\varphi)^2}\right)$$

$$\cancel{1+\frac{\varphi(1-\varphi)(2\varphi-1)}{2\varphi(1+\varphi)}}$$

$$1-\frac{\varphi(1-\varphi)^2}{2\varphi(1+\varphi)}+[\varphi(1-\varphi)]^2\left(\frac{1}{(1+\varphi)^2}-\frac{1}{2\varphi(1+\varphi)}\right)$$

dta $\quad -(\varphi(1-\varphi))^2\,\dfrac{1-\varphi-(1+\varphi)}{2\varphi(1+\varphi)^2}$

If Corr.

N_1 strike back

N_2 strikes and black

~~N_2 Pr~~

orange not more than

$$N_2 - N_1\varphi > N_1 - N_1\varphi$$

obs. by more black

~~If the sphere then the probability that~~ ~~of this more [illegible] reached~~

$$(N_1 - \varphi)N_1$$

orange or more

how many will be eventually be absorbed

If black now leave ~~$N_1 - N_1\varphi$~~

$$N_1(1-\varphi)\,x = N_1\varphi$$

proof of obs.

N_1 xN_1 strikes second time

x^2N_1 third time

$N_1 + xN$ $N_1 \leq x^n$ lots

N_1 black $N_1 \leq$ NXy prob for escape & prob for return