Losses in Slowing Neutrons.

Neutrons slowed in an absorbing medium by elastic collisions arrive at or below any given energy with a probability t which is less than one. The logarithm of 1/t varies in certain cases of interest inversely as the square root of the concentration of scattering atoms relative to absorbing atoms. Measurements of the activation of a detector when a neutron source is placed at various points in a medium of finite extent allow one to deduce both the transmission t for an infinite medium and the mean square distance from a point source at which the neutrons become thermal.

1. Introduction.

Neutrons slowed principally by elastic collisions arrive at, or below, any given energy with a probability, t, which is less than one and which depends on (a) the concentration of absorbing nuclei, (b) the variation with energy of the absorption cross-section, (c) diffusion losses (if the slowing medium is of finite extent). When the neutrons are slowed in a hydrogenic medium, their distribution in energy and hence the transmission can be calculated on the basis of the general theory given by Fermi.[1]

1. E. Fermi, Zeeman Festschrift (1935), p. 128.

The theory leads in general to a complicated mathematical connection between the transmission and the structure of the absorption spectrum of the capturing atoms. Within a certain approximation, however, it is possible to show that this connection assumes a very simple form (Section 2). It follows in particular for certain cases of actual physical interest (Section 3) that the logarithm of the transmission varies as the inverse square root of the dilution of absorbing atoms. A lower limit for the

constant of proportionality follows from measurements of self-absorption and of activation by capture. In Section 4 theory and observation are compared for the case of uranium and found to give reasonable agreemtnt.

Under suitable restrictions, and with the aid of a formula due to Placzek, the theory for hydrogen is extended to other slowing materials (Sections 5 and 6). In particular, measurements of the transmission when the absorber is dissolved in one slowing medium allow the calculation of the transmission for another medium.

In a finite medium, losses by diffusion reduce the transmission and are difficult to calculate. When, however, activation measurements are made with a detector at the center of a sphere and a point source of neutrons at various distances along a radius, the theory is simple (Sections 7 and 8). Such measurements carried out for spheres of two different radii allow one (a) to obtain the mean square of the distance of a slow neutron from its point of origin, and (b) to calculate for a sphere of any other radius the average chance t that a neutron will be slowed without loss (the average being taken in a certain simple way over various positions of the source in the interior of the sphere)(Section 9).

2. Connection between transmission and the absorption cross-section.

The probability t that neutrons of primary energy E_p shall be slowed without loss to energy E is defined for our purposes as the number of neutrons arriving per second in a unit energy interval at E expressed in relation to the corresponding number when there is no loss. From the standard theory[2] of the slowing of neutrons in hydrogenic media we find for

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2. See for example H. Bethe, Rev. Mod. Phys. 9, 123 (1937).

this ratio the expression

$$t = (s_p/s_p + a_p) \exp - \int_{\mathcal{E}}^{E_p} (a_E/a_E + s_E)\, dE/E. \qquad (1)$$

Here a_E is the probability per second that a neutron of energy E shall be lost, whether by capture alone, as we shall assume in the next sections, or by any other mechanism, such as diffusion, which we shall consider in Sections 7, 8 and 9. At present we may write a as the product $n_U v \sigma_U$ of concentration of absorbing atoms times neutron velocity times capture cross-section (or as the sum of such products if there are several species of absorbing atoms). Similarly, $s = n_H v \sigma_H$ is the probability per second of a collision with a proton. The concentration of hydrogen is supposed sufficient that such collisions alone are important in slowing neutrons. The chance, a_p, that a neutron of the primary energy shall be absorbed will be negligible compared to s_p in most cases of interest, so for convenience we shall drop the first factor in (1) hereafter; it is easily reintroduced in the subsequent formulae.

The ratio of absorption to scattering can be directly measured for neutrons of the primary energy E_p. Thus we can obtain from experiment and plot as in Fig. 1(i) the left hand side of an equation equivalent to (1):

$$\ell n(1/t) = \int_{\mathcal{E}}^{E_p} \frac{\sigma_U(E,\text{abs})/\sigma_H(E,\text{scatt})}{\sigma_U(E,\text{abs})/\sigma_H(E,\text{scatt}) + (n_H/n_U)}\, d(\ell n E). \qquad (2)$$

The right hand side of this equation is an integral of the type $\int y\, d\, x$. It represents the area under a curve which in general will possess a great many peaks due to resonance absorption. We will arrive at the same value for the area by calculating the sum, Σ, of the widths, $\Delta_i x$, of all peaks at a certain height, y, and calculating the integral $\int \Sigma\, dy$. Either by this argument or by partial integration we transform the right side of

Eq. (2) into the expression

$$\int_0^\infty \frac{(n_H/n_U)}{\left[(\sigma_U/\sigma_H) + (n_H/n_U)\right]^2} \sum d(\sigma_U/\sigma_H), \quad (3)$$

where σ_U/σ_H is now the independent variable and $\sum$ is an abbreviation for $\sum_i \Delta_i \ln E$. The integral has a simple interpretation. Plot the ratio of the cross-sections σ_U and σ_H as a function of the logarithm of energy over the interval E to E_o as in Fig. 1(iv). Superpose all these curves as in Fig. 1(iii) to obtain the total logarithmic energy width, $\Sigma = \sum_i \Delta_i \ln E$, for each value of the ratio σ_U/σ_H. Shade in a unit area of the resultant curve. Its contribution to the logarithm of 1/t is, according to (2),

$$(n_H/n_U)/\left[(\sigma_U/\sigma_H) + (n_H/n_U)\right]^2, \quad (3)$$

which is shown as a function of the dilution ratio (n_H/n_U) in Fig. 1(ii). The function has a maximum for $(n_H/n_U) = (\sigma_U/\sigma_H)$; moreover the area under the maximum of the curve (ii) is essentially equal to the area in curve (iii) whose contribution we are discussing (the maximum value of (ii) is $1/4(\sigma_U/\sigma_H)$ and the curve has a width of $3(\sigma_U/\sigma_H)$ at half maximum; the product is 3/4 as compared to the cross hatched area of unity). Add together many curves of the form (ii) corresponding to all elementary areas under the curve (iii). Then the resultant function is just the curve (i) which represents the logarithm of $s_o/(s_o + a_o)t$ as a function of the dilution, according to Eq. (2). From the manner in which curve (i) is built up from curve (iii), and in particular from the pronounced maximum in the intermediary curves (ii), we conclude:

Theorem:

The curve for the logarithmic level width of the nuclear absorption

levels is approximately identical with the curve obtained from measurements of the transmission probability as a function of dilution. The stated identity has to be qualified by the word "approximately" because the fine structure of (iii) is lost in the transformation to curve (i), and moreover curve (i) has for great dilutions a tail which is not present in (iii) and which falls off inversely as the dilution (as is to be expected!).

In order to give a mathematical proof of the approximate identity of curves (i) and (iii), it will be sufficient to show that a Fourier analysis gives the same amplitudes and phases for the low frequency or slowly varying components of both curves. Introduce new independent variables[3]

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3. I am indebted to Professor S. Bochner for suggesting the transformation of the kernel in (2) to a symmetric form.

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x and y defined by $(\sigma_U/\sigma_H) = e^x$, $(n_H/n_U) = e^y$ (i.e., make horizontal measurements in Fig. 1 on a logarithmic scale) and abbreviate $\ln s_o/(s_o + a_o)t$ by L. Then, from (2),

$$L(y) = \int_{-\infty}^{+\infty} \left[\exp\left(\frac{1}{2}x - \frac{1}{2}y\right) + \exp\left(\frac{1}{2}y - \frac{1}{2}x\right)\right]^{-2} \sum(x)dx. \qquad (4)$$

The symmetric function of x-y in the integrand is known to be representable as the expression

$$(1/2\pi)\int_{-\infty}^{+\infty} d\nu\, e^{i\nu y}\, (\pi\nu/\sinh \pi\nu) e^{-i\nu x}.$$

Therefore we may rewrite (4) in the form

$$L(y) = (1/2\pi) \int_{-\infty}^{+\infty} d\tau\, e^{i\tau y}\, (\pi\nu/\sinh \pi\nu) \quad e^{-i\nu x} \sum(x)dx. \qquad (5)$$

Take the Fourier transform of both sides of (5) and obtain the result

$$\int_{-\infty}^{+\infty} e^{-i\nu y} L(y)dy = (\pi\nu/\sinh \pi\nu) \int_{-\infty}^{+\infty} e^{-i\tau x} \sum(x)dx. \qquad (6)$$

If the expression $(\pi\nu/\sinh \pi\nu)$ were replaced by unity, Eq. (6) would state

that the amplitudes and phases of all Fourier components of the function L are identical with those of the function Σ ; the two functions would be exactly equal. The actual attenuation factor for components of frequency ν is $\pi\nu/\sinh \pi\nu$; it gives a very great decrement for high frequencies but almost perfect filtering of low frequencies, thus smoothing out curve (iii) to the form (i) (q.e.d.).

3. Dependence of transmission on concentration.

A second conclusion apparent either from Eqs. (1) and (2) or from the geometry of Fig. 1 is that curve (i) falls off monotonically with dilution at a rate which is never so rapid as inversely with the dilution. If we know the logarithm of the transmission probability for one value of (n_H/n_U), the value of this logarithm at twice the dilution will be less, but not less by as much as a factor of two; similarly at half the dilution the logarithm will be greater, but not double as much as it was for the given dilution.

We shall discuss in more detail the dependence of transmission on concentration, considering three cases: (a) the absorption is due to one resonance level with negligible Doppler broadening, (b) is due to many such levels, (c) the Doppler broadening is appreciable.

The cross-section due to one level of energy E_o and natural width Γ will be given as a function of energy by $\sigma_o / 1+(E-E_o/\frac{1}{2}\Gamma)^2$. This expression varies inversely as $(E-E_o)^2$ except near exact resonance. Therefore when the cross-section has fallen to a value σ_U small compared with σ_o, the width of the level will be $\Gamma(\sigma_o/\sigma_U)^{1/2}$. Divide this by E_o and obtain the logarithmic level width indicated by the arrow in Fig. 1 (iv). This logarithmic level width is proportional to the inverse square root of the argument (σ_U/σ_H), the constant of proportionality being

$$\Gamma\sigma_o^{1/2}/E_o\sigma_H^{1/2}. \qquad (7)$$

With the aid of the theorem stated above we therefore conclude that the logarithm of the neutron transmission probability (or of 1/t) varies inversely as the root, $(n_H/n_U)^{1/2}$, of the dilution of absorbing atoms, the constant of proportionality being given approximately by expression (7). An accurate calculation by direct substitution in (2) gives the result

$$(n_H/n_U)^{1/2}\, \ell n\, 1/t \;=\; (\pi\,\Gamma\sigma_o^{1/2}/2E_o\sigma_H^{1/2})(1+n_H\sigma_H/n_U\sigma_o)^{1/2}. \qquad (8)$$

Thus the stated dependence on dilution holds when the dilution, n_H/n_U, is small in comparison with the ratio of cross-sections, σ_o/σ_H, a result also directly evident from the construction of Fig. 1. Also the constant of proportionality differs from (7) by a factor $\pi/2$. This difference gives an impression as to the extent of the accuracy of the above theorem.

When the absorption is due to many levels, the transmission will be determined by the sum of a number of expressions similar to that on the right hand side of (8). When the factors of the form $(1 + n_H\sigma_H/n_U\sigma_o)^{1/2}$ are essentially unity (which will be the case for not too great dilutions), we shall therefore again have the inverse square root law for the transmission, with the proportionality constant $k = (n_H/n_U)^{1/2}\, \ell n(1/t)$ given by

$$\sigma_H^{1/2}k \;=\; \sum_i\, (\pi\,\Gamma_i/2E_i)\,\sigma_i^{1/2} \;=\; S_{1/2}. \qquad (9)$$

The level structure will not in general be known, but it will be possible to set a lower limit to the value of k by measurement of the following quantities: (a) the so-called resonance activation α, defined as the integral $\int\sigma_U(E)\, dE/E$, and determined by the equation

$$\alpha \;=\; \sum_i\, (\pi\,\Gamma_i/2E_i)\sigma_i \;=\; S_1; \qquad (10)$$

and (b) the cross-section, σ_{self}, per atom of a thin absorber for capture of the non-thermal neutrons which emerge from a paraffin block. This cross-section and the resonance activation α allow one to calculate the quantity

$$2\alpha\sigma_{self} = \sum_i (\pi\Gamma_i/2E_i)\sigma_i^2 = S_2. \qquad (11)$$

On the basis of Eqs. (9), (10) and (11), calculate the expression $S_{1/2}^2 S_2 - S_1^3$, in which each term is the product of three infinite series. The coefficient of $2(\pi\Gamma_i/2E_i)(\pi\Gamma_j/2E_j)(\pi\Gamma_k/2E_k)$ in this series will be $\sigma_i^2\sigma_j^{1/2}\sigma_k^{1/2} + \sigma_i^{1/2}\sigma_j^2\sigma_k^{1/2} + \sigma_i^{1/2}\sigma_j^{1/2}\sigma_k^2 - 3\sigma_i\sigma_j\sigma_k$, a quantity which never becomes negative and only vanishes when $\sigma_i = \sigma_j = \sigma_k$. We conclude that the expression in question is never negative and has zero as a lower bound (which is attainable). Consequently <u>the lower limit to the constant in the transmission law is given by the inequality</u>

$$(n_H/n_U)^{1/2}\ell n(1/t) = k = S_{1/2}/\sigma_H^{1/2} \geq S_1^{3/2}/S_2^{1/2}\sigma_H^{1/2} = \alpha/(2\sigma_{self}\sigma_H)^{1/2}. \qquad (12)$$

The equality sign will apply when only one level is important.

When there are many levels, we shall expect comparable values for the widths Γ_i, and values σ_i, for the cross-section at the successive resonances falling off on the average inversely as velocity. The sum $S_{1/2}$ will converge approximately as the series $\sum_i (E_o/E_i)^{5/4}$. If as a basis for discussion we assume exactly this law of convergence, and take all levels to be equally spaced with a separation D equal to the energy E_o of the first level, we find that the sum $S_{1/2}$ is 4.6 times as great as the contribution of the lowest resonance. Similarly S_1 is 2.6 times as great as for a single level, S_2 1.6 times larger. The transmission constant k is found to be greater by a factor 1.40 than the lower limit of Eq. (12). When the level spacing has twice

the value just assumed (i.e., when $D = 2E_o$), the factor in question is 1.38.

The results of the preceding paragraph show that even when there are many levels, at most the first few are responsible for much loss in the process of slowing neutrons. The cross-section at resonance for these levels will be of the same order of magnitude as σ_o, provided their energies are low enough for them to contribute at all. Therefore the inverse square root law of transmission will be valid, according to the argument following Eq. (8), up to a dilution of the order of one quarter (case of many levels) to one half (one level) of the value of the ratio, σ_o/σ_H, of lowest resonance cross-section to hydrogen cross-section. For greater dilutions the logarithm will fall off more rapidly, and for values of n_H/n_U considerably greater than σ_o/σ_H it will be inversely proportional to dilution. Very small values of n_H/n_U, on the other hand, will correspond to values of σ_U/σ_H (or of the capture cross-section) so small that they are realized, if at all, only in the region of overlap between resonance levels. The logarithm of the transmission will then vary less slowly than inversely as the root of the dilution.

Doppler broadening of an absorption line reduces the cross-section at exact resonance and widens the line at its peak, but does not change the shape of the absorption line near its base (the influence of the natural width falls off inversely as the square of the distance from resonance while that of the thermal motion falls off exponentially as the square of this distance). This fact and the construction of Fig. 1 allow a qualitative treatment of neutron losses when the Doppler broadening is large in comparison with the natural line width. The logarithm of the transmission will vary for large dilutions inversely as n_H/n_U, then will come a region where the variation is even slower than $(n_H/n_U)^{-1/2}$, then we will get to a con-

siderable range of concentrations where the natural line width predominates and the inverse root law holds, and finally we come to values of n_H/n_U so low that, if elastic collisions with hydrogen are still mainly responsible for the slowing and the above treatment continues to apply, then the logarithm of the transmission will vary quite slowly with concentration.

4. Comparison with the observations.

The probability, t, that a neutron will be slowed without absorption to an energy below the lowest resonance of uranium has been measured by von Halban, Joliot, Kowarski and Perrin[4] for dilutions, n_H/n_U, of 30, 65 and 140, with the results $\ell n(1/t) = 0.22$, 0.15 and 0.12, respectively. An expression of the type $k(n_H/n_U)^{-1/2}$ fits all three values within the estimated experimental error of ± 0.02. This accord with the inverse square root transmission law is reasonable (a) because the dilution is at most 140, while σ_o/σ_H is at least $11{,}000 \times 10^{-24}/21 \times 10^{-24}$ or over 500, according to the absorption measurements of Anderson[5] and the cross-section for neutron-proton scattering given by Hanstein[6]; and (b) because the Doppler width, though apparently comparable with the natural width[5], does not seem to be larger than the latter.

4. H. Halban, F. Joliot, L. Kowarski and F. Perrin, J. de Physique 10, 428 (1939).

5. H. L. Anderson, Phys. Rev. 58, (1940). I am indebted to Dr. Anderson for the opportunity to see his manuscript before publication.
6. H. B. Hanstein, Phys. Rev. 57, 1045 (1940).

The absolute magnitude of the absorption measured by the Paris group is such as to give a value $k = 1.27 \pm 0.2$ for the constant in the transmission law. The theoretical lower limit for k stated in Eq. (12) is already implicit

in the work of Anderson[5], who has measured the activation α (330×10^{-24} cm^2 ± 40 per cent) and the cross-section for self-indication σ_{self} (4600×10^{-24} cm^2 ± 15 per cent); these figures give $k_{min} = 0.75$. This minimum value will be attained, however, only when a single level is responsible, a situation which seems from various indications[5] not to be the case for uranium. If there are many levels with comparable properties and spaced at a distance of the order of the energy of the first level, k will be increased by a factor of approximately 1.4 to a value 1.1. The agreement between this number and the figure obtained from the Paris results should not be stressed, in view of the difficulties of the measurements.

Anderson is able to obtain indirectly by means of an argument due to Fermi the transmission for dilutions greater than those employed in the Paris experiments. His figures are smaller than those which would be obtained from the inverse square root law with a constant k of either 1.27 or 1.1. This result may indicate that the dilutions in question lie in the region of transition to the inverse first power law.

5. Slowing by elastic collisions in non-hydrogenic material.

An accurate formula for the chance of loss in slowing neutrons in a non-hydrogenic material will in general depend in a very complicated way on the position in the energy spectrum and strength of absorption regions. A simple generalization of Eq. (1) for hydrogen is, however, valid in at least two special cases and will be taken as sufficiently accurate for our purposes:

$$\ell n(1/t) = \int_{E}^{E_0} (a/a+s)\, f\, d(\ell n E). \qquad (13)$$

Here f is the number of elastic collisions which a neutron of mass m must make with nuclei of mass M in order that the logarithm of the neutron energy

shall be reduced on the average by the same amount (i.e., unity) by which it is reduced in one collision in hydrogen:

f = 1/average reduction of logarithm in one collision.

If the scattering is isotropic in a center of gravity frame of reference, one collision will leave a neutron of energy E with energy equally likely to lie at any point in the interval from $E_o(M-m)^2/(M+m)^2 = E_o/s$ to E_o, and f will be given by the expression $(s-1)/(s-1-\ell n s)$.

Table I

Nucleus	H^1	H^2	He^4	Be^9	C^{12}	O^{16}
f	1	1.378	2.35	4.85	6.35	8.35
$(M+m)^2/4Mm$	1	1.25	1.56	2.78	3.52	4.52
σ_H/σ_s [7]	1	5	13	4	4.2	5

7. See reference 6, also Dunning, Pegram, Fink and Mitchell, Phys. Rev. 48, 265 (1935); Carroll and Dunning, Phys. Rev. 54, 541 (1938); H. B. Hanstein and J. R. Dunning, Phys. Rev. 57, (1940).

The chance a/a+s of absorption per collision will be $n_U\sigma_U/n_U\sigma_U + n_s\sigma_s = (\sigma_U/\sigma_H)/(\sigma_U/\sigma_H) + (n_s\sigma_s/n_U\sigma_H)$, where n_s and σ_s represent the concentration and scattering power of the nuclei which slow neutrons. Compare this with the results for hydrogen: we see that curve (i) of Fig. 1 will apply to the new slowing medium, provided we rename the vertical coordinate $(1/f)\ell n(1/t)$ and replace the label n_H/n_U for the horizontal coordinate by $(\sigma_s/\sigma_H)(n_s/n_U)$. As an example let us calculate from Fig. 1 the amount of dilution, n_s/n_U by deuterium required to give a transmission of 70 per cent. For vertical coordinate we have $(1/1.38)\ell n(1/0.7) = 0.26$; according to the

curve this means a horizontal coordinate of 24, whence $(n_s/n_U) = 24(\sigma_H/\sigma_s) = 120$.

6. Validity of transmission formula.

In a slowing medium where there is no absorption and the scattering is isotropic the number of neutrons arriving per second in an energy interval dE at E will be QfdE/E, according to Placzek.[8] Here Q is the number

8. G. Placzek, Phys. Rev. 55, 1130 (1939).

of neutrons produced per second and it is required (except in the case of hydrogen) that E be small compared with the primary energy in order that the formula be accurate. In the case of weak absorption the number of neutrons lost per second in dE will be obtained in the first approximation by multiplying the above expression by the chance a/a+s that a neutron shall be absorbed on collision; integration over all energies leads then directly to Eq. (13) when we take into account that t is nearly unity. Eq. (13) will also be justified in the case the absorption per atom is arbitrarily strong but concentrated at energies E_n, E_{n-1}, ... in bands in such a way that the width of each band is small in comparison with 1/f times its energy and the energy of each band is small in comparison as well with the energy of the next higher band as with the primary energy. The number of neutrons per second absorbed in the highest band will then be

$$Q \int f(dE/E.$$

Denote the integral by I_n; according to our assumptions I_n will be small compared to unity. Then in addition to the source of Q neutrons per second of primary energy we have a "negative source" of $-QI_n$ neutrons of energy E_n. Since the energy E_{n-1} is assumed small in comparison with E_n we can apply

Placzek's formula for both sources, to obtain the number of neutrons arriving per second in the lower band. The effect of the absorption near E_n is to decrease this number by the factor $1-I_n$ which, being nearly one, may be written as $\exp(-I_n)$. We obtain similar factors for the probability of transmission through each of the lower bands and so arrive directly at a result equivalent to Eq. (13). When, however, E_{n-1} (for example) is not small compared to E_n, Placzek's formula does not apply to the "negative neutron source" at E_n; ~~the error is in such a direction as to decrease the factor f~~ ~~source" at E_n.~~ At an energy E comparable to that of the source, the true distribution law falls off more rapidly than the formula $QfdE/E$ and tends to a limiting expression $Q\left[(M+m)^2/4Mm\right]dE/E$. (Only for hydrogen are the two expressions equal; see Table I.) The result of this correction is that the absorption at an E_n does not lower so much as we have supposed the number of neutrons arriving at E_{n-1}. Consequently there will actually be somewhat more absorption than we have calculated at E_{n-1}. The same will be true at all lower levels. The errors will in general, however, not be great because, as we have seen earlier, the major part of the absorption occurs mainly in the lowest few (~ 4) levels, the spacing between which will ordinarily <u>not</u> be very small in comparison with their energies.

7. <u>Losses by diffusion</u>.

When the slowing material occupies a sphere of finite radius R, there will be some loss of neutrons by diffusion as well as by absorption. These losses will depend in a complicated way on the distribution of neutron sources in the interior but can be calculated in a simple way if (a) the mean free path between collisions is small compared to R and (b) the number of neutrons produced in a unit volume at a distance r from the center is

proportional to (1/r) sin(πr/2R). The importance of this particular distribution law has been pointed out by Perrin.[9] It has the very useful

9. F. Perrin, Comptes Rendus 208, 1394 (1939).

property that it is self-maintaining, being unaltered by diffusion and absorption. We can effectively obtain such a distribution if we keep at some fixed point in the sphere the detector whose activity we observe, make successive measurements with the source at various distances from the center, and weight each measurement in accordance with the given distribution law.

The rate of change with time of the density of neutrons of energy E will be given by the diffusion constant D times the divergence of the gradient of the density. For the distribution in question the diffusion results in a fractional decrease in the density which is the same at every point in the medium, so that the distribution law is unaltered. This fractional decrease is $(\pi/2R)^2 D_E$ per second; it represents the chance per second, d, that a neutron of the given energy will be lost by diffusion, and has to be added to the chance of capture, $c = n_U \sigma_U v_E$, to obtain the total chance of loss, a, which enters into the quotient a/a+s = c+d/c+d+s in Eq. (13). The chance at a given moment of diffusing out of a sphere of radius large in comparison with the mean free path is, however, always very small in comparison with the total chance of scattering, s, so that when the absorption by capture is also small it will be legitimate to rewrite the above quotient in the form (c/c+s) + (d/s). Eq. (13) can now be written in the form

$$\ln \{s_o/(s_o+c_o+d_o)t\} = \int_E^{E_o} (c/c+s) \; f d(\ln E) + \int_E^{E_o} (d/s) \; f \; d(\ln E). \quad (14)$$

In fact, those absorption regions where capture is actually comparable with scattering and the above decomposition into two terms breaks down are regions where the expression c/c+s by itself represents with sufficient accuracy the

total integrand; the logarithmic width of such regions must be small if neutrons are to have an appreciable chance of being slowed without loss; the second integrand (d/s) is, however, always very small compared with unity; and therefore no appreciable error is made in (14). It follows from (14) that the chance a neutron shall be slowed without loss is given by the corresponding figure for an infinite medium, diminished by the factor

$$F = \exp - \int_{E}^{E_o} (\pi/2R)^2 (D_E/s) f \, d(\ell n E), \qquad (15)$$

provided the neutron sources are distributed as we have assumed in a large sphere and provided, of course, also that the energy E is above the thermal region and the region where chemical binding is important. In $f\, d(\ell n E)$ collisions the logarithm of the neutron energy is reduced on the average by $d(\ell n E)$; this number of collisions divided by the chance per second, s, of a collision is the average time required for that reduction in energy; multiplication by the diffusion constant D gives (per definition) the increase in mean squared distance; and therefore we have the result

$$F = \exp - (\pi^2/4)(r_{av}^2/R^2) \qquad (16)$$

where r_{av}^2 is the mean square distance a neutron diffuses in an infinite medium in slowing down from energy E_o to the energy E.

8. Transmission to thermal energies.

To calculate the probability, t_{th}, that a primary neutron be slowed without loss and then captured at thermal energy by a particular species of absorbing atom will, in general, be impossible without a detailed treatment of losses in the region of energies extending from an energy E_c several times the molecular zero point energy down to thermal energies. In this energy

range we cannot apply the law of slowing valid for higher energies. When, however, capture is negligible in this region, the desired absorption probability will be given by the chance t_c of transmission to the energy E_c in a material of infinite extent, multiplied by $\exp-(\pi^2/4)(r_{av}^2/R^2)$ (where r_{av}^2 is now the mean square distance diffused in an infinite medium in slowing from the primary energy to <u>thermal</u> energy), multiplied by the fraction c_1/c_1+c_2+d (the ratio for thermal velocities between the chance per second of the given mode of capture and the chance of all forms of loss). In this fraction c_i has the value $n_i\sigma_i v_{th}$ and d is given by $(\pi^2/4R^2)$ times the diffusion constant for thermal energies: $D = (v_{th}/3)\cdot l$ (mean free path for scattering). Thus we have

$$t_{1,th} = t_c(n_1\sigma_1/n_1\sigma_1 + n_2\sigma_2 + \pi^2 l/12R^2) \exp - (\pi^2/4)(r_{av}^2/R^2). \quad (17)$$

Applied for example to a sphere of pure water 10 cm in radius, Eq. (17) gives

$$t_{1,th} = 1(0.019/0.019 + 0.0025) \exp - 7.18 = 8 \times 10^{-4},$$

when we adopt the values $n_1\sigma_1 = 6.7 \times 10^{22} \times 0.28 \times 10^{-24} = 1.9 \times 10^{-2}$ cm^{-1}, $l = 0.30$ cm, $r_{av}^2 = 290$ cm^2.

9. <u>Determination of diffusion distance</u>.

Little information is available about the diffusion distance r_{av}^2 for non-hydrogenic materials, but Eq. (17) offers the possibility of determining this quantity when we know (a) the mean free path l with respect to scattering for thermal neutrons (b) the ratio $t_{1,th}/t^*_{1,th}$ of the activities produced by absorption in the thermal region in the case of spheres of radii R and R^*. Knowing from other sources the value of the total capture probability $n_1\sigma_1 + n_2\sigma_2$, we can calculate $A = n_1\sigma_1 + n_2\sigma_2 + \pi^2 l/12R^2$ and solve for r_{av}^2:

$$(A/A^*)(t_{1,th}/t^*_{1,th}) = \exp(\pi^2/4)(R^{*-2} - R^{-2})r_{av}^2. \qquad (18)$$

The ratio of the quantities t may be found as follows: (a) measure in turn the activity produced in a standard detector located at the center of one sphere when the source is placed at a number of distances from the center, (b) plot the results as a function of r, (c) multiply every point by $(r/R^2)\sin(\pi r/2R)$ to obtain a new curve, and calculate the area under this curve, (d) take the ratio of the areas obtained in the two cases (radii R and R^*). (One is of course not forced to use a sphere; when a cylinder of radius R and height L is employed, however, the activity has to be measured at a larger number of points, multiplied by $(rdrdz/LR^2)\cos(\pi z/L)\, J_o(2.4048r/R)$, and integrated with respect to z and R. The exponents in the above equations also have to be changed to $[(\pi/L)^2 + (2.4048/R)^2]r_{av}^2$.)

Caption for Fig. 1.

Fig. 1. Geometrical relation between energy level structure (iv) and logarithm of the probability t of slowing without capture. In (iv) is plotted the capture cross-section, σ_U, of the dissolved absorbing atoms (relative to the scattering cross-section, σ_H, of hydrogen) as a function of the logarithm of energy. $\Delta_1 \ln E$ represents the logarithmic width of the first resonance peak, and is a function of σ_U/σ_H. The sum of all such widths gives curve (iii), which is shown in the text to be approximately the same curve as (i). The alternative labelling in (i) applies when elastic collisions with nuclei other than protons effect the slowing of the neutrons. Then n_s and σ_s represent the concentration and scattering cross-section of these nuclei, and f is a number given in Table I. Curve (i) summarizes the available information in the case of uranium.