LATTICE OF URANIUM SPHERES

Let us consider ~~If we have~~ now an infinitely large number of uranium spheres forming a lattice embedded in an infinite mass of carbon and ~~want to~~ calculate the ratio of the number of thermal neutrons ~~and resonance neutrons absorbed~~ absorbed by the uranium spheres, to the number of resonance neutrons produced. under the assumption ~~we shall again assume for the time being~~ that everywhere in the carbon the same number Q of neutrons enter the resonance region ~~and the thermal region~~ per c.c and second.

In the absence of uranium the thermal neutron density in the graphite is given by $\rho_0 = \frac{Q}{S(C)}$. If a lattice of uranium spheres is embedded in the carbon, the average neutron density $\bar{\rho}$ in the carbon is reduced by some factor α . (This reduction is due to the absorption of res. n and th. n. by U. –)

$$\bar{\rho} = \alpha \rho_0$$

Since the number of neutrons captured per second by carbon is proportionate to the average neutron density, and since in the absence of uranium all the neutrons produced are captured by carbon, the fraction of the neutrons which is captured by carbon in the presence of the uranium lattice is given by α . Correspondingly, the fraction of the neutrons which are absorbed by the uranium lattice is given by $1-\alpha$.

→(22) In order to determine the number of thermal neutrons $\mathcal{J}^{th}$ absorbed per second by one uranium sphere within the lattice, we may consider the following: a single uranium sphere, which is embedded in carbon, does not appreciably affect the thermal neutron density at distances which are large compared to R. Equation No. 5 shows that even for a "black" uranium sphere at a distance of 2 R from the center of the sphere ρ has already reached the value of $1/2\, \rho_0$. For this reason, the uranium spheres within the lattice affect each other with respect to their thermal neutron absorption only in so far as the presence of these spheres in the carbon determines the average neutron density, and we have

$$\mathcal{J}^{th} = \alpha J^{th}$$

~~(20)~~

Further, since the distance L between neighboring uranium spheres within the lattice will be large compared to B, the range of the resonance neutrons in carbon, we have for $\mathcal{J}^{res}$, the number of resonance neutrons absorbed by a uranium sphere within the lattice

(21) $$\mathcal{J}^{res} = J^{res}$$

→ Ins. In (20) we have left out of consideration that on account of the res. abs. the number of thermal n produced per c.c. and sec. is not the same

From this it follows that q, ~~giving~~ the fraction of ~~all~~ the neutrons which originally emitted ~~are absorb~~ reach the thermal region and are absorbed as thermal neutrons by uranium ~~ed by the uranium spheres in the thermal region alone is given by~~ is represented by

(22) $$q = \frac{J^{th}\alpha}{J^{th}\alpha + J^{res}}(1-\alpha)$$

or

(23) $$q = \frac{\varepsilon\alpha}{1+\varepsilon\alpha}(1-\alpha)$$

The value of α ~~depends~~ ~~to~~ is dependent on the ratio of U and carbon in the "mixture"

This expression has its maximum value for $\alpha = \alpha_m$

(24) $$\alpha_m = \frac{-1+\sqrt{1+\varepsilon}}{\varepsilon}$$

and for the maximum value for q we have $q - q_m$

(25)* $$\alpha_m = \frac{1-q_m}{2}$$

(26)* $$q_m = 1 - 2\,\frac{-1+\sqrt{1+\varepsilon}}{\varepsilon}$$

or

(27) $$\varepsilon = \frac{4q_m}{(1-q_m)^2}$$

CORRECTED VALUE

By calculating the value of q from ε we have neglected the effect of the absorption of resonance neutrons (by the uranium spheres ~~embedd~~ in the lattice) on the production on thermal neutrons in the carbon in the neighborhood of the uranium spheres. The absorption of resonance neutrons reduces in reality the value of Q near the spheres below the average value of Q and accordingly the correct number J^{th}_{corr} of thermal neutrons absorbed by the uranium spheres per sphere will be smaller than J^{th}. In order to find the value for the difference $(J^{th}_{corr} - J^{th})$ we may ~~reason~~ in the following way:

If the uranium spheres in the lattice, by an act of God, stopped absorbing resonance neutrons then a neutron which reaches a given sphere at least once while its energy is in the resonance region would have some probability γ of reaching the sphere at least once after it had been slowed down to the thermal region. On the other hand, the probability that the sphere which is

.... Not "black" for thermal neutrons absorbs a thermal neutron which reaches the sphere at least once is given by φ . (φ was originally defined as the ratio of the number of thermal neutrons which a sphere of uranium absorbs and the number of thermal neutrons which an equally "black" sphere of uranium would absorb under the same previously specified conditions. It is easy to see though that φ can also be defined as the ratio of the number of thermal neutrons which are absorbed by the uranium sphere and the number of thermal neutrons which reach the surface of the uranium sphere at least once during their life time. That these two definitions of φ are identical simply follows from the fact that a "black" uranium sphere absorbs all the thermal neutrons which reach its surface during their lifetime.) It follows that we have

$$J^{th}_{corr} = J^{th} - \nu\varphi J^{res} \quad ; \quad 0 < \nu < 1$$

correspondingly we have

$$q_{corr} = \frac{J^{th} - \nu\varphi J^{res}}{J^{res} + J^{th} - \nu\varphi J^{res}}(1-\alpha)$$

or

$$q_{corr} = \frac{\varepsilon\alpha - \nu\varphi}{1 + \varepsilon\alpha - \nu\varphi}(1-\alpha)$$

and by introducing the value of q_m from equations No 25 and 27 we find

(28) $$q_{corr} = q_m \frac{1 - \frac{\nu\varphi(1-q_m)}{2q_m}}{1 - \frac{\nu\varphi(1-q_m)}{1+q_m}}$$

If we neglect terms which contain powers higher than the second of $\frac{\nu\varphi(1-q_m)}{1+q_m}$ we obtain

(29) $$q_{corr} = q_m\left\{1 - \frac{\nu\varphi(1-q_m)^2}{2q_m(1+q_m)}\left(1+\nu\varphi\frac{1-q_m}{1+q_m}\right)\right\}$$

Taking into consideration that we have $\nu < 1$ and that for the uranium spheres which we shall consider φ has a value of about $\varphi \cong 0.5$ we ~~shall~~ may assume $\nu\varphi \leq 0.5$. ~~From equation No. we find that for and we have :~~ and find then for $q_m > 0.5$ from No. 28 :

(30) $$q_{corr} > q_m \times 0.9$$

With increasing q_m q_{corr}/q_m approaches 1; for $\varphi \leq 0.667$; $q_m \geqq 0.68$. we have $q_{corr}/q_m > 0.97$. For $\varphi \leqq 0.645$; $q_m \geqq 0.73$ we have $q_{corr}/q_m > 0.98$

If we have now an infinitely large number of uranium spheres forming a lattice embedded in an infinite mass of carbon and want to calculate the ratio of the number of thermal neutrons and resonance neutrons absorbed by the uranium spheres, we shall again assume for the time being that everywhere in the carbon the same number Q of neutrons enter the resonance region and the thermal region per c.c and second.

In the absence of uranium the thermal neutron density in the graphite is given by $\rho_0 = \frac{Q}{S(C)}$. If a lattice of uranium spheres is embedded in the carbon, the average neutron density $\bar{\rho}$ in the carbon is reduced by some factor α .

$$\bar{\rho} = \alpha \rho_0$$

Since the number of neutrons captured per second by carbon is proportionate to the average neutron density, and since in the absence of uranium all the neutrons produced are captured by carbon, the fraction of the neutrons which is captured by carbon in the presence of the uranium lattice is given by α . Correspondingly, the fraction of the neutrons which are absorbed by the uranium lattice is given by $1-\alpha$.

In order to determine the number of thermal neutrons J^{th} absorbed per second by one uranium sphere within the lattice, we may consider the following: a single uranium sphere, which is embedded in carbon, does not appreciably affect the thermal neutron density at distances which are large compared to R. Equation No. 5 shows that even for a "black" uranium sphere at a distance of 2 R from the center of the sphere ρ has already reached the value of $\frac{1}{2}\rho_0$. For this reason, the uranium spheres within the lattice affect each other with respect to their thermal neutron absorption only in so far as the presence of these spheres in the carbon determines the average neutron density, and we have

(20) $$J^{th} = \alpha J^{th}$$

Further, since the distance L between neighboring uranium spheres within the lattice will be large compared to B, the range of the resonance neutrons in carbon, we have for J^{res}, the number of resonance neutrons absorbed by a uranium sphere within the lattice

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From this it follows that q the fraction of all the neutrons which are absorbed by the uranium spheres in the thermal region alone is given by

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$$q = \frac{J^{th}}{J^{th} + J^{res}} (1-\alpha)$$

or

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$$q = \frac{\varepsilon\alpha}{1+\varepsilon\alpha} (1-\alpha)$$

This expression has its maximum value for $\alpha = \alpha_m$

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$$\alpha_m = \frac{-1+\sqrt{1+\varepsilon}}{\varepsilon}$$

and for the maximum value for q we have $q - q_m$

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<u>CORRECTED VALUE</u>

By calculating the value of q from ε we have neglected the effect of the absorption of resonance neutrons (by the uranium spheres ~~embedd~~ in the lattice) on the production on thermal neutrons in the carbon in the neighborhood of the uranium spheres. The absorption of resonance neutrons reduces in reality the value of Q near the spheres below the average value of Q and accordingly the correct number J^{th}_{corr} of thermal neutrons absorbed by the uranium spheres per sphere will be smaller than J^{th}. In order to find the value for the difference $(J^{th}_{corr} - J^{th})$ we may ~~proceed~~ in the following way:

If the uranium spheres in the lattice, ~~[illegible]~~, stopped absorb~~ing~~ resonance neutrons then a neutron which reaches a given sphere at least once while its energy is in the resonance region would have some probability p of reaching the sphere at least once after it had been slowed down to the thermal region. On the other hand, the probability that the sphere which is

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It follows that we have

$$J^{th}_{corr} = J^{th} - \nu\varphi J^{res} \quad ; \quad 0 < \nu < 1$$

correspondingly we have

$$q_{corr} = \frac{J^{th} - \nu\varphi J^{res}}{J^{res} + J^{th} - \nu\varphi J^{res}}(1-\alpha)$$

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$$q_{corr} = \frac{\varepsilon\alpha - \nu\varphi}{1 + \varepsilon\alpha - \nu\varphi}(1-\alpha)$$

and by introducing the value of q_m from equation No 25 and 27, we find

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~~xxxxx~~ If we neglect terms which contain powers higher than the second of $\frac{\nu\varphi(1-q_m)}{1+q_m}$ we obtain ~~xxxx~~

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Taking into consideration that we have $\nu < 1$ and that for the uranium spheres which we shall consider φ has a value of ab ut $\varphi \cong 0.5$ we ~~shall~~ may assume $\nu\varphi \leqq 0.5$. ~~From equation No. we find that for and~~ and find then for $q_m > 0.5$ from No. 28 :
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With increasing q_m q_{corr}/q_m approaches 1; For $\varphi \leqq 0.667$; $q_m \geqq 0.68$. we have $q_{corr}/q_m > 0.97$. For $\varphi \leqq 0.645$; $q_m \geqq 0.73$ we have $q_{corr}/q_m > 0.98$

LATTICE OF URANIUM SPHERES

We shall now consider an infinitely large number of uranium spheres ~~wchi~~ which may form, for instance, a close-packed lattice embedded in an infinite mass of carbon and we shall determine q the ratio of the number of ~~the~~ thermal neutrons absorbed by the uranium spheres to the number of resonance neutrons produced in the system. We assume that the production of resonance neutrons is homogeneous throughout the whole mass of the carbon. The number of thermal neutrons produced in the carbon will be smaller than the number of resonance neutrons produced because a fraction of the latter is absorbed by the uranium spheres.

In the absence of the uranium spheres in the carbon the thermal neutron density ρ would have everywhere the same value ρ_0. In the presence of uranium spheres in the carbon the average neutron density $\bar{\rho}$ in the carbon is reduced by some factor

$$\bar{\rho} = \alpha \rho_0 \qquad \alpha < 1$$

This reduction of the average thermal neutron density in the carbon by the uranium spheres is partly due to the absorption of resonance neutrons by the uranium spheres and partly to the absorption of the thermal neutrons by the uranium spheres. Since the number of neutrons captured per second by carbon is proportionate to the average neutron density $\bar{\rho}$, and since in the absence of uranium all the neutrons produced are captured by carbon, the fraction of the neutrons which is captured by carbon in the presence of the uranium lattice is given by α. Correspondingly, the fraction of the neutrons which are absorbed by the uranium lattice is given by $1-\alpha$. We can therefore write for q

$$q = (1-\alpha)\frac{J^{th}}{J^{th} + J^{res}} \tag{22}$$

where J^{th} and J^{res} are the number of thermal neutrons and the number of resonance neutrons absorbed per second by one uranium sphere in the lattice.

We shall now calculate the values of J^{th} and J^{res} from the values of $\mathcal{J}^{th}$ and $\mathcal{J}^{res}$ which were previously defined for a single uranium sphere in carbon. In doing so we shall neglect for the ~~time being~~ moment the fact that the production of the thermal neutrons in the carbon which contains a lattice of uranium spheres is not homogeneous ~~as it is~~ being smaller in the proximity of the uranium spheres than elsewhere. This effect which is due to the absorption of resonance neutrons in the uranium spheres, will be taken into account later and will lead to a small correction which has to be applied to the final result.

If the distance L of the neighboring uranium spheres in the lattice is large compared to the radius of the spheres the effect of the thermal neutron absorption of <u>one</u> uranium sphere on the thermal neutron absorption of ~~one of~~ its neighbors will be negligible. Nevertheless, if the average thermal neutron density $\bar{\rho}$ may be greatly reduced in the carbon by the presence of the uranium spheres in particular if the range A of the thermal neutrons in carbon is large compared to the distance L. Under such conditions the average thermal neutron density $\bar{\rho}$ determines with good approximation the number of thermal neutrons absorbed by one sphere in the lattice and we can write

(20) $$J^{th} = \alpha J^{th}$$

In reality J^{th} the thermal neutron absorption will be somewhat higher so that No. 20 presents a conservative value but the correction is small* if the volume of the uranium spheres is small compared to the volume of the carbon. This can be seen, for instance, from Equation No. 5 which shows that for large values of T/R the thermal neutron density is close to ρ_∞ , even for a uranium sphere which is black for thermal neutrons.(2)

Appendix: the approximation is good only for small spheres and would give a ~~for~~ ~~[illegible]~~ too large thermal absorption for ~~diving results for~~ large spheres (such as would have to be considered if one used U oxide in place of U metal) for small spheres however ~~the correct form~~ there was an ~~is~~ advantage ~~in~~ using at this stage of our knowledge the ~~correct~~ approximation formulae which are more complicated). This is so particularly if one consider only surface resonance abs and not volume res. abs because the correct treatment shifts the optimum size of the spheres towards larger R whereas the volume res abs which is also neglected requires a shift of the optimum size towards smaller R. –

LATTICE OF URANIUM SPHERES

Let us consider now an infinitely large number of uranium spheres embedded in in an infinite mass of carbon. The uranium spheres may form for instance, a close-packed lattice. We shall assume that everywhere in the carbon the same number of neutrons enter the resonance region per c.c. and second and we wish to determine q the ratio of the number of thermal neutrons absorbed by the uranium spheres to the number of resonance neutrons produced in the system.

In the absence of the uranium spheres in the carbon the thermal neutron density would have everywhere the same value . If in the presence of uranium spheres in the carbon the average neutron density in the carbon is reduced by some factor

This reduction of the average thermal neutron density in the carbon by the uranium spheres is due to two causes, i.e. to the absorption of resonance neutrons by the uranium spheres and to the absorption of the thermal neutrons by the uranium sphe

Since the number of neutrons captured per second by carbon is proportionate to the average neutron density, and since in the absence of uranium all the neutrons produced are captured by carbon, the fraction of the neutrons which is captured by carbon in the presence of the uranium lattice is given by . Correspondingly, the fraction of the neutrons which are absorbed by the uranium lattice is given by . We can therefore write for q

where J and J are the number of thermal neutrons and the number of resonance neutrons absorbed per second by one uranium sphere in the lattice.

We shall now relate the values of J and J to the values of and which were previously defined for a single uranium sphere in carbon. In doing so we shall neglect for the time being the fact that the thermal neutrons in the carbon which contains a lattice of uranium spheres is not homogeneous but the production of the thermal ~~neutrons~~ neutrons in the neighborhood of the uranium spheres is reduced by virtue of the absorption of resonance neutrons by the uranium spheres. This effect will be taken into account later and lead to a small correction which has to be applied to the final result.