REMARKS TO PAPER July 26, 1941

The properties of a system of small uranium metal spheres embedded in graphite are investigated in this paper under certain simplifying assumptions.

The resonance absorption of uranium is calculated from a diffusion equation and the simple treatment is made possible by the assumption that the uranium is black for resonance neutrons in an energy interval and transparent outside this energy interval. If such an assumption is made the resonance neutrons have certain range in carbon which is assumed to be 6.5 cm. This corresponds to about 15 collisions of the neutrons with carbon or a resonance band stretching for instance, from 10 to 100 volts. This assumption would be far too pessimistic if we had only the one single resonance absorption line in the neighborhood of 10 volts but there may be other absorption lines at low energies.

The paper does not take into account the possibility of ~~their~~ higher energies which may make the chance of a chain reaction somewhat worse, and also ~~this~~ regards the liberation of neutrons through fission caused by the fission neutrons themselves which will somewhat improve the chance of a chain reaction. Inasmuch as the paper considers all resonance absorption as surface absorption wheras part of the absorption ought to be treated as mass absorption ~~in the next approximation~~, the paper ~~does not emphasize sufficiently the importance of having a low ratio of uranium to carbon.~~

The generation of fast neutrons is considered as uniform throughout the carbon in which these fast neutrons are slowed down without absorption until they reach the resonance region of uranium. Accordingly, we have a uniform generation of resonance neutrons throughout the carbon. The generation of thermal neutrons is also considered in the first approximation as uniform throughout the carbon but correction is applied ~~later~~ in order to take into account the fact that a smaller number of thermal neutrons are generated per c.c. in the neighborhood of the uranium spheres.

The fraction of thermal neutrons absorbed by the uranium spheres is calculated from a diffusion equation. Such a treatment assumes that the radius of the uranium spheres is large compared with the main free path for scattering in graphite but no serious deviations have to be feared down to a radius of 3 cm. for the uranium sphere. ~~On the other hand~~, the diffusion equation for thermal neutrons ~~should~~ must not be applied to the

interior of the uranium sphere for a radius smaller than 5 cm. and for spheres smaller than that the value of $\frac{d\varphi}{dr}/\varphi$ in the graphite at the surface of the uranium sphere has to be obtained by other methods.

The calculations for the thermal neutron absorption for the uranium spheres are carried out in an approximation ~~in which the fraction of the thermal neutrons absorbed by the uranium spheres can be considered as proportionate to the average thermal neutron density. This approximation~~ which leads to extraordinarily simple formulae and which is used throughout the whole paper. It leads to somewhat pessimistic results inasmuch as it exaggerates the fraction of the neutrons which are captured by carbon. However, for uranium spheres of 3 to 5 cm. radius the error introduced is well within the limits of error which are due to the experimental error of the nuclear values involved. Nevertheless, it should be ~~in order that~~ emphasized that ~~instead of uranium metal the~~ uranium is used in some ~~other~~ other than metal form ~~at which its density is low so that~~ the optimum radius of the uranium spheres ~~becomes~~ is large (for instance, if a loose ~~part~~ powder of uranium oxide is used) the formulae of this paper ~~can no longer~~ must not be applied ~~without seriously~~ as they would lead to ~~a~~ erroneous overestimating the absorption of carbon.

The following nuclear values have been used in this paper: For the carbon cross-section the published upper limit of 10^{-27} cm² for which the chain reaction appeared already possible if the graphite is ~~captured~~ kept at a higher temperature. For the ratio of the capture cross-section and the scattering cross-section of uranium the ratio of 1/2 which was chosen by ~~rather~~ deliberately disregarding the published values for the total capture cross-section of uranium which would have led to a more pessimistic value of 1/3. The higher value for this ratio was chosen because of the ~~view~~ improved that impurities in particular hydrogen were responsible for the higher total cross-section variously reported in the literature. For the uranium capture cross-section the rather pessimistic value of 4.5 was used. For the number of fast neutrons liberated from uranium per absorbed thermal neutron the paper used the value of $2 \pm .2$ which was considerably more optimistic than the value of 1.5 previously published by Anderson, Fermi, and Szilard. Allowances were made for a decrease of the chance of the

~~of the~~ chain reaction by non-uniform production of the thermal neutrons and other similar effects by making a reduction of 10% in the final result. Using these values the conclusion was reached that for a carbon capture cross-section of 10^{-27} cm² the chain reaction should be just possible under the most favorable conditions whereas ~~for a carbon~~ if the capture cross-section for carbon has ~~value~~ magnitude 1/2 this it should be possible to reach the point of divergence with an amount of graphite of the order of magnitude of 100 tons.

CONTROL OF THE CHAIN REACTION

July 19, 1941

In discussing the question of stabilizing ~~such~~ the a chain reaction we have to distinguish between purely laboratory set-ups in which the chain reaction is maintained at a low intensity so that the carbon and uranium remain cold; and industrial set-ups in which the chain reaction is maintained at an intensity at which the temperature rise is considerable. In the case of laboratory set-ups questions of thermal stability do not arise since the intensity of the chain reaction has to be limited artificially in order to avoid that the radiations emitted should become too intense and consequently artificial stabilization is not only necessary but has also to be absolutely reliable in order to avoid accidents. In the case of industrial set-ups one also has to use artificial stabilization but there are certain automatic safe-guard in existence which would be brought into play incase there should be a break-down in the control. These safe-guards consist in the fact that while the system does not possess thermal stability of a static nature it does possess thermal stability of a dynamic nature. This should perhaps be explained more clearly by stating the following: If the temperature of the system is gradually raised so that heat conduction equalizes the temperature of the uranium and the graphite then we have static conditions and the chain reaction is unstable in the sense that if both the temperature in the uranium and the carbon increase simultaneously the chain reaction becomes more intense. On the other hand, if there is a certain change ~~with~~in the intensity of the chain reaction then the temperature of the carbon will rise much more slowly than the temperature of the uranium. The system has a dynamic stability inasmuch that the intensity of the chain reaction decreases if the temperature of the carbon is kept constant and the temperature of the uranium is increased. In practice, this will amount to the following: If the controls break down the intensity of the chain reaction will increase very slowly and the rate of increase will be determined by the rate of heat transfer from the uranium to the carbon. Two different kinds of controls should be applied to the chain reaction: one type of control which responds fairly fast for instance, shifting absorbing sheets like

cadmium and the like which will take care of certain variations but which would not be economical ifit were applied for the purpose of adjusting the conditions to slow changes which take place in the composition of the uranium bodies. For this reason a second type of control is advisable which consists in moving the uranium bodies so as to change the uranium carbon ratio. The best mode of operation consists in working with the uranium carbon ratiox and reduce the efficiency of the chain reaction as the chain reaction becomes more and more efficient in the course of time.Owing to the production of fissionable elements in the uranium the uranium carbon ratio is further increased thereby reducing this efficiency in an economic way and bringing the system back close to the point of divergence of the chain reaction.

Memo on stabilizing the chain reaction.

Supplement to previous memo. July 22, 1941

If we consider a system composed of uranium and carbon and assume that the system is heat insulated, then, for any given fixed intensity of the chain reaction, the temperature will be uniform within the system after an initial period during which the temperature increases. In this stationary case we have then two factors, one working in the direction of thermal stability and the other working in the direction of thermal instability. Since the diffusion velocity of the thermal neutrons is higher for higher temperatures the fraction of thermal neutrons which is lost across the boundary of the system without reacting within the system increases and this works for thermal stability. On the other hand, the ratio of the number of thermal neutrons reacting with uranium and the number reacting with carbon also increases since the uranium spheres are about seventy to eighty per cent black. This second point works for thermal instability, and may very well over-compensate the first point so that we may have a thermal instable system.

The situation, however, is rather different if we have a system which is cooled, for instance, by a circulating liquid of fixed temperature. Since most of the heat is produced in the uranium spheres or cylinders the over temperature of the uranium will be higher than the over temperature of the carbon. If, for instance, the cooling is so arranged that the uranium graphite boundaries are kept at a fixed temperature, then the over temperatures in the uranium and the carbon will both be proportionate to the intensity of the chain reaction. The ratio of the absolute temperatures of the uranium and the carbon will accordingly increase with increasing intensity of the chain reaction and correspondingly the ratio of the thermal neutrons absorbed in uranium and in carbon may very well have a maximum for a certain intensity of the chain reaction. So that there may be a certain high rate of the chain reaction at which it becomes stable for thermal reasons.

GENERAL REMARKS TO THE ENCLOSED PAPER

The values for the nuclear constants used in the enclosed paper are all based on measurements prior to February 1940. Many of these nuclear constants were at that time only inaccurately measured and I used in each case what I considered the most probable value. I have in all cases used what I considered a rather conservative value with one single exception. I took for the ratio of the absorption cross-section of uranium and the scattering cross-section of uranium the value of $\frac{1}{2}$ instead of the less favorable value of less than 1/3 which would have seemed to be better agreement with the accepted value for the total cross-section of uranium. For this total cross-section values of 20 to 23 were published which I took the liberty of disregarding in the absence of chemical tests for purity particularly regarding hydrogen content. The resonance absorption of uranium was taken into account by assuming that uranium is black for resonance neutrons in an energy interval between a lower energy E 1 and a higher energy E 2 and by postulating .If uranium had only one singel resonance line which has been well investigated the above assumption would be extremely conservative. However, uranium may have several absorption lines somewhere between 10 and 100 volts and probably also higher energies which make up for the difference. If uranium has an appreciable absorption at higher energies this absorption can not be taken into account by changing the constants used in the paper by requiring a different treatment. The paper assumes that for small uranium spheres for which the calculations are made the fact of the high absorption lines of uranium can be neglected. This assumption was based on some unpublished observations which I made at Oxford in investigating the capture of photo neutrons from a radium-beryllium source which seemed to show that elements of even atomic number have a surprisingly low capture cross-section. The example of bismuth a heavy element of even atomic number which appears to show extremely capture has also encouraged me to neglect the effect of higher resonances in the case of uranium.

The method used leads to exceedingly simple formulae which make it very easy to see how the various factors affect the efficiency of the system for a chain reaction. This simplicity is due to the fact that the paper uses an approximation which is varied only in the case of small uranium metal spheres the radius of which being about 5 cms. This approximation leads to a conservative estimate inasmuch as it exaggerates the fraction of the neutrons which are absorbed in carbon. However, the deviation from the correct values is well within the limits of the experimental uncertainty of the nuclear constants used. It should be ~~xxpointed~~ pointed out ~~xhxx~~ however, the approximation used is not permissible if large spheres of uranium are being considered such as one would use if uranium oxide at low density is used in place of uranium metal.

From observations made by Dr. Zinn and myself in March and April 1939 it would seem that a considerable fraction of the fission neutrons are sufficiently fast to cause fission in uranium and by refraining from taking into account this ~~x~~ effect the paper further deviates from the factors on the conservative side. Finally, a very conservative correction of 10% is applied in the paper under the title of taking into account the inhomogenity of the production of thermal neutrons in the graphite. This correction is certainly ample to take care not only of this effect by also of the inhomogenity of the production of resonance neutrons.

While Anderson, Fermi, and Szilard found a value of 1.5 for the number of fast neutrons emitted per thermal neutron absorbed by uranium the paper adopts a value of 2 ± 0.2 which is considerably more favorable for a chain reaction. I derived this more favorable value from certain experimental data published by Halban, Joliot, Kowarski, and Perrin and adopted it because I believed their ~~xxx~~ experiment to be better than ours, while this and perhaps some other of the values which I used might be slightly on the optimistic side. For this reason I felt it safe to conclude in the paper that a chain reaction can be made to work in a system of uranium metal spheres embedded in graphite even if the carbon absorption cross-section were as high as the accepted upper limit for its value. I therefore considered ever since February 1934 the possibility of making the chain reaction

work in such a system pretty much as a foregone conclusion and I did not fail to state this whenever an opportunity arose. I personally regret very much that such action which was taken was not put in the past and is not being put at present on this basis.

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