

Prof. Daniels

NOTES

Added in May, 1946

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"If we have a closed spherical layer of material in which the chain reaction takes place the inner radius (r) of which is large compared with the mean free path of the neutrons which maintain the chain, the density (s) of the neutrons will with good approximation be given as a function of the radius (r) by the following equation:

$$D \frac{d^2(rs)}{dr^2} + A(rs) = 0$$

D and A are determined by: the mean free path of the neutrons a ; the mean velocity of the neutrons w ; the factor of the multiplying action f which says how many collisions of neutron are needed in the average in order to produce one new neutron.

$$A = w/af ; D = aw/3 ; \sqrt{\frac{D}{A}} = \frac{af}{\sqrt{3}}$$

"We are interested in the critical thickness of the spherical layer for which the gradient of the density s vanishes. If the thickness ($r_2 - r_1$) approaches we can maintain with a very weak source of initial radiation in the interior of the inner surface of the spherical layer a very strong chain reaction and we can easily get one thousand or more times more neutrons emerging from the chain reaction layer than the number of the neutrons forming the chain reaction layer than the number of the neutrons forming the initial radiation. If the outer surface ($r = r_2$) of the spherical layer were to stand free in space the density s would be zero for that surface and the critical value l would be given by $l = \pi/2 \sqrt{D/A}$. If the outer surface is covered by some material, for instance if the transmutation layer is immersed into water or covered by lead, the critical value l is reduced. This is due to the back scattering by water or lead and also to the fact that the neutrons are slowed down in the water and their mean free path is thereby reduced.

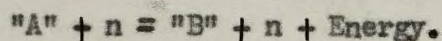
"It is important to prevent neutrons from escaping out of the interior of the inner surface of the spherical layer and also from being absorbed in the interior. If the initial radiation is generated by apparatus placed into the interior of the sphere the material used should be so selected as to lead to a minimum of absorption.

"If the thickness is larger than the critical value l we can produce an explosion.

We shall now discuss the composition of the matter in which the chain reaction is to be maintained. We wish to distinguish three main types of chains.

(a) Pure neutron chains, in which the links of the chain are formed by neutrons of the mass number 1 alone. Such chains are only possible in the presence of a metastable element. A metastable element is an element the mass of which (packing fraction) is sufficiently high to allow its disintegration into its parts under liberation of energy. Elements like uranium and thorium are examples of such metastable elements; these two elements reveal their metastable nature by emitting alpha particles. Other elements may be metastable without revealing their nature in this way. Whether an element is metastable or not can be determined by means of the mass spectrograph. If, for instance, the value obtained by Bainbridge for beryllium by means of the mass spectrograph, which appears to be generally accepted at present, is really valid, we have to conclude that beryllium is a metastable element and can disintegrate into parts with the liberation of energy, one of the parts set free in its disintegration being a neutron.

"If we have an element which is metastable but the disintegration of which is inhibited and if this inhibition can be lifted in a collision with a neutron we shall call such an element an inhibited metastable element. If an inhibited metastable element "A" is exposed to neutrons, we may have the following reaction.



"The element "A" transmutes into an element "B" which has the same atomic number and mass number and energy is transmitted to the neutron. The element "B" may break up into an element "C" and a neutron, the element "C" having the same atomic number as "B". The element "C" may or may not break up into further parts. If the interaction of a neutron with the element "A" leads in this way to an increase in the number of neutrons (the newly created neutrons would together with the original neutrons continue to interact with the elements "A" thereby forming the links of a chain reaction.)"

*21. It ought to be remembered at this point that in natural uranium essentially only thermal neutrons which are absorbed lead to fission and thus to neutron emission. Neutrons which are absorbed at higher energies are "absorbed at resonance" without leading to neutron emission, and are lost from the point of view of a chain reaction. Thus if ^afast neutron emitted in a uranium-carbon system is slowed down, a certain fraction is absorbed at resonance and the rest reaches the thermal region. Of the neutrons which reach the thermal region, some are absorbed as thermal neutrons by the carbon and the

rest is absorbed as thermal neutrons by uranium, leading to neutron emission. The paper assumed that no neutrons other than thermal neutrons are absorbed by carbon.

*22. This would be a conservative assumption for a single absorption line which obeys the Breit-Wigner formula. For such an absorption line the absorption falls off with $1/v$ in the thermal region and reaches a minimum at $0.2 E_0$ where E_0 is the resonance energy. Beyond the resonance energy the absorption falls rapidly and becomes negligible for $2E_0$ if E_0 is not too close to the thermal region. If E_0 is higher than five volts and if the temperature of the thermal neutrons does not exceed $1/10$ of a volt then the absorbing cross-section beyond $2E_0$ is less than $1/10$ of the absorbing cross-section for the thermal neutrons.

By postulating $E_2 = 10 E_1$ we have made a seemingly arbitrary pessimistic assumption with respect to the magnitude and the character of the resonance absorption. As far as magnitude is concerned the choice was justified by the following consideration: Results obtained by Joliot, Halban, Kowarski, and Fernin's paper, ⁽¹³⁾ indicate that a water-uranium system comes close to be chain reacting but certainly does not go much beyond being at best just about chain reacting. If we replace H with C it takes 6.5 collisions with carbon to slow down as much as by one collision by H, and since the scattering cross-section of carbon is 4 as compared with 18 for H, therefore we have to replace 1 H atom by 30 carbon atoms, if we wish to have the same resonance absorption in both systems. The absorption of H for thermal neutrons is about .3, and therefore, if the absorption of carbon for thermal neutrons is 30 times smaller, i.e., if $\sigma_c(C) = 0.01$, the loss of thermal neutrons by absorption will also be the same for the carbon as for the H system. In calculating the uranium-carbon system we had to use a number of constants which were only very inaccurately known but we chose a set of values in such a manner as to make the carbon-uranium system just about chain reacting (under the most favorable conditions) for the value of $\sigma_c(C) = 0.01$. For the set of constants used this condition was fulfilled by postulating that neutrons between energies E_1 and E_2 are absorbed at resonance if they come into contact with uranium and by setting $E_2 = 10 E_1$.

As to the character of the resonance absorption which we thus assumed to be pure surface absorption, this was of course a misleading assumption. It was partly made because it introduced only one parameter and led to very simple formulae, and partly

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*25. Equation 25 means that we have optimum conditions when half of the fast neutrons which are emitted but not absorbed by the uranium thermal region are absorbed by the carbon. That means that the other half must be absorbed by uranium at resonance so that we may interpret equation 25 as saying that we have optimum conditions when an equal fraction of the neutrons is lost through resonance absorption in uranium and through thermal absorption in carbon.

Equation 26 shows that we get optimum conditions by having ξ as large as possible.

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*26. We may in place of calculating μ , rather calculate the multiplication factor μq . Where q represents the fraction of the neutrons which is absorbed in the thermal region by uranium. Obviously we have

$$q = (1-p) \frac{\sigma_a(U)}{\sigma_a(U) + n\sigma_c(H)}$$

Thus using for μ its value from equation 38, we find

($\mu q = 0.85$)
This gives for $n = 3$, $p = 0.5$ and $\frac{\sigma_a(U)}{n\sigma_c(H)} = 5$ $\mu q = 1 - \frac{1}{5} \frac{1 - I_{ext}/I_0}{I_{int}/I_0} (1-p) \frac{1}{\frac{\sigma_a(U)}{n\sigma_c(H)} + 1}$

For $q = 1$ we could just maintain a chain reaction in the system. By comparing the experimental results obtained for $n = 1$, $n = 2$, and $n = 3$, we can extrapolate to larger values of n and thus can see that μq goes even closer to 1, if we increase n beyond 3.

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It may thus be seen that in spite of the uncertainty of the value p , the experiment shows that a homogeneous water-uranium system can come very close to be chain reacting.

The value of μ of course is very sensitive to the value of p and for $p = 0.3$ we would have $\mu = 1.42$ in place of $\mu = 2$. An earlier experiment yielded the value $\mu = 1.5$, and if we had chosen this value for the purpose of this paper it would have been necessary to assume a resonance absorption band narrower than the one corresponding to $E_2 = 10 E_1$.

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corresponding to 30 tons of uranium is a numerical error. The correct value is .022, corresponding to about 21 tons of uranium.

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Kerr L.

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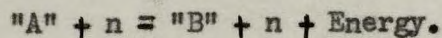
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