Spacing of the lattice.

Formula No. 26 giving the value of q_m was derived under the assumption that there is a uniform production of resonance neutrons throughout the whole mass of carbon into which the lattice of uranium spheres is embedded. We have to verify ~~that~~ this assumption ~~is correct~~. For this reason we have to compare the distance $\sqrt{\overline{r^2}}$ to which a fast neutron "diffuses" away from a uranium sphere from which it is emitted before it is slowed down to thermal energies with the distance L between two neighboring uranium spheres in a close-packed hexagonal or cubic lattice. If the value of L which corresponds to the maximum value of q were large compared to $\sqrt{\overline{r^2}}$ then obviously equation No. 26 giving the value of q_m could not be used.

In graphite of density 1.7 we have $\sqrt{\overline{r^2}}$ about 50 cm. and we shall see that the values of L which correspond to the maximum value of q are smaller for all values of $\sigma_c(C)$ which we are going to discuss in this paper.

In order to estimate L as well as for other reasons we shall calculate the volume V of carbon per uranium sphere in the lattice.

In the lattice of uranium spheres from the Q V neutrons which are slowed down per second within the volume V to resonance energies, the carbon absorbs $\alpha Q V$ neutrons and the uranium spheres in the lattice absorb $(1-\alpha) Q V$ neutrons. Accordingly we have $J^{th} + J^{res} = (1-\alpha) Q V$

and from this we find

31 $$V = 4\pi \varphi \frac{\alpha}{q} A^2 R(1 + R/A)$$

and if q has its maximum value q_m we have
$$\frac{\alpha}{q} = \frac{1-q_m}{2q_m}$$

and ~~it is~~ we have

32 $$V = 4\pi \varphi \frac{1-q_m}{2q_m} A^2 R(1 + R/A)$$

This gives for the ratio of the volumes of carbon and uranium

33 $$\frac{V}{\frac{4\pi}{3}R^3} = 3\varphi \frac{1-q_m}{2q_m} \frac{A^2}{R^2} (1 + R/A)$$

or using (14) and (27) → 33a

and for L the distance between neighboring uranium spheres in a hexagonal or cubic close-packed lattice we have

34 $$L^3 = \left(V + \frac{4\pi}{3}R^3\right)\sqrt{2}$$

33a $$\frac{4\pi}{3}R^3 / V = \frac{1-q_m}{6} \frac{R^2}{B^2} \cdot \frac{1}{(1 + R/B)}$$

or

$$(35) \quad L = 1.81R \sqrt[3]{1 + 3\varphi \frac{1-q_m}{2q_m} \frac{A^2}{R^2} \left(1 + R/A\right)}$$

For large values of ε we can write

$$\frac{1-q_m}{2q_m} \cong \frac{1}{\sqrt{\varepsilon}} = \frac{1}{\sqrt{\varphi \varepsilon_0}}$$

so that we would have for $\varepsilon \gg 1$

$$(36) \quad V \cong 4\pi ABR\sqrt{\varphi(1+R/A)(1+R/B)}$$

From No. 35 we find for R = 8 cm., $\sigma_c(C) = 0.0033$, A = 75.5, $q_m = 0.68$

$\varphi = 0.65$,: [redacted].

$L = 51$ cm