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by L. S.

In memory of Leo Szilard, who passed away on May 30, 1964, we present an English translation of his classical paper *Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen*, which appeared in the *Zeitschrift für Physik*, 1929, 53, 840–856. The publication in this journal of this translation was approved by Dr. Szilard before he died, but he never saw the copy. At Mrs. Szilard's request, Dr. Carl Eckart revised the translation.

This is one of the earliest, if not the earliest paper, in which the relations of physical entropy to information (in the sense of modern mathematical theory of communication) were rigorously demonstrated and in which Maxwell's famous demon was successfully exorcised: a milestone in the integration of physical and cognitive concepts.

ON THE DECREASE OF ENTROPY IN A THERMODYNAMIC SYSTEM BY THE INTERVENTION OF INTELLIGENT BEINGS

by Leo Szilard

Translated by Anatol Rapoport and Mechthilde Knoller from the original article "Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen." Zeitschrift für Physik, 1929, 53, 840–856.

The objective of the investigation is to find the conditions which apparently allow the construction of a perpetual-motion machine of the second kind, if one permits an intelligent being to intervene in a thermodynamic system. When such beings make measurements, they make the system behave in a manner distinctly different from the way a mechanical system behaves when left to itself. We show that it is a sort of a memory faculty, manifested by a system where measurements occur, that might cause a permanent decrease of entropy and thus a violation of the Second Law of Thermodynamics, were it not for the fact that the measurements themselves are necessarily accompanied by a production of entropy. At first we calculate this production of entropy quite generally from the postulate that full compensation is made in the sense of the Second Law (Equation [1]). Second, by using an inanimate device able to make measurements—however under continual entropy production—we shall calculate the resulting quantity of entropy. We find that it is exactly as great as is necessary for full compensation. The actual production of entropy in connection with the measurement, therefore, need not be greater than Equation (1) requires.

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THERE is an objection, already historical, against the universal validity of the Second Law of Thermodynamics, which indeed looks rather ominous. The objection is embodied in the notion of Maxwell's demon, who in a different form appears even nowadays again and again; perhaps not unreasonably, inasmuch as behind the precisely formulated question quantitative connections seem to be hidden which to date have not been clarified. The objection in its original formulation concerns a demon who catches the fast molecules and lets the slow ones pass. To be sure, the objection can be met with the reply that man cannot in principle foresee the value of a thermally fluctuating parameter. However, one cannot deny that we can very well measure the value of such a fluctuating parameter and therefore could certainly gain energy at the expense of heat by arranging our intervention according to the results of the measurements. Presently, of course, we do not know whether we commit an error by not including the intervening man into the system and by disregarding his biological phenomena.

Apart from this unresolved matter, it is known today that in a system left to itself no "perpetuum mobile" (perpetual motion machine) of the second kind (more exactly, no "automatic machine of continual finite work-yield which uses heat at the lowest temperature") can operate in spite of the fluctuation phenomena. A perpetuum mobile would have to be a machine which in the long run could lift a weight at the expense of the heat content of a reservoir. In other words, if we want to use the fluctuation phenomena in order to gain energy at the expense of heat, we are in the same position as playing a game of chance, in which we may win certain amounts now and then, although the expectation value of the winnings is zero or negative. The same applies to a system where the intervention from outside is performed strictly periodically, say by periodically moving machines. We consider this as established (Szilard, 1925) and intend here only to consider the difficulties that occur when intelligent beings intervene in a system. We shall try to discover the quantitative relations having to do with this intervention.

Smoluchowski (1914, p. 89) writes: "As far as we know today, there is no automatic, permanently effective perpetual motion machine, in spite of the molecular fluctuations, but such a device might, perhaps, function regularly if it were appropriately operated by intelligent beings. . . ."

A perpetual motion machine therefore is possible if—according to the general method of physics—we view the experimenting man as a sort of *deus ex machina*, one who is continuously and exactly informed of the existing state of nature and who is able to start or interrupt the macroscopic course of nature at any moment without expenditure of work. Therefore he would definitely not have to possess the ability to catch single molecules like Maxwell's demon, although he would definitely be different from real living beings in possessing the above abilities. In eliciting any physical effect by action of the sensory as well as the motor nervous systems a degradation of energy is always involved, quite apart from the fact that the very existence of a nervous system is dependent on continual dissipation of energy.

Whether—considering these circumstances—real living beings could continually or at least regularly produce energy at the expense of heat of the lowest temperature appears very doubtful, even though our ignorance of the biological phenomena does not allow a definite answer. However, the latter questions lead beyond the scope of physics in the strict sense.

It appears that the ignorance of the biological phenomena need not prevent us from understanding that which seems to us to be the essential thing. We may be sure that intelligent living beings—insofar as we are dealing with their intervention in a thermodynamic system—can be replaced by nonliving devices whose "biological phenomena" one could follow and determine whether in fact a compensation of the entropy decrease takes place as a result of the intervention by such a device in a system.

In the first place, we wish to learn what circumstance conditions the decrease of entropy which takes place when intelligent living beings intervene in a thermodynamic system. We shall see that this depends on a certain type of coupling between different parameters of the system. We shall consider an unusually simple type of these ominous couplings.[1] For brevity we shall talk about a "measurement," if we succeed in coupling the value of a parameter y (for instance the position co-ordinate of a pointer of a measuring instrument) at one moment with the simultaneous value of a fluctuating parameter x of the system, in such a way that, from the value y, we can draw conclusions about the value that x had at the moment of the "measurement." Then let x and y be uncoupled after the measurement, so that x can change, while y retains its value for some time. Such measurements are not harmless interventions. A system in which such measurements occur shows a sort of memory

[1] The author evidently uses the word "ominous" in the sense that the possibility of realizing the proposed arrangement threatens the validity of the Second Law.—*Translator*

faculty, in the sense that one can recognize by the state parameter y what value another state parameter x had at an earlier moment, and we shall see that simply because of such a memory the Second Law would be violated, if the measurement could take place without compensation. We shall realize that the Second Law is not threatened as much by this entropy decrease as one would think, as soon as we see that the entropy decrease resulting from the intervention would be compensated completely in any event if the execution of such a measurement were, for instance, always accompanied by production of $k \log 2$ units of entropy. In that case it will be possible to find a more general entropy law, which applies universally to all measurements. Finally we shall consider a very simple (of course, not living) device, that is able to make measurements continually and whose "biological phenomena" we can easily follow. By direct calculation, one finds in fact a continual entropy production of the magnitude required by the above-mentioned more general entropy law derived from the validity of the Second Law.

The first example, which we are going to consider more closely as a typical one, is the following. A standing hollow cylinder, closed at both ends, can be separated into two possibly unequal sections of volumes V_1 and V_2 respectively by inserting a partition from the side at an arbitrarily fixed height. This partition forms a piston that can be moved up and down in the cylinder. An infinitely large heat reservoir of a given temperature T insures that any gas present in the cylinder undergoes isothermal expansion as the piston moves. This gas shall consist of a single molecule which, as long as the piston is not inserted into the cylinder, tumbles about in the whole cylinder by virtue of its thermal motion.

Imagine, specifically, a man who at a given time inserts the piston into the cylinder and somehow notes whether the molecule is caught in the upper or lower part of the cylinder, that is, in volume V_1 or V_2. If he should find that the former is the case, then he would move the piston slowly downward until it reaches the bottom of the cylinder. During this slow movement of the piston the molecule stays, of course, above the piston. However, it is no longer constrained to the upper part of the cylinder but bounces many times against the piston which is already moving in the lower part of the cylinder. In this way the molecule does a certain amount of work on the piston. This is the work that corresponds to the isothermal expansion of an ideal gas—consisting of one single molecule—from volume V_1 to the volume $V_1 + V_2$. After some time, when the piston has reached the bottom of the container, the molecule has again the full volume $V_1 + V_2$ to move about in, and the piston is then removed. The procedure can be repeated as many times as desired. The man moves the piston up or down depending on whether the molecule is trapped in the upper or lower half of the piston. In more detail, this motion may be caused by a weight, that is to be raised, through a mechanism that transmits the force from the piston to the weight, in such a way that the latter is always displaced upwards. In this way the potential energy of the weight certainly increases constantly. (The transmission of force to the weight is best arranged so that the force exerted by the weight on the piston at any position of the latter equals the average pressure of the gas.) It is clear that in this manner energy is constantly gained at the expense of heat, insofar as the biological phenomena of the intervening man are ignored in the calculation.

In order to understand the essence of the man's effect on the system, one best imagines that the movement of the piston is performed mechanically and that the man's activity consists only in determining the altitude of the molecule and in pushing a lever (which steers the piston) to the right or left, depending on whether the molecule's height requires a down- or upward movement. This means that the intervention of the human being consists only in the coupling of two position co-ordinates, namely a co-ordinate x, which determines the altitude of the molecule, with another co-ordinate y, which determines the position of the lever and therefore also whether an upward or downward motion is imparted to the piston. It is best to imagine the mass of the piston as large and its speed sufficiently great, so that the thermal agitation of the piston at the temperature in question can be neglected.

In the typical example presented here, we wish to distinguish two periods, namely:

1. The period of *measurement* when the piston has just been inserted in the middle of the cylinder and the molecule is trapped either in the upper or lower part; so that if we choose the origin of co-ordinates appropriately, the x-co-ordinate of the molecule is restricted to either the interval $x > 0$ or $x < 0$;

2. The period of *utilization of the measurement*, "the period of decrease of entropy," during which the piston is moving up or down. During this period the x-co-ordinate of the molecule is certainly not restricted to the original interval $x > 0$ or $x < 0$. Rather, if the molecule was in the upper half of the cylinder during the period of measurement, i.e., when $x > 0$, the molecule must bounce on the downward-moving piston in the lower part of the cylinder, if it is to transmit energy to the piston; that is, the co-ordinate x has to enter the interval $x < 0$. The lever, on the contrary, retains during the whole period its position toward the right, corresponding to downward motion. If the position of the lever toward the right is designated by $y = 1$ (and correspondingly the position toward the left by $y = -1$) we see that during the period of measurement, the position $x > 0$ corresponds to $y = 1$; but afterwards $y = 1$ stays on, even though x passes into the other interval $x < 0$. We see that in the utilization of the measurement the coupling of the two parameters x and y disappears.

We shall say, quite generally, that a parameter y "measures" a parameter x (which varies according to a probability law), if the value of y is directed by the value of parameter x at a given moment. A measurement procedure underlies the entropy decrease effected by the intervention of intelligent beings.

One may reasonably assume that a measurement procedure is fundamentally associated with a certain definite average entropy production, and that this restores concordance with the Second Law. The amount of entropy generated by the measurement may, of course, always be greater than this fundamental amount, but not smaller. To put it precisely: we have to distinguish here between two entropy values. One of them, $\bar{S}_1$, is produced when during the measurement y assumes the value 1, and the other, $\bar{S}_2$, when y assumes the value -1. We cannot expect to get general information about $\bar{S}_1$ or $\bar{S}_2$ separately, but we shall see that *if* the amount of entropy produced by the "measurement" is to compensate the entropy decrease affected by utilization, the relation must always hold good.

$$e^{-\bar{S}_1/k} + e^{-\bar{S}_2/k} \leqq 1 \tag{1}$$

One sees from this formula that one can make one of the values, for instance $\bar{S}_1$, as small as one wishes, but then the other value $\bar{S}_2$ becomes correspondingly greater. Furthermore, one can notice that the magnitude of the interval under consideration is of no consequence. One can also easily understand that it cannot be otherwise.

Conversely, as long as the entropies $\bar{S}_1$ and $\bar{S}_2$, produced by the measurements, satisfy the inequality (1), we can be sure that the expected decrease of entropy caused by the later utilization of the measurement will be fully compensated.

Before we proceed with the proof of inequality (1), let us see in the light of the above mechanical example, how all this fits together. For the entropies $\bar{S}_1$ and $\bar{S}_2$ produced by the measurements, we make the following Ansatz:

$$\bar{S}_1 = \bar{S}_2 = k \log 2 \tag{2}$$

This ansatz satisfies inequality (1) and the mean value of the quantity of entropy produced by a measurement is (of course in this special case independent of the frequencies w_1, w_2 of the two events):

$$\bar{S} = k \log 2 \tag{3}$$

In this example one achieves a decrease of entropy by the isothermal expansion:[2]

$$\begin{aligned} -\bar{s}_1 &= -k \log \frac{V_1}{V_1 + V_2}; \\ -\bar{s}_2 &= -k \log \frac{V_2}{V_1 + V_2}, \end{aligned} \tag{4}$$

[2] The entropy generated is denoted by $\bar{s}_1$, $\bar{s}_2$.

depending on whether the molecule was found in volume V_1 or V_2 when the piston was inserted. (The decrease of entropy equals the ratio of the quantity of heat taken from the heat reservoir during the isothermal expansion, to the temperature of the heat reservoir in question). Since in the above case the frequencies w_1, w_2 are in the ratio of the volumes V_1, V_2, the mean value of the entropy generated is (a negative number):

$$\bar{s} = w_1 \cdot (+\bar{s}_1) + w_2 \cdot (+\bar{s}_2) = \frac{V_1}{V_1 + V_2} k \log \frac{V_1}{V_1 + V_2} + \frac{V_2}{V_1 + V_2} k \log \frac{V_1}{V_1 + V_2} \tag{5}$$

As one can see, we have, indeed

$$\frac{V_1}{V_1 + V_2} k \log \frac{V_1}{V_1 + V_2} + \frac{V_2}{V_1 + V_2} \cdot k \log \frac{V_2}{V_1 + V_2} + k \log 2 \geqq 0 \tag{6}$$

and therefore:

$$\bar{S} + \bar{s} \geqq 0. \tag{7}$$

In the special case considered, we would actually have a full compensation for the decrease of entropy achieved by the utilization of the measurement.

We shall not examine more special cases, but instead try to clarify the matter by a general argument, and to derive formula (1). We shall therefore imagine the whole system—in which the co-ordinate x, exposed to some kind of thermal fluctuations, can be measured by the parameter y in the way just explained—as a multitude of particles, all enclosed in one box. Every one of these particles can move freely, so that they may be considered as the molecules of an ideal gas, which, because of thermal agitation, wander about in the common box independently of each other and exert a certain pressure on the walls of the box—the pressure being determined by the temperature. We shall now consider two of these molecules as chemically different and, in principle, separable by semipermeable walls, if the co-ordinate x for one molecule is in a preassigned interval while the corresponding co-ordinate of the other molecule falls outside that interval. We also shall look upon them as chemically different, if they differ only in that the y co-ordinate is $+1$ for one and -1 for the other.

We should like to give the box in which the "molecules" are stored the form of a hollow cylinder containing four pistons. Pistons A and A' are fixed while the other two are movable, so that the distance BB' always equals the distance AA', as is indicated in Figure 1 by the two brackets. A', the bottom, and B, the cover of the container, are impermeable for all "molecules," while A and B' are semipermeable; namely, A is permeable only for those "molecules" for which the parameter x is in the preassigned interval, i.e., (x_1, x_2), B' is only permeable for the rest.

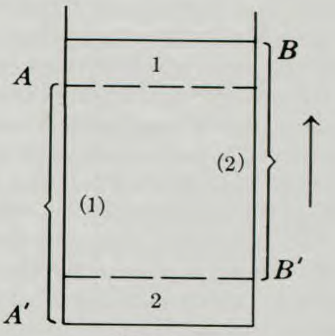

FIG. 1

In the beginning the piston B is at A and therefore B' at A', and all "molecules" are in the space between. A certain fraction of the molecules have their co-ordinate x in the preassigned interval. We shall designate by w_1 the probability that this is the case for a randomly selected molecule and by w_2 the probability that x is outside the interval. Then $w_1 + w_2 = 1$.

Let the distribution of the parameter y be over the values $+1$ and -1 in any proportion but in any event independent of the x-values. We imagine an intervention by an intelligent being, who imparts to y the value 1 for all "molecules" whose x at that moment is in the selected interval. Otherwise the value -1 is assigned. If then, because of thermal fluctuation, for any "molecule," the parameter x should come out of the preassigned interval or, as we also may put it, if the "molecule" suffers a monomolecular chemical reaction with regard to x (by which it is transformed from a species that can pass the semipermeable piston A into a species for which the piston is impermeable), then the parameter y retains its value 1 for the time being, so that the "molecule," because of the value of the parameter y, "remembers" during the whole following process that x originally was in the preassigned interval. We shall see immediately what part this memory may play. After the intervention just discussed, we move the piston, so that we separate the two kinds of molecules without doing work. This results in two containers, of which the first contains only the one modification and the second only the other. Each modification now occupies the same volume as the mixture did previously. In one of these containers, if considered by itself, there is now no equilibrium with regard to the two "modifications in x." Of course the ratio of the two modifications has remained $w_1:w_2$. If we allow this equilibrium to be achieved in both containers independently and at constant volume and temperature, then the entropy of the system certainly has increased. For the total heat release is 0, since the ratio of the two "modifications in x" $w_1:w_2$ does not change. If we accomplish the equilibrium distribution in both containers in a reversible fashion then the entropy of the rest of the world will decrease by the same amount. Therefore the entropy increases by a negative value, and, the value of the entropy increase per molecule is exactly:

$$\bar{s} = k(w_1 \log w_1 + w_2 \log w_2). \qquad (9)$$

(The entropy constants that we must assign to the two "modifications in x" do not occur here explicitly, as the process leaves the total number of molecules belonging to the one or the other species unchanged.)

Now of course we cannot bring the two gases back to the original volume without expenditure of work by simply moving the piston back, as there are now in the container—which is bounded by the pistons BB'—also molecules whose x-co-ordinate lies outside of the preassigned interval and for which the piston A is not permeable any longer. Thus one can see that the calculated decrease of entropy (Equation [9]) does not mean a contradiction of the Second Law. *As long as we do not use the fact that the molecules in the container BB', by virtue of their co-ordinate y, "remember" that the x-co-ordinate for the molecules of this container originally was in the preassigned interval, full compensation exists for the calculated decrease of entropy,* by virtue of the fact that the partial pressures in the two containers are smaller than in the original mixture.

But now we can use the fact that all molecules in the container BB' have the y-co-ordinate 1, and in the other accordingly -1, to bring all molecules back again to the original volume. To accomplish this we only need to replace the semipermeable wall A by a wall A^*, which is semipermeable not with regard to x but with regard to y, namely so that it is permeable for the molecules with the y-co-ordinate 1 and impermeable for the others. Correspondingly we replace B' by a piston B'^*, which is impermeable for the molecules with $y = -1$ and permeable for the others. Then both containers can be put into each other again without expenditure of energy. The distribution of the y-co-ordinate with regard to 1 and -1 now has become statistically independent of the x-values and besides we are able to re-establish the original distribution over 1 and -1. Thus we would have gone through a complete cycle. The only change that we have to register is the resulting decrease of entropy given by (9):

$$\bar{s} = k(w_1 \log w_1 + w_2 \log w_2). \qquad (10)$$

If we do not wish to admit that the Second Law has been violated, we must conclude *that the intervention which establishes the coupling between y and x, the measurement of x by y, must be accompanied by a production of entropy.* If a definite way of achieving this coupling is adopted and if the quantity of entropy that is inevitably produced is designated by S_1 and S_2, where S_1 stands for the mean increase in entropy that occurs when y acquires the value 1, and accordingly S_2 for the increase that occurs when y acquires the value -1, we arrive at the equation:

$$w_1 S_1 + w_2 S_2 = \bar{S} \qquad (11)$$

In order for the Second Law to remain in force, this quantity of entropy must be greater than the decrease of entropy $\bar{s}$, which according to (9) is produced by the utilization of the measurement. Therefore the following unequality must be valid:

$$\bar{S} + \bar{s} \geqq 0$$
$$w_1 S_1 + w_2 S_2 \qquad (12)$$
$$+ k(w_1 \log w_1 + w_2 \log w_2) \geqq 0$$

This equation must be valid for any values of w_1 and w_2,[3] and of course the constraint $w_2 + w_2 = 1$ cannot be violated. We ask, in particular, for which w_1 and w_2 and given S-values the expression becomes a minimum. For the two minimizing values w_1 and w_2 the inequality (12) must still be valid. Under the above constraint, the minimum occurs when the following equation holds:

$$\frac{S_1}{k} + \log w_1 = \frac{S_2}{k} + \log w_2 \qquad (13)$$

But then:

$$e^{-S_1/k} + e^{-S_2/k} \leqq 1. \qquad (14)$$

This is easily seen if one introduces the notation

$$\frac{S_1}{k} + \log w_1 = \frac{S_2}{k} + \log w_2 = \lambda; \quad (15)$$

then:

$$w_1 = e^{\lambda} \cdot e^{-S_1/k}; \quad w_2 = e^{\lambda} \cdot e^{-S_2/k}. \quad (16)$$

If one substitutes these values into the inequality (12) one gets:

$$\lambda e^{\lambda}(e^{-S_1/k} + e^{-S_2/k}) \geqq 0. \qquad (17)$$

Therefore the following also holds:

$$\lambda \geqq 0. \qquad (18)$$

If one puts the values w_1 and w_2 from (16) into the equation $w_1 + w_2 = 1$, one gets

$$e^{-S_1/k} + e^{-S_2/k} = e^{-\lambda}. \qquad (19)$$

And because $\lambda \geqq 0$, the following holds:

$$e^{-S_1/k} + e^{-S_2/k} \leqq 1. \qquad (20)$$

This equation must be universally valid, if thermodynamics is not to be violated.

As long as we allow intelligent beings to perform the intervention, a direct test is not possible. But we can try to describe simple nonliving devices that effect such coupling, and see if indeed entropy is generated and in what quantity. Having already recognized that the only important factor is a certain characteristic type of coupling, a "measurement," we need not construct any complicated models which imitate the intervention of living beings in detail. We can be satisfied with the construction of this particular type of coupling which is accompanied by memory.

In our next example, the position co-ordinate of an oscillating pointer is "measured" by the energy content of a body K. The pointer is supposed to connect, in a purely mechanical way, the body K—by whose energy content the position of the pointer is to be measured—by heat conduction with one of two intermediate pieces, A or B. The body is connected with A as long as the co-ordinate—which determines the position of the pointer—falls into a certain preassigned, but otherwise arbitrarily large or small interval a, and otherwise if the co-ordinate is in the interval b, with B. Up to a certain moment, namely the moment of the "measurement," both intermediate pieces will be thermally connected with a heat reservoir at temperature T_0. At this moment the insertion A will be cooled reversibly to the temperature T_A, e.g., by a periodically functioning mechanical device. That is, after successive contacts with heat reservoirs of intermediate temperatures, A will be brought into contact with a heat reservoir of the temperature T_A. At the same time the insertion B will be heated in the same way to temperature T_B. Then the intermediate pieces will again be isolated from the corresponding heat reservoirs.

We assume that the position of the pointer changes so slowly that all the operations that we have sketched take place while the position of the pointer remains unchanged. If the position co-ordinate of the pointer fell in the preassigned interval, then the body was connected with the insertion A during the above-mentioned operation, and consequently is now cooled to temperature T_A.

In the opposite case, the body is now heated to temperature T_B. Its energy content becomes—according to the position of

[3] The increase in entropy can depend only on the types of measurement and their results but not on how many systems of one or the other type were present.

the pointer at the time of "measurement"—small at temperature T_A or great at temperature T_B and will retain its value, even if the pointer eventually leaves the preassigned interval or enters into it. After some time, while the pointer is still oscillating, one can no longer draw any definite conclusion from the energy content of the body K with regard to the momentary position of the pointer but one can draw a definite conclusion with regard to the position of the pointer at the time of the measurement. Then the measurement is completed.

After the measurement has been accomplished, the above-mentioned periodically functioning mechanical device should connect the thermally isolated insertions A and B with the heat reservoir T_0. This has the purpose of bringing the body K—which is now also connected with one of the two intermediate pieces—back into its original state. The direct connection of the intermediate pieces and hence of the body K—which has been either cooled to T_A or heated to T_B—to the reservoir T_0 consequently causes an increase of entropy. This cannot possibly be avoided, because it would make no sense to heat the insertion A reversibly to the temperature T_0 by successive contacts with the reservoirs of intermediate temperatures and to cool B in the same manner. After the measurement we do not know with which of the two insertions the body K is in contact at that moment; nor do we know whether it had been in connection with T_A or T_B in the end. Therefore neither do we know whether we should use intermediate temperatures between T_A and T_0 or between T_0 and T_B.

The mean value of the quantity of entropy S_1 and S_2, per measurement, can be calculated, if the heat capacity as a function of the temperature $\bar{u}(T)$ is known for the body K, since the entropy can be calculated from the heat capacity. We have, of course, neglected the heat capacities of the intermediate pieces. If the position co-ordinate of the pointer was in the preassigned interval at the time of the "measurement," and accordingly the body in connection with insertion A, then the entropy conveyed to the heat reservoirs during successive cooling was

$$\int_{T_A}^{T_0} \frac{1}{T} \frac{d\bar{u}}{dT}. \tag{21}$$

However, following this, the entropy withdrawn from the reservoir T_0 by direct contact with it was

$$\frac{\bar{u}(T_0) - \bar{u}(T_A)}{T_0}. \tag{22}$$

All in all the entropy was increased by the amount

$$S_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \int_{T_A}^{T_0} \frac{1}{T} \frac{d\bar{u}}{dT} dT. \tag{23}$$

Analogously, the entropy will increase by the following amount, if the body was in contact with the intermediate piece B at the time of the "measurement":

$$S_B = \frac{\bar{u}(T_B) - \bar{u}(T_0)}{T_0} + \int_{T_B}^{T_0} \frac{1}{T} \frac{d\bar{u}}{dT} dT. \tag{24}$$

We shall now evaluate these expressions for the very simple case, where the body which we use has only two energy states, a lower and a higher state. If such a body is in thermal contact with a heat reservoir at any temperature T, the probability that it is in the lower or upper state is given by respectively:

$$\left.\begin{aligned} p(T) &= \frac{1}{1 + ge^{-u/kT}} \\ q(T) &= \frac{ge^{-u/kT}}{1 + ge^{-u/kT}} \end{aligned}\right\} \tag{25}$$

Here u stands for the difference of energy of the two states and g for the statistical weight. We can set the energy of the lower state equal to zero without loss of generality. Therefore:[4]

$$\left.\begin{aligned} S_A &= q(T_A)\, k \log \frac{q(T_A)\, p\,(T_0)}{q(T_0)\, p\,(T_A)} \\ &\qquad + k \log \frac{p(T_A)}{p(T_0)} \\ S_B &= p(T_B)\, k \log \frac{q(T_0)\, p\,(T_B)}{q(T_B)\, p\,(T_0)} \\ &\qquad + k \log \frac{q(T_B)}{q(T_0)} \end{aligned}\right\}. \tag{26}$$

Here q and p are the functions of T given

[4] See the Appendix.

by equation (25), which are here to be taken for the arguments T_0, T_A, or T_B.

If (as is necessitated by the above concept of a "measurement") we wish to draw a dependable conclusion from the energy content of the body K as to the position co-ordinate of the pointer, we have to see to it that the body surely gets into the lower energy state when it gets into contact with T_B. In other words:

$$p(T_A) = 1, q(T_A) = 0; \quad p(T_B) = 0, q(T_B) = 1. \tag{27}$$

This of course cannot be achieved, but may be arbitrarily approximated by allowing T_A to approach absolute zero and the statistical weight g to approach infinity. (In this limiting process, T_0 is also changed, in such a way that $p(T_0)$ and $q(T_0)$ remain constant.) The equation (26) then becomes:

$$S_A = -k \log p(T_0); \quad S_B = -k \log q(T_0) \tag{28}$$

and if we form the expression $e^{-S_A/k} + e^{-S_B/k}$, we find:

$$e^{-S_A/k} + e^{-S_B/k} = 1. \tag{29}$$

Our foregoing considerations have thus just realized the smallest permissible limiting care. The use of semipermeable walls according to Figure 1 allows a complete utilization of the measurement: inequality (1) certainly cannot be sharpened.

As we have seen in this example, a simple inanimate device can achieve the same essential result as would be achieved by the intervention of intelligent beings. We have examined the "biological phenomena" of a nonliving device and have seen that it generates exactly that quantity of entropy which is required by thermodynamics.

APPENDIX

In the case considered, when the frequency of the two states depends on the temperature according to the equations:

$$p(T) = \frac{1}{1 + ge^{-u/kT}}; q(T) = \frac{ge^{-u/kT}}{1 + ge^{-u/kT}} \tag{30}$$

and the mean energy of the body is given by:

$$\bar{u}(T) = uq(T) = \frac{uge^{-u/kT}}{1 + ge^{-u/kT}}, \tag{31}$$

the following identity is valid:

$$\frac{1}{T}\frac{d\bar{u}}{dT} = \frac{d}{dT}\left\{\frac{\bar{u}(T)}{T} + k \log\left(1 + e^{-u/kT}\right)\right\}. \tag{32}$$

Therefore we can also write the equation:

$$B_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \int_{T_A}^{T_0} \frac{1}{T}\frac{d\bar{u}}{dT} dT \tag{33}$$

as

$$S_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \left\{\frac{\bar{u}(T)}{T} + k \log(1 + ge^{-u/kT})\right\}_{T_A}^{T_0}, \tag{34}$$

and by substituting the limits we obtain:

$$S_A = \bar{u}(T_A)\left(\frac{1}{T_0} - \frac{1}{T_A}\right) + k \log \frac{1 + ge^{-u/kT_0}}{1 + ge^{-u/kT_A}}. \tag{35}$$

If we write the latter equation according to (25):

$$1 + ge^{-u/kT} = \frac{1}{p(T)} \tag{36}$$

for T_A and T_0, then we obtain:

$$S_A = \bar{u}(T_A)\left(\frac{1}{T_0} - \frac{1}{T_A}\right) + k \log \frac{p(T_A)}{p(T_0)} \tag{37}$$

and if we then write according to (31):

$$\bar{u}(T_A) = uq(T_A) \tag{38}$$

we obtain:

$$S_A = q(T_A)\left(\frac{u}{T_0} - \frac{u}{T_A}\right) + k \log \frac{p(T_A)}{p(T_0)}. \tag{39}$$

If we finally write according to (25):

$$\frac{u}{T} = -k \log \frac{q(T)}{gp(T)} \tag{40}$$

for T_A and T_0, then we obtain:

$$S_A = q(T_A)\, k \log \frac{p(T_0)}{q(T_0)} \frac{q(T_A)}{p(T_A)} + k \log \frac{p(T_A)}{p(T_0)}. \tag{41}$$

We obtain the corresponding equation for S_B, if we replace the index A with B. Then we obtain:

$$S_B = q(T_B)\,k\log\frac{p(T_0)}{q(T_0)}\frac{q((T_B)}{p((T_B)} + k\log\frac{p(T_B)}{p(T_0)}. \qquad (42)$$

Formula (41) is identical with (26), given, for S_A, in the text.

We can bring the formula for S_B into a somewhat different form, if we write:

$$q(T_B) = 1 - p(T_B), \qquad (43)$$

expand and collect terms, then we get

$$S_B = p(T_B)\,k\log\frac{q(T_0)}{p(T_0)}\frac{p(T_B)}{q(T_B)} + k\log\frac{q(T_B)}{q(T_0)}. \qquad (44)$$

This is the formula given in the text for S_B.

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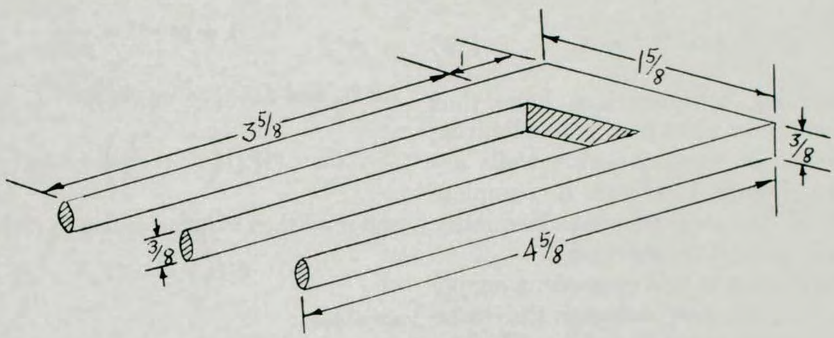

American Foreign Policy

ON THE DECREASE OF ENTROPY IN A THERMODYNAMIC SYSTEM

BY THE INTERVENTION OF INTELLIGENT BEINGS

by Leo Szilard

Translated by Anatol Rapoport and Mechthilde Knoller from the original article "Uber die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen." Zeitschrift fur Physik, 1929, 53, 840-856.

1. The author evidently uses the word "ominous" in the sense that the possibility of realizing the proposed arrangement threatens the validity of the Second Law.--Translator

2. The entropy generated is denoted by $\overline{s}_1$, $\overline{s}_2$.

3. The increase in entropy can depend only on the types of measurement and their results but not on how many systems of one or the other type were present.

4. See the Appendix.

Abstract

The objective of the investigation is to find the conditions which apparently allow the construction of a perpetual-motion machine of the second kind, if one permits an intelligent being to intervene in a thermodynamic system. When such beings make measurements, they make the system behave in a manner distinctly different from the way a mechanical system behaves when left to itself. We show that it is a sort of a memory faculty, manifested by a system where measurements occur, that

might cause a permanent decrease of entropy and thus a violation of the Second Law of Thermodynamics, were it not for the fact that the measurements themselves are necessarily accompanied by a production of entropy. At first we calculate this production of entropy quite generally from the postulate that full compensation is made, in the sense of the Second Law (Equation (1)). Second, by using an inaminate device able to make measurements--however under continual entropy production--we shall calculate the resulting quantity of entropy. We find that it is exactly as great as is necessary for full compensation. The actual production of entropy in connection with the measurement, therefore, need not be greater than Equation (1) requires.

There is an objection, already historical, against the universal validity of the Second Law of Thermodynamics, which indeed looks rather ominous. The objection is embodied in the notion of Maxwell's demon, who in a different form appears, even nowadays again and again; perhaps not unreasonably, inasmuch as behind the precisely formulated question, quantitative connections seem to be hidden, which to date have not been clarified. The objection in its original formulation concerns a demon who catches the fast molecules and lets the slow ones pass. To be sure, the objection can be met with the reply that man cannot, in principle, foresee the value of a thermally fluctuating parameter. However, one cannot deny that we can very well measure the value of such a fluctuating parameter and therefore could certainly gain energy at the expense of heat by arranging our intervention according to the result of the measurement. Presently, of course, we do not know whether we commit an error by not including the intervening man in the system, and by disregarding his biological phenomena.

Apart from this unresolved matter, it is known today that in a system left to itself no "perpetuum mobile" (perpetual motion machine) of the second kind (more exactly, no "automatic machine of continual finite work-yield which uses heat at the lowest temperature") can operate, fluctuation phenomena not withstanding. A perpetuum mobile would have to be a machine which in the long run could lift a weight at the expense of the heat content of a reservoir. In other words, if we want to use the fluctuation phenomena in order to gain energy at the expense of heat, we are in the same position as when playing a game of chance, in which we may win certain amounts now and then, although the expectation value of the winnings is zero or negative. The same applies to a system where the intervention from outside is performed strictly periodically, say, by periodically moving machines. We consider this as established (Szilard, 1925) and intend here only to consider the difficulties that occur when intelligent beings intervene in a system. We shall try to discover the quantitative relations having to do with this intervention.

Smoluchowski (1914, p. 89) writes: "As far as we know today, there is no automatic, continually effective perpetual motion machine, in spite of the molecular fluctuations, but such a device might, perhaps, function regularly if it were appropriately operated by intelligent beings...."

A perpetual motion machine therefore is possible if--according to the general method of physics--we view the experimenting man as a sort of deus ex machina, one who is continuously and exactly informed of the existing state of nature and who is able to start or interrupt the macroscopic course of nature at any moment without expenditure of work. Therefore he would definitely not have to possess the ability to catch single molecules like Maxwell's demon, although he would definitely be different from real living beings in possessing the above abilities. In eliciting any physical effect by action of the sensory as well as the motor nervous systems, a degradation of energy is always involved, quite apart from the fact that the very existence of a nervous system is dependent on continual dissipation of energy.

Whether--considering these circumstances--real living beings could continually or at least regularly produce energy at the expense of heat of the lowest temperature appears very doubtful, even though our ignorance of the biological phenomena does not allow a definite answer. However the latter questions lead beyond the scope of physics in the strict sense....

It appears that ~~the~~ ignorance of the biological phenomena need not prevent us from understanding that which seems to us to be the essential thing. We may be sure that intelligent living beings--in so far as we are dealing with their intervention in a thermodynamic system--can be replaced by nonliving devices whose "biological phenomena" one could follow and determine whether in fact, a compensation of the entropy decrease takes place, as a result of the intervention by such a device in a system.

In the first place, we wish to learn what circumstance conditions the decrease of entropy which takes place when intelligent living beings intervene in a thermodynamic system. We shall see that this depends on a certain type of coupling between different parameters of the system. We shall consider an unusually simple type of these ominous couplings[1]. For brevity, we shall talk about a "measurement," if we succeed in coupling the value of a parameter y (for instance the position co-ordinate of a pointer of a measuring instrument) at one moment with the simultaneous value of a fluctuating parameter x of the system, in such a way that, from the value y, we can draw conclusions about the value that x had at the moment of the "measurement." Then let x and y be uncoupled after the measurement, so that x can change, while y retains its value for some time. Such measurements are not harmless interventions. A system in which such measurements occur shows a sort of memory faculty, in the sense that one can recognize by the state parameter y what value another state parameter x had at an earlier moment, and we shall see that simply because of such a memory the Second Law would be violated, if the measurement could take place without compensation. We shall realize that the Second Law is not threatened as much by this entropy decrease as one would think, as soon as we see that the entropy decrease resulting from the intervention would be compensated completely, in any event, if the execution of such a measurement were, for instance, always accompanied by production of $k \log 2$ units of entropy. In that case it will be possible to find a more general entropy law, which applies universally to all measurements. Finally we shall consider a very simple (of course, not living) device, that is able to make measurements continually and whose "biological phenomena" we can easily follow. By direct calculation, one finds in fact a continual entropy production of the magnitude required by the above-mentioned more general entropy law, which is derived from the validity of the Second Law.

The first example, which we are going to consider more closely as a typical one, is the following. A standing hollow cylinder, closed at both ends, can be separated into two possibly unequal sections of volumes V_1 and V_2, respectively, by inserting a partition from the side at an arbitrarily fixed height. This partition forms a piston that can be moved up and down in the cylinder. An infinitely large heat reservoir of a given temperature T insures that any gas present in the cylinder undergoes isothermal expansion as the piston moves. This gas shall consist of a single molecule which, as long as the piston is not inserted into the cylinder, tumbles about in the whole cylinder by virtue of its thermal motion.

Imagine, specifically, a man who at a given time inserts the piston into the cylinder and somehow notes whether the molecule is caught in the upper or lower part of the cylinder, that is, in volume V_1 or V_2. If he should find that the former is the case, then he would move the piston slowly downward until it reaches the bottom of the cylinder.

During this slow movement of the piston the molecule stays, of course, above the piston. However, it is no longer constrained to the upper part of the cylinder but bounces many times against the piston which is already moving in the lower part of the cylinder. In this way the molecule does a certain amount of work on the piston. This is the work that corresponds to the isothermal expansion of an ideal gas--consisting of one single molecule--from volume V_1 to the volume $V_1 + V_2$. After some time, when the piston has reached the bottom of the container, the molecule has again the full volume $V_1 + V_2$ to move about in, and the piston is then removed. The procedure can be repeated as many times as desired. The man moves the piston up or down depending on whether the molecule is trapped in the upper or lower half of the cylinder. In more detail, this motion may be caused by a weight, that is to be raised, by means of a mechanism that transmits the force from the piston to the weight, in such a way that the latter is always displaced upwards. In this way the potential energy of the weight certainly increases constantly. (The transmission of force to the weight is best arranged so that the force exerted by the weight on the piston at any position of the latter equals the average pressure of the gas). It is clear that in this manner energy is constantly gained at the expense of heat, in so far as the biological phenomena of the intervening man are ignored in the calculation.

In order to understand the essence of the man's effect on the system, one best imagines that the movement of the piston is performed mechanically and that the man's activity consists only in determining the altitude of the molecule and in pushing a lever (which steers the piston) to the right or left, depending on whether the height requires a down- or upward movement. This means that the intervention of the human being consists only in the coupling of two position co-ordinates, namely a co-ordinate x , which determines the altitude of the molecule, with another co-ordinate y , which determines the position of the lever and therefore also whether an upward or downward motion is imparted to the piston. It is best to imagine the mass of the piston as large and its speed sufficiently great, so that the thermal agitation of the piston at the temperature in question can be neglected.

In the typical example presented here, we ~~wish to~~ can distinguish two periods, namely:

1. The period of <u>measurement</u> when the piston has just been inserted in the middle of the cylinder and the molecule is trapped either in the upper or lower part; so that if we choose the origin of co-ordinates appropriately, the x-co-ordinate of the molecule is restricted to either the interval $x > 0$ or $x < 0$;

2. The period of <u>utilization of the measurement</u>, "the period of decrease of entropy," during which the piston is moving up or down. During this period the x-co-ordinate of the molecule is certainly not restricted to the original interval $x > 0$ or $x < 0$. Rather, if the molecule was in the upper half of the cylinder during the period of measurement i.e., when x was > 0 , the molecule must bounce on the downward-moving piston in the lower part of the cylinder, if it is to transmit energy to the piston; that is, the co-ordinate x has to enter the interval $x < 0$. The lever, on the contrary, retains

during the whole period its position toward the right, corresponding to downward motion. If the position of the lever toward the right is designated by $y = 1$ (and correspondingly the position toward the left by $y = -1$) we see that during the period of measurement, the position $y = 1$ corresponds to $x > 0$; but afterwards $y = 1$ stays on, even though x passes into the other interval $x < 0$. We see that in the utilization of the measurement the coupling of the two parameters x and y disappears.

We shall say, quite generally, that a parameter y "measures" a parameter x (which varies according to a probability law), if the value of y is determined by the value of parameter x at a given moment. A measurement procedure underlies the entropy decrease effected by the intervention of intelligent beings.

One may reasonably assume that a measurement procedure is fundamentally associated with a certain definite average entropy production, and that this restores concordance with the Second Law. The amount of entropy generated by the measurement may, of course, always be greater than this fundamental amount, but not smaller. To put it precisely: we have to distinguish here between two entropy values. One of them, $\bar{S}_1$, is produced when during the measurement y assumes the value 1, and the other, $\bar{S}_2$, when y assumes the value -1. We cannot expect to get general information about $\bar{S}_1$ or $\bar{S}_2$ separately, but we shall see that <u>if</u> the amount of entropy produced by the "measurement" is to compensate the entropy decrease affected by utilization, then the relation

$$\exp(-\bar{S}_1/k) + \exp(-\bar{S}_2/k) \leq 1 \tag{1}$$

must always hold good.

One sees from this formula that one can make one of the values, for instance $\bar{S}_1$, as small as one wishes, but then the other value $\bar{S}_2$ becomes correspondingly greater. Furthermore one can notice that the magnitude of the interval under consideration is of no consequence. One can also easily understand that it cannot be otherwise.

Conversely, as long as the entropies $\bar{S}_1$ and $\bar{S}_2$, produced by the measurements, satisfy the inequality (1), we can be sure that the expected decrease of entropy caused by the later utilization of the measurement will be fully compensated.

Before we proceed with the proof of inequality (1), let us see in the light of the above mechanical example, how all this fits together. For the entropies $\bar{S}_1$ and $\bar{S}_2$ produced by the measurements, we make the following ansatz:

$$\bar{S}_1 = \bar{S}_2 = k \log 2 \tag{2}$$

This ansatz satisfies inequality (1) and the mean value of the quantity of entropy produced by a measurement is (of course in this special case independent of the frequencies w_1, w_2 of the two events):

$$\bar{S} = k \log 2. \tag{3}$$

In this example one achieves a decrease of entropy, by the isothermal expansion:[2]

$$
\begin{aligned}
-\bar{s}_1 &= -k \log [V_1/(V_1+V_2)], \\
-\bar{s}_2 &= -k \log [V_2/(V_1+V_2)],
\end{aligned}
\qquad (4)
$$

depending on whether the molecule was found in volume V_1 or V_2 when the piston was inserted. (The decrease of entropy equals the ratio of the quantity of heat taken from the heat reservoirs during the isothermal expansion, to the temperature of the heat reservoir in question). Since in the above case the frequencies w_1, w_2, of the two alternatives, are in the ratio of the volumes V_1, V_2, the mean value of the entropy generated is (a negative number):

$$
\begin{aligned}
\bar{s} &= w_1 \cdot (+\bar{s}_1) + w_2 \cdot (+\bar{s}_2) \\
&= [V_1/(V_1+V_2)]\, k \log [V_1/(V_1+V_2)] \\
&\quad + [V_2/(V_1+V_2)]\, k \log [V_2/(V_1+V_2)].
\end{aligned}
\qquad (5)
$$

As one can see, we have, indeed

$$
\begin{aligned}
&[V_1/(V_1+V_2)]\, k \log [V_1/(V_1+V_2)] \\
&+ [V_2/(V_1+V_2)]\, k \log [V_2/(V_1+V_2)] \\
&+ k \log 2 \geq 0,
\end{aligned}
\qquad (6)
$$

and therefore

$$\bar{S} + \bar{s} \geq 0. \tag{7}$$

In the special case considered, we would actually have a full compensation for the decrease of entropy achieved by the utilization of the measurement.

We shall not examine more special cases, but instead try to clarify the matter by a general argument, and to derive formula (1). We shall therefore imagine the whole system--in which the co-ordinate x, subject to some kind of thermal fluctuations, can be measured by the parameter y in the way just explained--as a multitude of particles, all enclosed in one box. Every one of these particles can move freely, so that they may be considered as the molecules of an ideal gas, which, because of thermal agitation, wander about in the common box independently of each other and exert a certain pressure on the walls of the box--the pressure being determined by the temperature. We shall now consider two of these molecules as chemically different and, in principle, separable by semipermeable walls, if the co-ordinate x for one molecule is in a preassigned interval while the corresponding co-ordinate of the other molecule falls outside that interval. We also shall look upon them as chemically different, if they differ only in that the y co-ordinate is +1 for one and -1 for the other.

We will suppose that the box in which the "molecules" are stored has the form of a hollow cylinder containing four pistons (see Fig. 1). Pistons A and A' are fixed while B and B' are movable, so that the distance BB'

always equals the distance AA', as is indicated in Figure 1 by the two brackets. A', the bottom, and B, the cover of the container, are impermeable for all "molecules," while A and B' are semipermeable; namely, A is permeable only for those "molecules" for which the parameter x is in the preassigned interval, i.e., $(x_1 \leq x \leq x_2)$, B is only permeable for the rest.

In the beginning the piston B is at A and therefore B' at A', and all "molecules" are in the space between. A certain fraction of the molecules have their co-ordinate x in the preassigned interval. We shall designate by w_1, the probability that this is the case for a randomly selected molecule and by w_2, the probability that x is outside the interval. Then $w_1 + w_2 = 1$. Let the distribution of the parameter y be over the values +1 and -1 in any proportion but in any event independent of the x values. We imagine an intervention by an intelligent being, who imparts to y the value 1 for all "molecules" whose x at that moment is in the selected interval. Otherwise the value -1 is assigned. If then, because of thermal fluctuation, for any "molecule," the parameter x should come out of the preassigned interval or, as we also may put it: if the "molecule" suffers a monomolecular chemical reaction with regard to x (by which it is transformed from a species that can pass the semipermeable piston A into a species for which the piston is impermeable) then the parameter y retains its value 1 for the time being, so that the "molecule", because of the value of the parameter y, "remembers" during the whole following process that x originally was in the preassigned interval. We shall see immediately what part this memory may play. After the intervention just discussed, we move the piston, so that we separate the two kinds of molecules without doing work. This results in two containers, of which the first contains only the one modification and the second only the other. Each modification now occupies the same volume as the mixture did previously. In one of these containers, ~~if~~ considered by itself, there is now no equilibrium with regard to the two "modifications in x." Of course the ratio of the two modifications has remained $w_1 : w_2$. If we allow this equilibrium to be achieved in both containers independently and at constant volume and temperature, then the entropy of the system certainly has increased. For the total heat release is 0, since the ratio of the two "modifications in x" does not change. If we accomplish the equilibrium distribution in both containers in a reversible fashion then the entropy of the rest of the world will decrease by the same amount. Therefore the entropy increases by a negative value, and the value of the entropy increase per molecule is exactly:

$$\bar{s} = k\left(w_1 \log w_1 + w_2 \log w_2\right). \tag{9}$$

(The entropy constants that we must assign to the two "modifications in x" do not occur here explicitly, as the process leaves the total number of molecules belonging to the one or the other species unchanged.)

Now of course we cannot bring the two gases back to the original volume without expenditure of work by simply moving the piston back, as there are now in the container--which is bounded by the pistons BB' -- also molecules whose x co-ordinate lies outside of the preassigned interval and for which the piston A is not permeable any longer. Thus one can see that the calculated decrease of entropy (Equation (9)) does not mean a contradiction of the Second Law. As long as we do not use the fact that the molecules in the container BB', by virtue of their co-ordinate y, "remember" that the x co-ordinate for the molecules of this container originally was in the preassigned interval, full compensation exists for the calculated decrease of entropy, by virtue of the fact that the partial pressures in the two containers are smaller than in the original mixture.

But now we can use the fact that all molecules in the container BB' have the y co-ordinate 1, and in the other accordingly -1, to bring all molecules back again to the original volume. To accomplish this we only need to replace the semipermeable wall A by a wall A^*, which is semipermeable not with regard to x but with regard to y, namely so that it is permeable for the molecules with the y co-ordinate 1 and impermeable for the others. Correspondingly we replace B' by a piston B'^*, which is impermeable for the molecules with $y = 1$ and permeable for the others. Then both containers can be put into each other again without expenditure of energy. The distribution of the y co-ordinate with regard to 1 and -1 now has become statistically independent of the x values and besides we are able to re-establish the original distribution over 1 and -1. Thus we would have gone through a

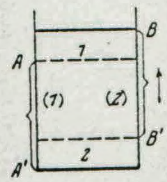

Fig. 1.

complete cycle. The only change that we have to register is the resulting decrease of entropy given by (9):

$$\bar{s} = k(w_1 \log w_1 + w_2 \log w_2) \tag{10}$$

If we do not wish to admit that the Second Law has been violated, we must conclude that the intervention which established the coupling between y and x, the measurement of x by y, must be accompanied by a production of entropy. If a definite way of achieving this coupling is adopted and if the quantity of entropy that is inevitably produced is designated by S_1 and S_2, where S_1 stands for the mean increase in entropy that occurs when y acquires the value 1, and correspondingly ~~accordingly~~ S_2 for the increase that occurs when y acquires the value -1, we arrive at the equation:

$$w_1 S_1 + w_2 S_2 = \bar{S}. \tag{11}$$

In order for the Second Law to remain in force, this quantity of entropy must be greater than the decrease of entropy $\bar{s}$, which according to (9) is produced by the utilization of the measurement. Therefore the following inequality must be valid:

$$\bar{S} + \bar{s} \geq 0,$$

$$w_1 S_1 + w_2 S_2 + k(w_1 \log w_1 + w_2 \log w_2) \geq 0. \tag{12}$$

This inequality must be valid[3] for any values of ϖ_1 and ϖ_2, ~~and~~ but of course the constraint $\varpi_1 + \varpi_2 = 1$ may ~~can~~not be violated. We ask, in particular, for which ϖ_1 and ϖ_2 and given S- values the expression becomes a minimum. For the two minimizing values ϖ_1 and ϖ_2 the inequality (12) must still be valid. Under the above constraint, the minimum occurs when the following equation holds:

$$(S_1/k) + \log \varpi_1 = (S_2/k) + \log \varpi_2 . \tag{13}$$

But then:

$$\exp(-S_1/k) + \exp(-S_2/k) \leq 1 . \tag{14}$$

This is easily seen if one introduces the notation

$$(S_1/k) + \log \varpi_1 = (S_2/k) + \log \varpi_2 = \lambda; \quad (15);$$

then:

$$\begin{aligned} \varpi_1 &= \exp(\lambda - S_1/k), \\ \varpi_2 &= \exp(\lambda - S_2/k). \end{aligned} \tag{16}$$

If one substitutes these values into the inequality (12) one gets:

$$\lambda \exp(\lambda)\left[\exp(-S_1/k) + \exp(-S_2/k)\right] \geq 0. \tag{17}$$

Therefore:

$$\lambda \geqslant 0. \tag{18}$$

If one puts the values w_1 and w_2 from (16) into the equation $w_1 + w_2 = 1$, one gets

$$\exp(-S_1/k) + \exp(-S_2/k) = \exp(-\lambda). \tag{19}$$

And because $\lambda \geqslant 0$:

$$\exp(-S_1/k) + \exp(-S_2/k) \leq 1. \tag{20}$$

This equation must be universally valid, if thermodynamics is not to be violated.

As long as we allow intelligent beings to perform the intervention, a direct test is not possible. But we can try to describe simple non-living devices that effect such coupling, and see if indeed entropy is generated and in what quantity. Having already recognized that the only important factor is a certain characteristic type of coupling, a "measurement," we need not construct any complicated models, which imitate the intervention of living beings in detail. We can be satisfied with the construction of this particular type of coupling which is accompanied by memory.

As an example of this, the position co-ordinate of an oscillating pointer is "measured" by the energy content of a body K. The pointer is supposed to connect, in a purely mechanical way, the body K --by whose energy content the position of the pointer is to be measured--by heat conduction with one of two intermediate pieces, A or B. The body is connected with A as long as the co-ordinate--which determines the position of the pointer--falls into a certain preassigned, but otherwise arbitrarily large or small interval a, and otherwise, when ~~if~~ the co-ordinate is in the interval b, with B. Up to a certain moment, namely the moment of the "measurement," both intermediate pieces will be thermally connected with a heat reservoir at temperature T_0. At this moment the intermediate piece A will be cooled reversibly to the temperature T_A e.g., by a periodically functioning mechanical device. That is, after successive contacts with heat reservoirs of intermediate temperatures, A will be brought into contact with a heat reservoir of the temperature T_A. At the same time the intermediate piece B will be heated in the same way to temperature T_B. Then the intermediate pieces will again be isolated from the corresponding heat reservoirs.

We assume that the position of the pointer changes so slowly that all the operations that we have sketched take place while the position of the pointer remains unchanged. If the position co-ordinate of the pointer fell in the preassigned interval, then the body was connected with the insertion A during the above-mentioned operation, and consequently is now cooled to temperature T_A. In the opposite case, the body is now heated to temperature T_B. Its energy content becomes--according to the position of the pointer at the time of "measurement"--small at temperature T_A or great at temperature T_B and will retain its value, even if the pointer eventually leaves the preassigned interval or enters into it. After some time, while the pointer is still oscillating, one can no longer draw any definite conclusion from the energy content of the body K with regard to the momentary position of the pointer but one can draw a definite conclusion with regard to the position of the pointer at the time of the measurement. Then the measurement is completed.

After the measurement has been accomplished, the above-mentioned periodically functioning mechanical device should connect the thermally isolated intermediate pieces A and B with the heat reservoir T_0. This has the purpose of bringing the body K --which is now also connected with one of the two intermediate pieces--back into its original state. The direct connection of the intermediate pieces and hence of the body K --which has been either cooled to T_A or heated to T_B -- to the reservoir T_0 consequently causes an increase of entropy. This cannot possibly be avoided, because it would make no sense to heat the intermediate piece~~s~~ A reversibly to the temperature T_0 by successive contacts with the reservoirs of intermediate temperatures and to cool B in the same manner. After the measurement we do not know with which of the two intermediate pieces the body K is in contact at that moment; nor do we know whether it had been in connection with T_A or T_B in the end. Therefore neither do we know whether we should use intermediate temperatures between T_A and T_0 or between T_0 and T_B .

The mean value of the quantity of entropy $\bar{S}_1$ and $\bar{S}_2$, per measurement, can be calculated, if the heat capacity , $\bar{u}(T)$, of K is known as a function of the temperature, ~~is known for the body~~ since the entropy can be calculated from the heat capacity. We have, of course, neglected the heat capacities of the intermediate pieces. If the position coordinate of the pointer was in the preassigned interval at the time of the "measurement," and accordingly the body in connection with piece A , then the entropy conveyed to the heat reservoirs during the successive cooling was

$$\int_{T_A}^{T_0} (1/T)(d\bar{u}/dT)\,dT. \tag{21}$$

However, following this, the entropy withdrawn from the reservoir T_0 by direct contact with it was

$$[\bar{u}(T_0) - \bar{u}(T_A)]/T_0 . \tag{22}$$

All in all the entropy was increased by the amount

$$\begin{aligned} S_A = {} & [\bar{u}(T_A) - \bar{u}(T_0)]/T_0 \\ & + \int_{T_A}^{T_0} (1/T)(d\bar{u}/dT)\,dT. \end{aligned} \tag{23}$$

Analogously, the entropy will increase by the following amount, if the body was in contact with the intermediate piece B at the time of the "measurement":

$$S_B = [\bar{u}(T_B) - \bar{u}(T_0)]/T_0$$
$$+ \int_{T_B}^{T_0} (1/T)(d\bar{u}/dT)\, dT. \tag{24}$$

We shall now evaluate these expressions for the very simple case, where the body which we use has only two energy states, a lower and a higher state. If such a body is in thermal contact with a heat reservoir at any temperature T, the probability that it is in the lower or upper state is given by

$$p(T) = 1/[1 + g\exp(-u/kT)],$$
or
$$q(T) = g\exp(-u/kT)/[1 + g\exp(-u/kT)]. \tag{25}$$

Here u stands for the difference of energy of the two states and g for the statistical weight. We can set the energy of the lower state equal to zero without loss of generality. Therefore:[4]

$$S_A = q(T_A)k \log \frac{q(T_A)\,p(T_0)}{q(T_0)\,p(T_A)}$$
$$+ k \log \frac{p(T_A)}{p(T_0)}, \tag{26}$$
and
$$S_B = p(T_B)k \log \frac{q(T_0)\,p(T_B)}{q(T_B)\,p(T_0)}$$
$$+ k \log \frac{q(T_B)}{q(T_0)}.$$

Here q and p are the functions of T given by equation (25), and ~~which~~ are ~~here~~ to be taken for the arguments T_0, T_A, T_B.

If (as is necessitated by the above concept of a "measurement") we wish to draw a dependable conclusion from the energy content of the body K as to the position co-ordinate of the pointer, we have to see to it that the body surely gets into the lower energy state when it gets into contact with T_A, and surely into the upper energy state when it gets into contact with T_B. In other words: that

$$p(T_A) = 1, \quad q(T_A) = 0,$$
$$p(T_B) = 0, \quad q(T_B) = 1. \tag{27}$$

This, of course, cannot be achieved, but may be arbitrarily well approximated by allowing T_A to approach absolute zero, and the statistical weight factor to approach infinity. (In this limiting process, T_0 is also to be changed in such a way that $p(T_0)$ and $q(T_0)$ remain constant.) The equation (26) then becomes

$$S_A = -k \log p(T_0);$$
$$S_B = -k \log q(T_0), \tag{28}$$

and consequently

$$\exp(-S_A/k) + \exp(-S_B/k) = 1 \tag{29}$$

Our foregoing considerations have thus just realized the smallest permissible limiting care. The use of semipermeable walls according to also

Figure 1 allows a complete utilization of the measurement: inequality (1) certainly cannot be sharpened.

As we have seen in this example, a simple inanimate device can achieve the same essential result as would be achieved by the intervention of intelligent beings. We have examined the "biological phenomena" of a non-living device and have seen that it generates exactly that quantity of entropy which is required by thermodynamics.

APPENDIX

In the case considered, when the frequency of the two states depends on the temperature according to the equations:

$$p(T) = 1/[1 + g \exp(-u/kT)],$$
$$q(T) = g \exp(-u/kT)/[1 + g \exp(-u/kT)], \tag{30}$$

so that
~~and~~ the mean energy of the body is given by:

$$\begin{aligned} \bar{u}(T) &= u\, q(T) \\ &= u g \exp(-u/kT)/[1 + g \exp(-u/kT)], \end{aligned} \tag{31}$$

the following identity is valid:

$$\frac{1}{T}\frac{d\bar{u}}{dT} = \frac{d}{dT}\left\{ \frac{\bar{u}(T)}{T} + k \log\left[1 + g \exp(-u/kT)\right]\right\}. \tag{32}$$

Therefore we can also write the equation:

$$S_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \int_{T_A}^{T_0} \frac{1}{T}\frac{d\bar{u}}{dT}\, dT \tag{33}$$

as

$$\begin{aligned} S_A = &\frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} \\ &+ \left\{ \frac{\bar{u}(T)}{T} + k \log\left[1 + g \exp(-u/kT)\right]\right\}_{T_A}^{T_0}, \end{aligned} \tag{34}$$

and by substituting the limits we obtain:

$$S_A = \bar{u}(T_A)\left(\frac{1}{T_0} - \frac{1}{T_A}\right) + k \log \frac{1 + g \exp(-u/kT_0)}{1 + g \exp(-u/kT_A)} . \tag{35}$$

If we insert in this equation, according to (25):

$$1 + g \exp(-u/kT) = 1/p(T) \tag{36}$$

for T_A and T_0, then we obtain:

$$S_A = \bar{u}(T_A)\left(\frac{1}{T_0} - \frac{1}{T_A}\right) + k \log [p(T_A)/p(T_0)], \tag{37}$$

and if we then write according to (31):

$$\bar{u}(T_A) = u q(T_A) \tag{38}$$

then we obtain:

$$S_A = q(T_A)\left(\frac{u}{T_0} - \frac{u}{T_A}\right) + k \log [p(T_A)/p(T_0)]. \tag{39}$$

If we finally write according to (25):

$$u/T = -k \log [q(T)/g p(T)] \tag{40}$$

for T_A and T_0, then we obtain:

$$S_A = q(T_A) k \log \frac{p(T_0)\, q(T_A)}{q(T_0)\, p(T_A)} + k \log \frac{p(T_A)}{p(T_0)} . \tag{41}$$

We obtain the corresponding equation for S_B, if we replace the index A with B:

$$S_B = q(T_B)\, k \log \frac{p(T_0)\, q(T_B)}{q(T_0)\, q(T_B)} + k \log \frac{p(T_B)}{p(T_0)}. \tag{42}$$

Formula (41) is identical with (26), for S_A. ~~in the text.~~

We can also bring the formula for S_B into a somewhat different form, if we write:

$$q(T_B) = 1 - p(T_B), \tag{43}$$

expand and collect terms:

$$S_B = p(T_B)\, k \log \frac{q(T_0)\, p(T_B)}{p(T_0)\, q(T_B)} + k \log \frac{q(T_B)}{q(T_0)}. \tag{44}$$

This is the formula given in the text for S_B.

REFERENCES

Smoluchowski, . Vortrage uber die kinetische Theorie der Materie u. Elektrizitat. Leipzig: 1914.

Szilard, [illegible] Zeitschrift fur Physik, 1925, 32, 753-788.

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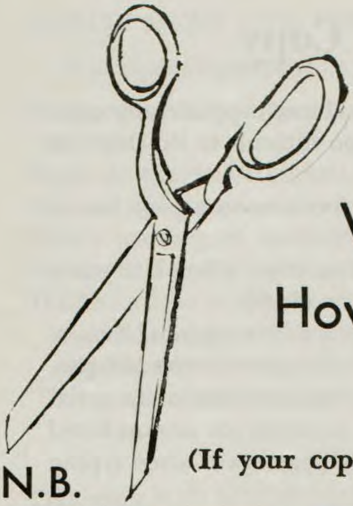

Why Copy is Cut and How to Deal with Cut Copy

N.B. **(If your copy is *not* cut disregard what follows)**

It is certainly disconcerting, when an author gets his MS back with his galleys, to find that the MS has been cut up and rearranged. Seems like a pointless impertinence, but certainly there is reason in it.

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Following is a sample of marked up proof and a list of the proof readers' marks in general use.

X Change bad letter	Comma	One-em dash
Push down space	:/ Colon	¶ Paragraph
Turn over	;/ Semicolon	no ¶ No paragraph—run in
Take out	Apostrophe	w.f. Wrong font
∧ Left out—insert	Quotation	...... Let it stand
# Insert space	= Hyphen	stet Let it stand
✓ Equalize spacing		tr. Transpose
Less space	Superior figure	cap. Capital letters
Close up	Inferior figure	sm. cap. Small caps
Raise (show in some manner how much to raise or where to raise)	Move to left	l.c. Lower case
	Move to right	ital. Italics
Lower	out, s.c. Out, see copy (be sure MS is returned if this is used)	rom. Roman
Straighten lines		b.f. Bold face
⊙ Period	□ Em quad space	

Many of the proof readers' marks may also be utilized in marking manuscripts

Capitals	Comparative Data	=	COMPARATIVE DATA
Caps and Small Caps	COMPARATIVE DATA	=	COMPARATIVE DATA
Small Caps	Comparative Data	=	COMPARATIVE DATA
Italic	Current Notes	=	*Current Notes*
Italic Caps	Current Notes	=	*CURRENT NOTES*
Bold Face	News and Notes	=	**News and Notes**
Bold Face Italic	News and Notes	=	***News and Notes***
Bold Face Italic Caps	News and Notes	=	***NEWS AND NOTES***

To change Caps to Caps and lower case CONCLUSIONS

To change Caps to Caps and small caps CONCLUSIONS

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B E H A V I O R A L S C I E N C E
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Please direct all communications concerning editorial and business matters to the Mental Health Research Institute, University of Michigan, Ann Arbor, Michigan.

Received July 29

July 22, 1964

Mrs. Leo Szilard
c/o Dupont Plaza Hotel
Washington, D. C.

Dear Mrs. Szilard:

As you know, we are publishing an English translation of your late husband's paper "On the Decrease of Entropy in a Thermodynamic System by the Intervention of Intelligent Beings" in the October issue of Behavioral Science. Although Dr. Szilard had approved this publication, he did not have an opportunity to read the translation.

We are having galley proofs sent to you, and would welcome any comments or suggestions you may wish to offer.

On behalf of myself and of our Board of Editors, may I extend to you our very deepest sympathy for your loss.

Sincerely yours,

James G. Miller per ss

James G. Miller, M. D., Director
Mental Health Research Institute

Editor

JGM:cjt

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In memory of Leo Szilard, who passed away on May 30, 1964 we present an English translation of his classical paper *Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen,* which appeared in the *Zeitschrift für Physik,* 1929, 53, 840–856. The publication in this journal of this translation was approved by Dr. Szilard before he died, but he never saw the copy. X

This is one of the earliest, if not the earliest paper, in which the relations of physical entropy to information (in the sense of modern mathematical theory of communication) were rigorously demonstrated and in which Maxwell's famous demon was successfully exorcised: a milestone in the integration of physical and cognitive concepts.

ON THE DECREASE OF ENTROPY IN A THERMODYNAMIC SYSTEM BY THE INTERVENTION OF INTELLIGENT BEINGS

by Leo Szilard

Translated by Anatol Rapoport and Mechthilde Knoller from the original article "Uber die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen." Zeitschrift fur Physik, 1929, 53, 840–856.

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[1] The author evidently uses the word "ominous" in the sense that the possibility of realizing the proposed arrangement threatens the validity of the Second Law.—*Translator*

[2] The ~~produced~~ entropy generated is denoted ~~with~~ by $\bar{s}_1$, $\bar{s}_2$.

[3] The increase in entropy can depend only on the types of measurement and their results but not on how many systems of one or the other type were present.

[4] See the Appendix.

xx delete apostrophes

6 pt. (Abstract)

The objective of the ~~following~~ investigation is to find the conditions which apparently allow the construction of a perpetual-motion "machine" of ~~a~~ the second ~~type~~ kind, if one permits an intelligent being to intervene in a thermodynamic system. When such beings make measurements, they make the system behave in a manner ~~quite~~ distinctly different from the way a mechanical system behaves when left to itself. We ~~shall~~ will show that it is a sort of a memory faculty, manifested by a system where measurements occur, that might cause a permanent decrease of entropy and thus a violation of the Second Law of Thermodynamics, were it not for the fact that the measurements themselves are necessarily accompanied by a production of entropy. At first we ~~shall~~ will calculate this production of entropy quite generally from the postulate that full compensation is made in the sense of the Second Law (Equation [1]). Second, by using an inanimate device able to make measurements—however under continual entropy production—we shall calculate the resulting quantity of entropy. We ~~shall~~ will find that it is exactly as great as is necessary for full compensation. The ~~real~~ actual production of entropy in connection with the measurement, therefore, need not be greater than Equation (1) requires.

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There is an objection, already historical, against the universal validity of the Second Law of Thermodynamics, which indeed looks rather ominous. The objection is embodied in the notion of Maxwell's demon, who in a different form appears even nowadays again and again; perhaps not unreasonably, inasmuch as behind the precisely formulated question quantitative connections seem to be hidden which to date have not been clarified. The objection in its original formulation concerns a demon who catches the fast molecules and lets the slow ones pass. To be sure, the objection can be met with the reply that man cannot in principle ~~estimate~~ foresee the value of a thermally fluctuating parameter. However, one cannot deny that we can very well measure the value of such a fluctuating parameter and therefore could certainly gain energy at the expense of heat by arranging our intervention according to the results of the measurements. Presently, of course, we do not know whether we commit an error by not including the intervening man into the system and by disregarding his biological ~~functions~~ phenomena.

known today that in a system left to itself no "perpetuum mobile" (perpetual motion machine) of the second kind (more exactly, no "automatic machine of continual finite work-yield which uses heat at the lowest temperature") can operate in spite of the fluctuation phenomena. A perpetuum mobile would have to be a machine which in the long run could lift a weight at the expense of the heat content of a reservoir. In other words, if we want to use the fluctuation phenomena in order to gain energy at the expense of heat, we are in the same position as playing a game of chance, in which we may win certain amounts now and then, although the expect~~ed~~ ation value of the winnings is zero or negative. The same applies to a system where the intervention from outside is performed strictly periodically, say, by periodically moving machines. We consider this as established (Szilard, 1925) and intend here only to consider the difficulties that occur when intelligent beings intervene in a system. We shall try to discover the quantitative ~~dependencies~~ relations having to do with this intervention.

Smoluchowski (1914, p. 89) writes: "As far as we know today, there is no automatic, permanently effective perpetual motion machine, in spite of the molecular fluctuations, but such a device ~~could~~ might, perhaps, function regularly if it were appropriately operated by intelligent beings....."

A perpetual motion machine therefore is possible if—according to the general method of physics—we view the experimenting man as a sort of *deus ex machina*, one who is continuously and exactly informed of the existing state of nature and who is able to start or interrupt the macroscopic course of nature at any moment without expenditure of work. Therefore he would definitely not have to possess the ability to catch single molecules like Maxwell's demon, although he would definitely be different from real living beings in possessing the above abilities. In eliciting any physical effect by ~~use~~ action of the sensory as well as the motor nervous systems ~~an expenditure~~ a degradation of energy is always involved, quite apart from the fact that the very existence of ~~living beings~~ a nervous system is dependent on continual dissipation of energy.

Whether — considering these circumstances—real living beings could continually or at least ~~periodically~~ regularly produce energy at the expense of heat ~~at~~ of the lowest temperature appears very doubtful, even though our ignorance of the biological ~~conditions~~ phenomena does not allow a definite answer. However the latter questions lead beyond the scope of physics in the strict sense.

It appears that the ignorance of the biological ~~conditions~~ phenomena need not ~~trouble us in order to recognize what~~ prevent us from understanding that which seems to us to be the essential thing. We may be sure that intelligent living beings—~~inasmuch~~ in so far as we are dealing with their intervention in a thermodynamic system—can be replaced by nonliving devices whose "biological ~~conditions~~ phenomena" one could ~~study in order to see~~ whether in fact a compensation of the entropy decrease takes place as a result of the intervention by such a device in a system.

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In the first place, we wish to ~~understand~~ ~~how~~ the decrease of entropy takes place when intelligent living beings intervene in a thermodynamic system. We shall see that this depends on a certain type of coupling between different parameters of the system. We shall consider an unusually simple type of these "ominous" couplings.[1] ~~In short~~ we shall talk about a "measurement," if we succeed in coupling the value of a parameter y (for instance the position co-ordinate of a pointer of a measuring instrument) at one moment with the ~~momentary~~ value of a fluctuating parameter x of the system, in such a way that we can draw conclusions from the value y, ~~to~~ the value that x had at the moment of the "measurement." ~~In this procedure~~ x and y ~~will~~ be uncoupled after the measurement, so that x can change, while y retains its value for some time. Such measurements are not harmless inter~~actions.~~ A system in which such measurements occur shows a sort of memory faculty, in the sense that one can recognize by the state parameter y what value another state parameter x had at an earlier moment, and we shall see that ~~just by virtue~~ of such memory the Second Law could be violated, if the measurement ~~were to~~ take place without compensation. We shall realize that the Second Law is not ~~violated~~ as much by this entropy decrease as one would think, ~~when~~ we see that the entropy decrease resulting from the inter~~action~~ would be compensated completely in any event if the ~~procedure~~ of such a measurement were, for instance, always accompanied by production of $k \log 2$ units of entropy. In that case it ~~would~~ be possible to find a more general entropy law, which ~~can be~~ universally ~~applied~~ to all measurements. Finally we shall consider a very simple (of course, not living) device, that is able to make measurements continually and whose "biological ~~conditions~~" we can easily ~~study.~~ By direct calculation, one finds in fact a continual entropy production of the ~~value~~ required by the above-mentioned entropy law ~~defined~~ from the validity of the Second Law.

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The first example, which we are going to consider more closely as a typical one, is the following. A standing hollow cylinder, closed at both ends, can be separated into two unequal sections of volumes V_1 and V_2 respectively by inserting a partition from the side at an arbitrarily fixed height. This partition forms a piston that can be moved up~~wards~~ and down~~wards~~ in the cylinder. An ~~enor-~~ mously large heat reservoir of a given t…

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by virtue of its thermal motion.

Imagine, specifically, a man who at a given time inserts the piston into the cylinder and somehow notes whether the molecule is caught in the upper or lower part of the cylinder, that is, in volume V_1 or V_2. If he should find that the former is the case, then he would ~~push~~ the piston slowly downward until it reaches the bottom of the cylinder. During this slow movement of the piston the molecule stays, of course, above the piston. However, it is no longer constrained to the upper ~~half~~ of the cylinder but bounces many times against the piston which is already in the lower ~~half~~ of the cylinder. In this way the molecule does a certain amount of work on the piston. This is the work that corresponds to the isothermal expansion of an ideal gas—consisting of one single molecule—from volume V_1 to the volume $V_1 + V_2$. After some time, when the piston has reached the bottom of the container, the molecule has again the full volume $V_1 + V_2$ to move about in, and the piston is removed. The procedure ~~may~~ be repeated many times. The man ~~pushes~~ up or down depending on whether the molecule is trapped in the upper or lower half of the piston. ~~To express it more exactly we have a procedure of coupling the transmission of force in such a way that a weight is always being lifted by the piston.~~ In this way the potential energy of the weight certainly increases constantly. (The transmission of force to the weight is best ~~performed in such a way~~ that the ~~force~~ exerted by the weight on the ~~piston~~ at any position of the latter equals the average pressure of the gas). It is clear that in this manner energy is constantly gained at the expense of heat, ~~as long~~ as the biological ~~functions~~ of the intervening man are ignored in the calculation.

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In more detail, this motion may be caused by a weight, that is to be raised, through a mechanism that transmits the force from the piston to the weight, in such a way that the latter is always displaced upwards.

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In order to ~~recognize how the man really affects~~ the system, one best imagines that the movement of the piston is performed mechanically and that the man's activity consists only in ~~registering~~ the altitude of the molecule and in pushing a lever (which ~~controls the movement of~~ the piston) to the right or left, depending on whether the ~~registered~~ height ~~necessitates~~ a down or upward movement. This means that the intervention of the human being consists only in the coupling of two position co-ordinates, namely a co-ordinate x, which determines the altitude of the molecule, with another co-ordinate y, which determines the position of the lever and ~~which determines~~ an upward or downward motion ~~of~~ the piston. It is best to imagine the mass of the piston as large and its speed sufficiently great, so that the thermal agitation of the piston at the temperature in question can be neglected.

understand the essence of the man's effect on

phenomena

determining

down- or

requires

molecule's

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therefore also whether

is imparted to

In the typical example presented here, we wish to distinguish two periods, namely:

1. The period of *measurement* when the piston has just been inserted in the middle of the cylinder and the molecule is trapped

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either in the upper or lower ~~half~~ part; so that if we choose the origin of co-ordinates appropriately, the x-co-ordinate of the molecule is restricted to either the interval $x > 0$ or $x < 0$;

2. The period of *utilization of the measurement*, "the period of decrease of entropy," during which the piston is moving up or down. During this period the x-co-ordinate of the molecule is certainly not restricted to the original interval $x > 0$ or $x < 0$. Rather, if the molecule was in the upper half of the cylinder during the period of measurement i.e., when $x > 0$, the molecule must bounce on the downward-moving piston in the lower part of the cylinder, if it is to transmit energy to the piston; that is, the co-ordinate x has to ~~be extended to~~ enter the interval $x < 0$. The lever, on the contrary, retains during the whole period its position ~~on~~ toward the right, ~~because of the~~ corresponding ~~movement~~ motion downward, to If the position of the lever ~~on~~ toward the right is designated by $y = 1$ (and correspondingly the position ~~on~~ toward the left by $y = -1$) we see that during the period of measurement, the position $x > 0$ corresponds to $y = 1$; but afterwards $y = 1$ stays on, even though x passes into the other interval $x < 0$. We see that in the utilization of the measurement~~s~~ the coupling of the two parameters x and y disappears.

We shall ~~generally~~ say, quite generally, that a parameter y "measures" a parameter x (which varies according to a probability law), if the value of y ~~depends~~ is directed by on the value of parameter x at a given moment. A measurement procedure underlies the entropy decrease effected by the intervention of intelligent beings.

One may reasonably assume that a measurement procedure is ~~in principle~~ fundamentally associated with a ~~very~~ certain definite average entropy production, and that this restores concordance with the Second Law. The amount of entropy ~~associated with~~ generated by the measurement may, of course, always be greater than this fundamental amount, but not smaller. To put it precisely: we have to distinguish here between two entropy values. One of them, $\bar{S}_1$, is produced when during the measurement y assumes the value 1, and the other, $\bar{S}_2$, when y assumes the value -1. We cannot expect to get general information about $\bar{S}_1$ or $\bar{S}_2$ ~~alone~~ separately, but we shall see that if the amount of entropy produced by the "measurement" is to compensate the entropy decrease affected by utilization, the ~~general following equation derives:~~ relation

$$e^{-\bar{S}_1/k} + e^{-\bar{S}_2/k} \leq 1 \qquad (1)$$

must always hold good.

One ~~can~~ sees from this ~~equation~~ formula that one can reduce one of the values, for instance make

as small as one wishes,

$\bar{S}_1$ ~~at will~~, but then the other value $\bar{S}_2$ becomes correspondingly greater. Furthermore one can notice that the magnitude of the ~~considered~~ interval is of no consequence. One can also easily understand that it cannot be otherwise.

under consideration

~~On the other hand~~, as long as the entropies $\bar{S}_1$ and $\bar{S}_2$, produced by the measurements, satisfy ~~Equation~~ (1), we can be sure that the decrease of entropy ~~later effected~~ by the utilization of the measurement will be fully compensated ~~for~~.

expected

the inequality

Conversely

later

caused

Before we proceed with the proof of ~~Equation~~ (1), let us see in the light of the above mechanical example, how all this fits together. For the entropies $\bar{S}_1$ and $\bar{S}_2$ produced by the measurements, we ~~assume~~ the following ~~special case~~:

inequality

ansatz

make

$$\bar{S}_1 = \bar{S}_2 = k \log 2. \tag{2}$$

This ~~assumption~~ satisfies ~~U~~nequality (1) and the mean value of the quantity of entropy produced by a measurement is (of course in this special case) independent of the frequencies w_1, w_2 of the two events):

i.e

ansatz

$$\bar{S} = k \log 2 \tag{3}$$

In this example one achieves a decrease of entropy ~~with~~ the isotherm~~ic~~ expansion:[2]

al

took

by

$$- \bar{s}_1 = -k \log \frac{V_1}{V_1 - V_2};$$
$$- \bar{s}_2 = -k \log \frac{V_2}{V_1 + V_2}, \tag{4}$$

taken

depending on whether the molecule was found in volume V_1 or V_2 when the piston was inserted. (The decrease of entropy equals the ratio of the quantity of heat ~~lost~~ from the heat reservoir ~~in~~ isotherm~~ic~~ expansion to the temperature of the heat reservoir in question). Since in the above case the frequencies w_1, w_2 are ~~analogous to~~ the volumes V_1, V_2, the mean value of the ~~produced~~ entropy is (a negative number):

al

in the ratio of

during the

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$$\bar{s} = w_1 \cdot (+\bar{s}_1) + w_2 \cdot (+\bar{s}_2) = \frac{V_1}{V_1 + V_2} \tag{5}$$
$$k \log \frac{V_1}{V_1 + V_2} + \frac{V_2}{V_1 + V_2} k \log \frac{V_1}{V_1 + V_2}$$

Dr

generated

As one can see, we have, indeed

$$\frac{V_1}{V_1 + V_2} k \log \frac{V_1}{V_1 + V_2} + \frac{V_2}{V_1 + V_2}$$
$$\cdot k \log \frac{V_2}{V_1 + V_2} + k \log 2 \geqq 0 \tag{6}$$

And therefore:

l.c

$$\bar{S} + \bar{s} \geqq 0. \tag{7}$$

In the special case considered, we would ~~in effect~~ have a full compensation for the decrease of entropy achieved by the utilization of the measurement.

actually

took

We shall ~~now forgo examining further~~ special cases, ~~We shall~~ instead try ~~with the help of a general observation~~ to clarify the ~~existing conditions~~ and to derive ~~equation~~ (1). We shall therefore imagine the whole

not examine more

but

matter by a general argument

formers

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system—in which the co-ordinate x, exposed to some kind of thermal fluctuations, can be measured by the parameter y in the way just explained—as a multitude of particles, all enclosed in one box. Every one of these particles can move freely, so that they may be considered as the molecules of an ideal gas, which, because of thermal agitation, wander about in the common box independently of each other and exert a certain pressure on the walls of the box—the pressure being determined by the temperature. We shall now ~~recognize~~ consider two of these molecules as chemically different and ~~principally~~ separable by semipermeable walls, if the co-ordinate x for one molecule is in a preassigned interval while the corresponding co-ordinate of the other molecule falls outside that interval. We also shall look upon them as chemically different, if they differ only in that the y co-ordinate is $+1$ for one and -1 for the other.

We should like to give the box in which the "molecules" are stored the form of a hollow cylinder containing four pistons. Pistons A and A' are fixed while the other two are movable, so that the distance BB' always equals the distance AA', as is indicated in Figure 1 by the two brackets. A', the bottom, and B, the cover of the container, are impermeable for all "molecules," while A and B' are semipermeable; namely, A is permeable only for those "molecules" for which the parameter x is in the preassigned interval, i.e., (x_1, x_2), B' is only permeable for the rest.

This figure must be changed to conform to that on p 847 of the original

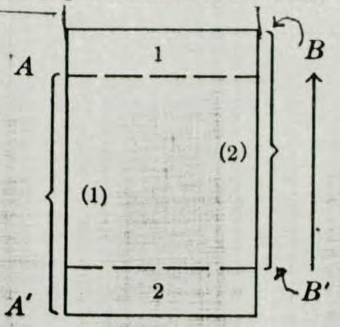

FIG. 1

(8)

CUT

In the beginning the piston B is at A and therefore B' at A', and all "molecules" are in the space between. A certain fraction of the molecules have their co-ordinate x in the ~~selected~~ preassigned interval. We shall designate by w_1 the probability that this is the case for a randomly selected molecule and by w_2 the probability that x is outside the interval. Then $w_1 + w_2 = 1$.

Let the distribution of the parameter y be over the values $+1$ and -1 in any proportion but in any event independent of the ~~value x~~ values. We imagine an intervention by an intelligent being, who imparts to y the value 1 for all "molecules" whose x at that moment is in the selected interval. Otherwise the value -1 is assigned. If then, because of thermal fluctuation, for any "molecule," the signed interval or, as we also may put it; if the "molecule" suffers a monomolecular chemical reaction with regard to x (by which it is transformed from a species that can pass the semipermeable piston A into a species for which the piston is impermeable), then the parameter y retains its value 1 for the time being, so that the "molecule", because of the volume of the parameter y, ~~is "reminded"~~ "remembers" during the whole following process that x originally was in the preassigned interval. We shall see immediately what part this ~~"reminiscence"~~ memory may play. After the intervention just discussed, we move the piston, so that we separate the two kinds of molecules without ~~the use of energy~~ doing work. This results in two containers, of which the first contains only the one ~~species~~ modification and the second only the other. Each ~~species~~ modification now ~~requires~~ occupies the same volume as the mixture did previously. In one of these containers, if considered by itself, there is now no equilibrium with regard to the two "~~species~~ modifications in x." Of course the ratio of the two ~~species~~ modifications has remained $w_1 : w_2$. If we ~~create~~ allow this equilibrium to be achieved in both containers independently and at ~~with~~ constant volume and temperature, then the entropy of the system certainly has increased. ~~because~~ For the total heat ~~change~~ release is 0, ~~while~~ since the ratio of the two "~~species~~ modifications in x" $w_1 : w_2$ does not change. If we accomplish the equilibrium distribution in both containers in a reversible fashion then the entropy of the rest of the world will decrease by the same amount. Therefore the entropy increases by a negative value, ~~e.g.~~ and, the value of the entropy increase per molecule is exactly:

$$\bar{s} = k(w_1 \log w_1 + w_2 \log w_2). \qquad (9)$$

(The entropy constants ~~in connection with~~ that we must assign to the two "~~species~~ modifications in x" do not occur here explicitly, as the process leaves ~~undisturbed~~ unchanged the total number of molecules belonging to the one or the other species).

Now of course we cannot bring the two gases back to the original volume without expenditure of work by simply moving the piston back, as there are now in the container—which is ~~bordered~~ bounded by the pistons BB'—also molecules whose x-co-ordinate lies outside of the preassigned interval and for which the piston A is not permeable any longer. Thus one can see that the calculated decrease of entropy (Equation [9]) does not mean a contradiction of the Second Law.

No ¶ *As long as we do not use the fact that the molecules in the container BB', by virtue of their co-ordinate y, "remember" that the x-co-ordinate for the molecules of this container originally was in the preassigned interval, full compensation exists for the calculated decrease of entropy, by virtue of the fact that the partial pressures in the two containers are smaller than in the original mixture.* Ital

¶ But now ~~Now~~ we can use the fact that all molecules in the container BB' have the y-co-ordinate 1, and in the other accordingly -1, to bring all molecules back again to the original volume. To accomplish this we only need to replace the semipermeable wall A by a wall A^*, which is semipermeable not with regard

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PROOF-READ & RETURN

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B'^*, which is impermeable for the molecules with $y = -1$ and permeable for the others. Then both containers can be put into each other again without expenditure of energy. The distribution of the y-co-ordinate with regard to 1 and -1 now has become statistically independent, of the x-values and besides we are able to re-establish the original distribution over 1 and -1. Thus we would have gone through a complete cycle. The only change that we have to register is the resulting decrease of entropy given by (9):

$$\bar{s} = k(w_1 \log w_1 + w_2 \log w_2). \quad (10)$$

If we do not wish to admit that the Second Law ~~principle~~ has been violated, we must conclude that the intervention which establishes the coupling between y and x, the measurement of x by y, must be accompanied ~~cannot be separated from~~ by a production of entropy. If a definite way of achieving ~~method is chosen to realize~~ this coupling is adopted and if the quantity of entropy that is inevitably produced is designated by ~~with~~ S_1 and S_2, where S_1 stands for the mean increase in entropy that occurs when y acquires ~~obtains~~ the value 1, and accordingly S_2 for the increase that occurs when y acquires ~~obtains~~ the value -1, we arrive at the equation:

$$w_1S_1 + w_2S_2 = \bar{S} \quad (11)$$

No ¶ In order for the Second Law to remain in force, this quantity of entropy must be greater than the decrease of entropy $\bar{s}$, which according to (9) is produced by the utilization of the measurement. Therefore the following unequality must be valid:

Center
$$\bar{S} + \bar{s} \geqq 0$$

Center
$$w_1S_1 + w_2S_2 + k(w_1 \log w_1 + w_2 \log w_2) \geqq 0 \quad (12)$$

This equation must be valid for any values of w_1 and w_2,[3] and of course the constraint $w_1 + w_2 = 1$ cannot be violated. We ask ~~are investigating~~, in particular, for ~~at~~ which w_1 and w_2 ~~for~~ given S-values the expression becomes a minimum. For the two minimizing values w_1 and w_2 the unequality (12) ~~(8)~~ must still be valid. Under the above constraint, the minimum occurs when the following equation holds:

$$\frac{S_1}{k} + \log w_1 = \frac{S_2}{k} + \log w_2 \quad (13)$$

No ¶ But then:

$$e^{-S_1/k} + e^{-S_2/k} \leqq 1. \quad (14)$$

and ~~It~~ is easily seen ~~that~~ if one introduces the notation

S_1 ... S_2

(no ¶) this

$$\frac{s_1}{k} + \log w_1 = \frac{s_2}{k} + \log w_2 = \lambda \quad (15)$$;

then:

$$w_1 = e^{\lambda} \cdot e^{-s_1/k};\ w_2 = e^{\lambda} \cdot e^{-s_2/k}. \quad (16)$$

no ¶ If one substitutes these values into ~~Equation~~ the inequality (12) one gets:

$$\lambda e^{\lambda}(e^{-s_1/k} + e^{-s_2/k}) \geqq 0. \quad (17)$$

no ¶ Therefore the following also holds:

$$\lambda \geqq 0. \quad (18)$$

no ¶ If one puts the values w_1 and w_2 from (16) into the equations $w_1 + w_2 = 1$, one ~~will get:~~ gets

$$e^{-s_1/k} + e^{-s_2/k} = e^{-\lambda}. \quad (19)$$

no ¶ And because $\lambda \geqq 0$, the following holds:

$$e^{-s_1/k} + e^{-s_2/k} \leqq 1. \quad (20)$$

no ¶ This equation ~~has to~~ must be universally valid, if ~~the Second Law of~~ Thermodynamics is not to be violated. / l.c.

As long as we allow intelligent beings to perform the intervention, a direct test is not possible. But we can try to ~~build~~ describe simple nonliving devices that effect such coupling, ~~Then we could~~ and see if indeed entropy is ~~produced~~ generated and in what quantity. Having already recognized that the only important factor is a certain characteristic type of coupling, a "measurement," we need not construct any complicated models, which imitate the intervention of living beings in detail. We can be satisfied with the construction of ~~a~~ this particular type of coupling ~~with~~ which is accompanied by memory.

In our next example, the position co-ordinate of ~~a~~ an oscillating pointer ~~going back and forth~~ is "measured" by the energy content of a body K. The pointer is supposed to ~~bring about that~~ connect, in a purely mechanical way, the body K—by whose energy content the position of the pointer is to be measured—~~is connected~~ by heat conduction with one of ~~the~~ two ~~insertions~~ intermediate pieces, A or B. The body is connected with A as long as the co-ordinate—which determines the position of the pointer—falls into a certain preassigned, but otherwise arbitrarily large or small interval a, and otherwise if the co-ordinate is in the interval b, with B. Up to a certain moment, namely the moment of the "measurement," both ~~insertions~~ intermediate pieces will be thermally connected with a heat reservoir at temperature T_0. At this moment the insertion A will be cooled reversibly to the temperature T_A, e.g., by a periodically functioning mechanical device. That is, after successive contacts with heat reservoirs of intermediate temperatures, A will be brought into contact with a heat reservoir of the temperature T_A. At the same time the insertion B will be ~~brought~~ heated in the same way to temperature T_B. Then the ~~insertions~~ intermediate pieces will again be isolated from the corresponding heat reservoirs. / l.c.

We assume that the position of the pointer changes so slowly that all the operations that we have sketched take place while the position of the pointer remains ~~the same.~~ unchanged. If the position co-ordinate of the pointer fell in the preassigned interval, then the body was connected with the insertion A

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In the opposite case

is now

~~Otherwise~~, the body ~~would be~~ heated to temperature T_B. Its energy content becomes—according to the position of the pointer at the time of "measurement"—small at temperature T_A or great at temperature T_B and will retain its value, even if the pointer eventually ~~moves out of~~ the preassigned interval or enters into it. After some time, while the pointer is still ~~moving,~~ one ~~cannot~~ draw ~~a~~ conclusion from the energy content of the body K with regard to the position of the pointer ~~at a given moment,~~ but one can draw a definite conclusion with regard to the position of the pointer at the time of the measurement. Then the measurement is completed.

leaves

oscillating;

any definite

can no longer

momentary

After the measurement has been accomplished, the above-mentioned periodically functioning mechanical device should connect the thermally isolated insertions A and B with the heat reservoir T_0. This has the purpose of bringing the body K—which is now also connected with one of the two ~~insertions~~—back into its original state. The direct connection of the ~~insertions~~ and hence of the body K—which has been either cooled to T_A or heated to T_B—to the reservoir T_0 consequently causes an increase of entropy. This cannot be avoided, because it would ~~not be of advantage~~ to heat the insertion A reversibly to the temperature T_0 by successive contacts with the reservoirs of intermediate temperatures and to cool B in the same manner. After the measurement we do not know with which of the two insertions the body K is in contact at that moment; nor do we know whether it had been in connection with T_A or T_B in the end. Therefore neither do we know whether we should use intermediate temperatures between T_A and T_0 or between T_0 and T_B.

intermediate pieces

possibly

make no sense

The mean value of the quantity of entropy S_1 and S_2 for each measurement can be calculated, if the heat capacity as a function of the temperature $\overline{u}(T)$ is known for the body K, since the entropy can be calculated from the heat capacity. We have, of course, neglected the heat capacities of the ~~insertions~~. If the position co-ordinate of the pointer was in the preassigned interval at the time of the "measurement," and accordingly the body in connection with insertion A, then the entropy conveyed to the heat reservoirs during successive cooling was

intermediate pieces

to the

$$\int_{T_A}^{T_0} \frac{1}{T}\frac{d\overline{u}}{dT}. \qquad (21)$$

However, following this, the entropy withdrawn from the reservoir T_0 by direct contact with it was

no ¶

Insert parenth.

$$\frac{\overline{u}(T_0) - \overline{u}(T_A)}{T_0}. \qquad (22)$$

All in all the entropy was increased by

No 91

the amount

$$S_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \int_{T_A}^{T_0} \frac{1}{T}\frac{d\bar{u}}{dT}\,dT. \quad (23)$$

No 91

~~Accordingly~~ the entropy will increase by the following amount, if the body was in contact with the ~~insertion~~ intermediate piece B at the time of the "measurement":

Analogously,

$$S_B = \frac{\bar{u}(T_B) - \bar{u}(T_0)}{T_0} + \int_{T_B}^{T_0} \frac{1}{T}\frac{d\bar{u}}{dT}\,dT. \quad (24)$$

We shall now ~~apply these equations to~~ evaluate these expressions for the very simple case, where ~~our~~ the body which we use has only two energy states, a lower and a higher state. If such a body is in thermal contact with a heat reservoir at any temperature ~~T~~ ϑ, the probability that it is in the lower or upper state (respectively) is given by: ϑ_1 ϑ:

$$\left.\begin{aligned} p(T) &= \frac{1}{1 + ge^{-u/kT}} \\ q(T) &= \frac{ge^{-u/kT}}{1 + ge^{-u/kT}} \end{aligned}\right\} \quad (25)$$

Here u stands for the difference of energy of the two states and g for the statistical weight~~ing~~. We can set the energy of the lower state equal to zero without loss of generality. Therefore:[4] ϑ

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$$\left.\begin{aligned} S_A &= q(T_A)\,k\log\frac{q(T_A)\,p(T_0)}{q(T_0)\,p(T_A)} \\ &\qquad + k\log\frac{p(T_A)}{p(T_0)} \\ S_B &= p(T_B)\,k\log\frac{q(T_0)\,p(T_B)}{q(T_B)\,p(T_0)} \\ &\qquad + k\log\frac{q(T_B)}{q(T_0)} \end{aligned}\right\}. \quad (26)$$

Here q and p are the functions of T given by equation (25), ~~The arguments of the function are to be~~ which are here to be taken for the arguments T_0, T_A, or T_B, ~~whichever applies.~~

If, as is necessitated by the above concept of a "measurement", we wish to draw a dependable conclusion from the energy content of the body K to the position co-ordinate of the pointer, we have to see to it that the body surely gets into the lower energy state when it gets into contact with T_A, and surely into the upper energy state when it gets into contact with T_B. In other words:

$$p(T_A) = 1,\ q(T_A) = 0;\ p(T_B) = 0,\ q(T_B) = 1. \quad (27)$$

This of course cannot be achieved, but may be arbitrarily approximated by ~~setting~~ allowing T_A ~~near~~ to approach absolute zero and ~~by making~~ the statistical weight~~ing~~ g ~~infinitely large when the threshold is crossed.~~ to approach infinity. T_0 is also changed, ~~so~~ in such a way that $p(T_0)$ and $q(T_0)$ ~~become fixed~~ remain constant. The equation (26) ~~accordingly~~ then becomes:

No 9

$$S_A = -k\log p(T_0); \qquad S_B = -k\log q(T_0) \quad (28)$$

?|0K as set or display?

i.e. And if we form the expression $(e^{-S_A/k} + e^{-S_B/k})$ we find:

$$e^{-S_A/k} + e^{-S_B/k} = 1. \quad (29)$$

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foregoing

¶ Our considerations have thus just realised the smallest permissible limiting case

$$\cancel{e^{-S_A/k} + e^{-S_B/k} = 1.} \qquad \cancel{(29)}$$

~~Hence the permissible limiting case could just be realized.~~ The use of semipermeable walls according to Figure 1 allows ~~full~~ a complete utilization of the measurement; inequality (1) certainly cannot be sharpened. l.c.

As we have seen in this example, a simple ~~nonliving~~ inanimate device can achieve the same essential result as would be achieved by the intervention of intelligent beings. We have examined the "~~living functions~~" "biological phenomena" of a nonliving device and have seen that ~~exactly the~~ quantity of entropy ~~is in it produced that~~ they generate exactly that which is required by thermodynamics.

APPENDIX

In the case considered, when the frequency of the two states depends on the temperature according to the equations:

$$p(T) = \frac{1}{1 + ge^{-u/kT}}\,;\, q(T) = \frac{ge^{-u/kT}}{1 + ge^{-u/kT}} \qquad (30)$$

and the mean energy of the body is given by:

$$\bar{u}(T) = uq(T) = \frac{uge^{-u/kT}}{1 + ge^{-u/kT}}, \qquad (31)$$

No ¶ l.c. The following identity ~~becomes~~ is valid:

$$\frac{1}{T}\frac{d\bar{u}}{dT} = \frac{d}{dT}\left\{\frac{\bar{u}(T)}{T} + k\log\left(1 + e^{-u/kT}\right)\right\}. \qquad (32)$$

Therefore we can also write the equation:

$$B_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \int_{T_A}^{T_0}\frac{1}{T}\frac{d\bar{u}}{dT}\,dT. \qquad (33)$$

~~Also this way:~~ as

$$S_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \left\{\frac{\bar{u}(T)}{T} + k\log(1 + ge^{-u/kT})\right\}_{T_A}^{T_0}, \qquad (34)$$

and by substituting the limits we obtain:

$$S_A = \bar{u}(T_A)\left(\frac{1}{T_0} - \frac{1}{T_A}\right) + k\log\frac{1 + ge^{-u/kT_0}}{1 + ge^{-u/kT_A}}. \qquad (35)$$

If we ~~express~~ write the latter equation according to (25):

$$1 + ge^{-u/kT} = \frac{1}{p(T)} \qquad (36)$$

No ¶ l.c For T_A and T_0, then we obtain:

$$S_A = \bar{u}(T_A)\left(\frac{1}{T_0} - \frac{1}{T_A}\right) + k\log\frac{p(T_A)}{p(T_0)} \qquad (37)$$

No ¶ l.c. And if we ~~thus~~ write according to (31):

$$\bar{u}(T_A) = uq(T_A) \qquad (38)$$

~~then~~ we obtain:

$$S_A = q(T_A)\left(\frac{u}{T_0} - \frac{u}{T_A}\right) + k\log\frac{p(T_A)}{p(T_0)}. \qquad (39)$$

No ¶ If i.e. ~~W~~e finally write according to (25):

$$\frac{u}{T} = -k \log \frac{q(T)}{gp(T)}. \tag{40}$$

No ¶ i.e. ~~F~~or T_A and T_0, then we obtain:

$$S_A = p(T_A)\, k \log \frac{p\,(T_0)}{q\,(T_0)} \frac{q(T_A)}{p(T_A)} + k \log \frac{p(T_A)}{p(T_0)}. \tag{41}$$

q

No ¶ We obtain the corresponding equation for S_B, if we ~~interchange~~ replace the index A with B. Then we obtain:

$$S_B = q(T_B)\, k \log \frac{p(T_0)}{q(T_0)} \frac{q((T_B)}{p((T_B)} + k \log \frac{p(T_B)}{p(T_0)}. \tag{42}$$

No ¶ Formula (41) is identical with (26), given, for S_A, in the text.

No ¶ We can ~~also obtain~~ bring the formula for S_B into a somewhat different form, if we write:

$$q(T_B) = 1 - p(T_B), \tag{43}$$

expand and collect terms, then we get .

$$S_B = p(T_B)\, k \log \frac{q(T_0)}{p(T_0)} \frac{p(T_B)}{q(T_B)} + k \log \frac{q(T_B)}{q(T_0)}. \tag{44}$$

No ¶ This is the formula given in the text for S_B.

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In memory of Leo Szilard, who passed away on May 30, 1964 we present an English translation of his classical paper *Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen,* which appeared in the *Zeitschrift für Physik,* 1929, 53, 840–856. The publication in this journal of this translation was approved by Dr. Szilard before he died, but he never saw the copy. X

This is one of the earliest, if not the earliest paper, in which the relations of physical entropy to information (in the sense of modern mathematical theory of communication) were rigorously demonstrated and in which Maxwell's famous demon was successfully exorcised: a milestone in the integration of physical and cognitive concepts.

ON THE DECREASE OF ENTROPY IN A THERMODYNAMIC SYSTEM BY THE INTERVENTION OF INTELLIGENT BEINGS

by Leo Szilard

Translated by Anatol Rapoport and Mechthilde Knoller from the original article "Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen." Zeitschrift für Physik, 1929, 53, 840–856.

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Corrections f/S

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Gal. 2, BSC, 1564, p. 6, 7-22-6 F, 533, 7-16-64

[1] The author evidently uses the word "ominous" in the sense that the possibility of realizing the proposed arrangement threatens the validity of the Second Law.—*Translator*

[2] The produced entropy is denoted with $\bar{s}_1$, $\bar{s}_2$.

[3] The increase in entropy can depend only on the types of measurement and their results but not on how many systems of one or the other type were present.

[4] See the Appendix.

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This paragraph in small print — this is the Summary of the paper

THE objective of the ~~following~~ investigation is to find the conditions which apparently allow the construction of a perpetual-motion "machine" of a second type, if one permits an intelligent being to intervene in a thermodynamic system. When such beings make measurements, they make the system behave in a manner ~~quite~~ distinctly different from the way a mechanical system behaves when left to itself. We ~~shall~~ show that ~~it is~~ already a sort of a memory faculty, manifested by a system where measurements occur, ~~that~~ might cause a permanent decrease of entropy and thus a violation of the Second Law of Thermodynamics, were it not for the fact that the measurements themselves are necessarily accompanied by a production of entropy. At first we ~~shall~~ calculate this production of entropy quite generally from the postulate that full compensation is made in the sense of the Second Law [Equation (1)]. Second, by using an inanimate device able to make measurements—however under continual entropy production—we ~~shall~~ calculate the resulting quantity of entropy. We ~~shall~~ find that it is exactly as great as is necessary for full compensation. The real production of entropy in connection with the measurement, therefore, need not be greater than Equation (1) requires.

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There is an objection, already historical, against the universal validity of the Second Law of Thermodynamics, which indeed looks rather ominous. The objection is embodied in the notion of Maxwell's demon, who in a different form appears even nowadays again and again; perhaps not unreasonably, inasmuch as behind the precisely formulated question quantitative connections seem to be hidden which to date have not been clarified. The objection in its original formulation concerns a demon who catches the fast molecules and lets the slow ones pass. To be sure, the objection can be met with the reply that man cannot in principle estimate a thermally fluctuating parameter. However, one cannot deny that we can very well measure the value of such a fluctuating parameter and therefore could certainly gain energy at the expense of heat by arranging our intervention according to the results of the measurements. Presently, of course, we do not know whether we commit an error by not including the intervening man into the system and by disregarding his biological functions.

Apart from this unresolved matter, it is known today that in a system left to itself no "perpetuum mobile" (perpetual motion machine) of the second kind (more exactly, no "automatic machine of continual finite work-yield which uses heat at the lowest temperature") can operate in spite of the fluctuation phenomena. A perpetuum mobile would have to be a machine which in the long run could lift a weight at the expense of the heat content of a reservoir. In other words, if we want to use the fluctuation phenomena in order to gain energy at the expense of heat, we are in the same position as playing a game of chance, in which we may win certain amounts now and then, although the expected value of the winnings is zero or negative. The same applies to a system where the intervention from outside is performed strictly periodically. We consider this as established (Szilard, 1925) and intend here only to consider the difficulties that occur when intelligent beings intervene in a system. We shall try to discover the quantitative dependencies having to do with this intervention.

Smoluchowski (1914, p. 89) writes: "As far as we know today, there is no automatic, permanently effective perpetual motion machine, in spite of the molecular fluctuations, but such a device could function regularly if it were appropriately operated by intelligent beings....."

A perpetual motion machine therefore is possible if—according to the general method of physics—we view the experimenting man as a sort of *deus ex machina*, one who is continuously and exactly informed of the existing state of nature and who is able to start or interrupt the macroscopic course of nature at any moment without expenditure of work. Therefore he would not have to possess the ability to catch single molecules like Maxwell's demon, although he would definitely be different from real living beings. In eliciting any physical effect by use of the sensory as well as the motor nervous system an expenditure of energy is always involved, apart from the fact that the very existence of living beings is dependent on continual dissipation of energy.

Whether — considering these circumstances—real living beings could continually or at least periodically produce energy at the expense of heat at lowest temperature appears very doubtful, even though our ignorance of the biological conditions does not allow a definite answer. However the latter questions lead beyond the scope of physics in the strict sense.

It appears that the ignorance of the biological conditions need not trouble us in order to recognize what seems to us to be the essential thing. We may be sure that intelligent living beings—inasmuch as we are dealing with their intervention in a thermodynamic system—can be replaced by nonliving devices whose "biological conditions" one could study in order to see whether in fact a compensation of the entropy decrease takes place as a result of the intervention by such a device in a system.

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In the first place, we wish to understand how the decrease of entropy takes place when intelligent living beings intervene in a thermodynamic system. We shall see that this depends on a certain type of coupling between different parameters of the system. We shall consider an unusually simple type of these "ominous" couplings.[1] In short we shall talk about a "measurement," if we succeed in coupling the value of a parameter y (for instance the position co-ordinate of a pointer of a measuring instrument) at one moment with the momentary value of a fluctuating parameter x of the system, in such a way that we can draw conclusions from the value y to the value that x had at the moment of the "measurement." In this procedure x and y will be uncoupled after the measurement, so that x can change, while y retains its value for some time. Such measurements are not harmless interactions. A system in which such measurements occur shows a sort of memory faculty, in the sense that one can recognize by the state parameter y what value another state parameter x had at an earlier moment, and we shall see that just by virtue of such memory the Second Law could be violated, if the measurement were to take place without compensation. We shall realize that the Second Law is not violated as much by this entropy decrease as one would think, when we see that the entropy decrease resulting from the interaction would be compensated completely in any event if the procedure of such a measurement were, for instance, always accompanied by production of $k \log 2$ units of entropy. In that case it would be possible to find a more general entropy law, which can be universally applied to all measurements. Finally we shall consider a very simple (of course, not living) device, that is able to make measurements continually and whose "biological conditions" we can easily study. By direct calculation, one finds in fact a continual entropy production of the value required by the above-mentioned entropy law—defined from the validity of the Second Law.

The first example, which we are going to consider more closely as a typical one, is the following. A standing hollow cylinder, closed at both ends, can be separated into two unequal sections of volumes V_1 and V_2 respectively by inserting a partition from the side at an arbitrarily fixed height. This partition forms a piston that can be moved upwards and downwards in the cylinder. An enormously large heat reservoir of a given temperature T insures that a possibly present gas in the cylinder goes through an isothermic expansion as the piston moves. This gas will consist of a single molecule which, as long as the piston is not inserted into the cylinder, moves about in the whole cylinder by virtue of its thermal motion.

Imagine, specifically, a man who at a given time inserts the piston into the cylinder and somehow notes whether the molecule is caught in the upper or lower part of the cylinder, that is, in volume V_1 or V_2. If he should find that the former is the case, then he would push the piston slowly downward until it reaches the bottom of the cylinder. During this slow movement of the piston the molecule stays, of course, above the piston. However, it is no longer constrained to the upper half of the cylinder but bounces many times against the piston which is already in the lower half of the cylinder. In this way the molecule does a certain amount of work on the piston. This is the work that corresponds to the isothermal expansion of an ideal gas—consisting of one single molecule—from volume V_1 to the volume $V_1 + V_2$. After some time, when the piston has reached the bottom of the container, the molecule has again the full volume $V_1 + V_2$ to move about in, and the piston is removed. The procedure may be repeated many times. The man pushes up or down depending on whether the molecule is trapped in the upper or lower half of the piston. To express it more exactly we have a procedure of coupling the transmission of force in such a way that a weight is always being lifted by the piston. In this way the potential energy of the weight certainly increases constantly. (The transmission of force to the weight is best performed in such a way that the force exerted by the weight on the piston at any position of the latter equals the average pressure of the gas). It is clear that in this manner energy is constantly gained at the expense of heat, as long as the biological functions of the intervening man are ignored in the calculation.

In order to recognize how the man really affects the system, one best imagines that the movement of the piston is performed mechanically and that the man's activity consists only in registering the altitude of the molecule and in pushing a lever (which controls the movement of the piston) to the right or left, depending on whether the registered height necessitates a down or upward movement. This means that the intervention of the human being consists only in the coupling of two position co-ordinates, namely a co-ordinate x, which determines the altitude of the molecule, with another co-ordinate y, which determines the position of the lever and which determines an upward or downward motion of the piston. It is best to imagine the mass of the piston as large and its speed sufficiently great, so that the thermal agitation of the piston at the temperature in question can be neglected.

In the typical example presented here, we wish to distinguish two periods, namely:

1. The period of *measurement* when the piston has just been inserted in the middle of the cylinder and the molecule is trapped

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either in the upper or lower half; so that if we choose the origin of co-ordinates appropriately, the x-co-ordinate of the molecule is restricted to either the interval $x > 0$ or $x < 0$;

2. The period of *utilization of the measurement*, "the period of decrease of entropy," during which the piston is moving up or down. During this period the x-co-ordinate of the molecule is certainly not restricted to the original interval $x > 0$ or $x < 0$. Rather, if the molecule was in the upper half of the cylinder during the period of measurement i.e., when $x > 0$, the molecule must bounce on the downward-moving piston in the lower part of the cylinder, if it is to transmit energy to the piston; that is, the co-ordinate x has to be extended to the interval $x < 0$. The lever, on the contrary, retains during the whole period its position on the right because of the corresponding movement downward. If the position of the lever on the right is designated by $y = 1$ (and correspondingly the position on the left by $y = -1$) we see that during the period of measurement, the position $x > 0$ corresponds to $y = 1$; but afterwards $y = 1$ stays on, even though x passes into the other interval $x < 0$. We see that in the utilization of the measurements the coupling of the two parameters x and y disappears.

We shall generally say that a parameter y "measures" a parameter x (which varies according to a probability law), if the value of y depends on the value of parameter x at a given moment. A measurement procedure underlies the entropy decrease effected by the intervention of intelligent beings.

One may reasonably assume that a measurement procedure is in principle associated with a very definite average entropy production, and this restores concordance with the Second Law. The amount of entropy associated with the measurement may, of course, always be greater, but not smaller. To put it precisely: we have to distinguish here between two entropy values. One of them, $\bar{S}_1$, is produced when during the measurement y assumes the value 1, and the other, $\bar{S}_2$, when y assumes the value -1. We cannot expect to get general information about $\bar{S}_1$ or $\bar{S}_2$ alone, but we shall see that if the amount of entropy produced by the "measurement" is to compensate the entropy decrease affected by utilization, the general following equation derives:

$$e^{-\bar{S}_1/k} + e^{-\bar{S}_2/k} \leqq 1 \qquad (1)$$

One can see from this equation that one can reduce one of the values, for instance $\bar{S}_1$ at will, but then the other value $\bar{S}_2$ becomes correspondingly greater. Furthermore one can notice that the magnitude of the considered interval is of no consequence. One can also easily understand that it cannot be otherwise.

On the other hand, as long as the entropies $\bar{S}_1$ and $\bar{S}_2$, produced by the measurements, satisfy Equation (1), we can be sure that the decrease of entropy later effected by the utilization of the measurement will be fully compensated for.

Before we proceed with the proof of Equation (1), let us see in the light of the above mechanical example, how all this fits together. For the entropies $\bar{S}_1$ and $\bar{S}_2$ produced by the measurements, we assume the following special case:

$$\bar{S}_1 = \bar{S}_2 = k \log 2 \qquad \cancel{(2)}$$

This assumption satisfies Unequality (1) and the mean value of the quantity of entropy produced by a measurement is (of course in this special case) independent of the frequencies w_1, w_2 of the two events:

$$\bar{S} = k \log 2 \qquad \cancel{(3)}$$

In this example one achieves decrease of entropy with the isothermic expansion:[2]

$$-\bar{s}_1 = -k \log \frac{V_1}{V_1 - V_2};$$

$$-\bar{s}_2 = -k \log \frac{V_2}{V_1 + V_2}, \qquad \cancel{(4)}$$

depending on whether the molecule was found in volume V_1 or V_2 when the piston was inserted. (The decrease of entropy equals the ratio of the quantity of heat lost from the heat reservoir in isothermic expansion to the temperature of the heat reservoir in question). Since in the above case the frequencies w_1, w_2 are analogous to the volumes V_1, V_2, the mean value of the produced entropy is (a negative number):

$$+\bar{s} = w_1 \cdot (+\bar{s}_1)\, w_2 \cdot (+\bar{s}_2) = \frac{V_1}{V_1 + V_2}$$

$$k \log \frac{V_1}{V_1 + V_2} + \frac{V_2}{V_1 + V_2} k \log \frac{V_1}{V_1 + V_2} \qquad \cancel{(5)}$$

As one can see, we have, indeed

$$\frac{V_1}{V_1 + V_2} k \log \frac{V_1}{V_1 + V_2} + \frac{V_2}{V_1 + V_2}$$

$$\cdot k \log \frac{V_2}{V_1 + V_2} + k \log 2 \geqq 0 \qquad \cancel{(6)}$$

And therefore:

$$\bar{S} + \bar{s} \geqq 0. \qquad \cancel{(7)}$$

In the special case considered, we would in effect have a full compensation for the decrease of entropy achieved by the utilization of the measurement.

We shall now forgo examining further special cases. We shall instead try with the help of a general observation to clarify the existing conditions and to derive equation (1). We shall therefore imagine the whole

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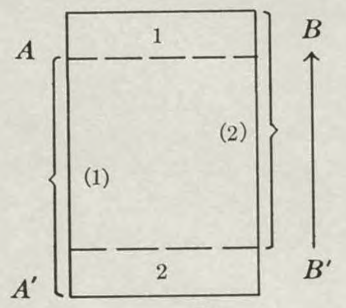
system—in which the co-ordinate x, exposed to some thermal fluctuations, can be measured by the parameter y in the way just explained—as a multitude of particles, all enclosed in one box. Every one of these particles can move freely, so that they may be considered as the molecules of an ideal gas, which, because of thermal agitation, wander about in the common box independently of each other and exert a certain pressure on the walls of the box—the pressure being determined by the temperature. We shall now recognize two of these molecules as chemically different and principally separable by semipermeable walls, if the co-ordinate x for one molecule is in a preassigned interval while the corresponding co-ordinate of the other molecule falls outside that interval. We also shall look upon them as chemically different, if they differ only in that the y co-ordinate is $+1$ for one and -1 for the other.

We should like to give the box in which the "molecules" are stored the form of a hollow cylinder containing four pistons. Pistons A and A' are fixed while the other two are movable, so that the distance BB' always equals the distance AA', as is indicated in Figure 1 by the two brackets. A', the bottom, and B, the cover of the container, are impermeable for all "molecules," while A and B' are semipermeable; namely, A is permeable only for those "molecules" for which the parameter x is in the preassigned interval, i.e., (x_1, x_2), B' is only permeable for the rest.



(8)

FIG. 1

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In the beginning the piston B is at A and therefore B' at A', and all "molecules" are in the space between. A certain fraction of the molecules have their co-ordinate x in the selected interval. We shall designate by w_1 the probability that this is the case for a randomly selected molecule and by w_2 the probability that x is outside the interval. Then $w_1 + w_2 = 1$.

Let the distribution of the parameter y be over the values $+1$ and -1 in any proportion but in any event independent of the value x. We imagine an intervention by an intelligent being, who imparts to y the value 1 for all "molecules" whose x at that moment is in the selected interval. Otherwise the value -1 is assigned. If then, because of thermal fluctuation, for any "molecule," the parameter x should come out of the preassigned interval or, as we also may put it, if the "molecule" suffers a monomolecular chemical reaction with regard to x (by which it is transformed from a species that can pass the semipermeable piston A into a species for which the piston is impermeable); then the parameter y retains its value 1 for the time being, so that the "molecule", because of the volume of the parameter y, is "reminded" during the whole following process that x originally was in the preassigned interval. We shall see immediately what part this "reminiscence" may play. After the intervention just discussed, we move the piston, so that we separate the two molecules without the use of energy. This results in two containers, of which the first contains only the one species and the second only the other. Each species now requires the same volume as the mixture did previously. In one of these containers, if considered by itself, there is now no equilibrium with regard to the two "species in x." Of course the ratio of the two species has remained $w_1{:}w_2$. If we create this equilibrium in both containers independently with constant volumes and temperature, then the entropy of the system certainly has increased because the total heat change is 0, while the ratio of the two "species in x" $w_1{:}w_2$ does not change. If we accomplish the equilibrium distribution in both containers in a reversible fashion then the entropy of the rest of the world will decrease by the same amount. Therefore the entropy increases by a negative value, e.g., the value of the entropy increase per molecule is:

$$\bar{s} = k(w_1 \log w_1 + w_2 \log w_2). \qquad \cancel{(9)}\ (2)$$

(The entropy constants in connection with the two "species in x" do not occur here explicitly, as the process leaves undisturbed the total number of molecules belonging to the one or the other species).

Now of course we cannot bring the two gases back to the original volume without expenditure of work by simply moving the piston back, as there are now in the container—which is bordered by the pistons BB'—also molecules whose x-co-ordinate lies outside of the preassigned interval and for which the piston A is not permeable any longer. Thus one can see that the calculated decrease of entropy (Equation [9]) does not mean a contradiction of the Second Law.

As long as we do not use the fact that the molecules in the container BB', by virtue of their co-ordinate y, "remember" that the x-co-ordinate for the molecules of this container originally was in the preassigned interval, full compensation exists for the calculated decrease of entropy, by virtue of the fact that the partial pressures in the two containers are smaller than in the original mixture.

Now we can use the fact that all molecules in the container BB' have the y-co-ordinate 1, and in the other accordingly -1, to bring all molecules back again to the original volume. To accomplish this we only need to replace the semipermeable wall A by a wall A^*, which is semipermeable not with regard to x but with regard to y, namely so that it is permeable for the molecules with the y-co-ordinate 1 and impermeable for the others. Correspondingly we replace B' by a piston

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B'^*, which is impermeable for the molecules with $y = -1$ and permeable for the others. Then both containers can be put into each other again without expenditure of energy. The distribution of the y-co-ordinate with regard to 1 and -1 now has become statistically independent, and besides we are able to re-establish the original distribution over 1 and -1. Thus we have gone through a complete cycle. The only change that we have to register is the resulting decrease of entropy given by (9):

$$\bar{s} = k(w_1 \log w_1 + w_2 \log w_2). \quad \cancel{(10)}$$

If we do not wish to admit that the second principle has been violated, we must conclude that the intervention which establishes the coupling between y and x, the measurement of x by y, cannot be separated from a production of entropy. If a definite method is chosen to realize this coupling and if the quantity of entropy that is inevitably produced is designated with S_1 and S_2, where S_1 stands for the mean increase in entropy that occurs when y obtains the value 1, and accordingly S_2 for the increase that occurs when y obtains the value -1, we arrive at the equation:

$$w_1 S_1 + w_2 S_2 = \bar{S} \quad \cancel{(11)}$$

In order for the Second Law to remain in force, this quantity of entropy must be greater than the decrease of entropy $\bar{s}$, which according to (9) is produced by the utilization of the measurement. Therefore the following unequality must be valid:

$$\bar{S} + \bar{s} \geqq 0$$

$$w_1 S_1 + w_2 S_2 \quad \cancel{(12)}$$

$$+ k(w_1 \log w_1 + w_2 \log w_2) \geqq 0 \quad (3)$$

This equation must be valid for any values of w_1 and w_2,[3] and of course the constraint $w_1 + w_2 = 1$ cannot be violated. We are investigating, in particular, at which w_1 and w_2 for given S-values the expression becomes a minimum. For the two minimizing values w_1 and w_2 the unequality (3) must still be valid. Under the above constraint, the minimum occurs when the following equation holds:

$$\frac{S_1}{k} + \log w_1 = \frac{S_2}{k} + \log w_2 \quad \cancel{(13)}$$

But then:

$$e^{-S_1/k} + e^{-S_2/k} \leqq 1. \quad \cancel{(14)}$$

It is easily seen that if one introduces the notation

$$\frac{S_1}{k} + \log w_1 = \frac{S_2}{k} + \log w_2 = \lambda, \quad \cancel{(15)}$$

then:

$$w_1 = e^{\lambda} \cdot e^{-S_1/k}; \; w_2 = e^{\lambda} \cdot e^{-S_2/k}. \quad (16)$$

If one substitutes these values into Equation (12) one gets:

$$\lambda e^{\lambda}(e^{-S_1/k} + e^{-S_2/k}) \geqq 0. \quad \cancel{(17)}$$

Therefore the following also holds:

$$\lambda \geqq 0. \quad (18)$$

If one puts the values w_1 and w_2 from (16) into the equations $w_1 + w_2 = 1$, one will get:

$$e^{-S_1/k} + e^{-S_2/k} = e^{-\lambda}. \quad (19)$$

And because $\lambda \geqq 0$, the following holds:

$$e^{-S_1/k} + e^{-S_2/k} \leqq 1. \quad (20)$$

This equation has to be universally valid, if the Second Law of Thermodynamics is not to be violated.

As long as we allow intelligent beings to perform the intervention, a direct test is not possible. But we can try to build simple nonliving devices that effect such coupling. Then we could see if indeed entropy is produced and in what quantity. Having recognized that the only important factor is a certain characteristic type of coupling, a "measurement," we need not construct any complicated models, which imitate the intervention of living beings. We can be satisfied with the construction of a particular type of coupling with memory.

In our example, the position co-ordinate of a pointer going back and forth is "measured" by the energy content of a body K. The pointer is supposed to bring about that the body K—by whose energy content the position of the pointer is to be measured—is connected by heat conduction with one of the two insertions, A or B. The body is connected with A as long as the co-ordinate—which determines the positions—falls into a certain preassigned, but otherwise arbitrarily large or small interval a, and otherwise if the co-ordinate is in the interval b, with B. Up to a certain moment, namely the moment of the "measurement," both insertions will be thermally connected with a heat reservoir at temperature T_0. At this moment the insertion A will be cooled reversibly to the Temperature T_A by a periodically functioning mechanical device. That is, after successive contacts with heat reservoirs of intermediate temperatures, A will be brought into contact with a heat reservoir of the temperature T_A. At the same time the insertion B will be brought in the same way to temperature T_B. Then the insertions will again be isolated from the corresponding heat reservoirs.

We assume that the position of the pointer changes so slowly that all the operations that we have sketched take place while the position of the pointer remains the same. If the position co-ordinate of the pointer fell in the preassigned interval, then the body was connected with the insertion A during the mentioned operation, and consequently was cooled to temperature T_A.

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Otherwise, the body would be heated to temperature T_B. Its energy content becomes—according to the position of the pointer at the time of "measurement"—small at temperature T_A or great at temperature T_B and will retain its value, even if the pointer eventually moves out of the preassigned interval or enters into it. After some time, while the pointer is still moving, one cannot draw a conclusion from the energy content of the body K with regard to the position of the pointer at a given moment, but one can draw a definite conclusion with regard to the position of the pointer at the time of the measurement. Then the measurement is completed.

After the measurement has been accomplished, the above-mentioned periodically functioning mechanical device should connect the thermally isolated insertions A and B with the heat reservoir T_0. This has the purpose of bringing the body K—which is now also connected with one of the two insertions—back into its original state. The direct connection of the insertions and hence of the body K—which has been either cooled to T_A or heated to T_B—to the reservoir T_0 consequently causes an increase of entropy. This cannot be avoided, because it would not be of advantage to heat the insertion A reversibly to the temperature T_0 by successive contacts with the reservoirs of intermediate temperatures and to cool B in the same manner. After the measurement we do not know with which of the two insertions the body K is in contact at that moment; nor do we know whether it had been in connection with T_A or T_B in the end. Therefore neither do we know whether we should use intermediate temperatures between T_A and T_0 or between T_0 and T_B.

The mean value of the quantity of entropy S_1 and S_2 for each measurement can be calculated, if the heat capacity as a function of the temperature $u(T)$ is known for the body K, since the entropy can be calculated from the heat capacity. We have, of course, neglected the heat capacities of the insertions. If the position co-ordinate of the pointer was in the preassigned interval at the time of the "measurement," and accordingly the body in connection with insertion A, then the entropy conveyed to the heat reservoirs during successive cooling was

$$\int_{T_A}^{T_0} \frac{1}{T}\frac{du}{dT}. \qquad (21)$$

However, following this, the entropy withdrawn from the reservoir T_0 by direct contact with it was

$$\frac{u(T_0) - uT_A)}{T_0}. \qquad (22)$$

All in all the entropy was increased by the amount

$$S_A = \frac{u(T_A) - u(T_0)}{T_0} + \int_{T_A}^{T_0} \frac{1}{T}\frac{du}{dT}dT. \quad (23)$$

Accordingly the entropy will increase by the following amount, if the body was in contact with the insertion B at the time of the "measurement":

$$S_B = \frac{u(T_B) - u(T_0)}{T_0} + \int_{T_B}^{T_0} \frac{1}{T}\frac{du}{dT}dT. \quad (24)$$

We shall now apply these equations to the very simple case, where our body has only two energy states, a lower and a higher state. If such a body is in thermal contact with a heat reservoir at any temperature T_1, the probability that it is in the lower or upper state respectively is given by:

$$\left.\begin{aligned} p(T) &= \frac{1}{1 + ge^{-u/kT}} \\ q(T) &= \frac{ge^{-u/kT}}{1 + ge^{-u/kT}} \end{aligned}\right\} \qquad (25)$$

Here u stands for the difference of energy of the two states and g for the statistical weighting. We can set the energy of the lower state equal to zero without loss of generality. Therefore:[4]

$$\left.\begin{aligned} S_A &= q(T_A)\, k \log \frac{q(T_A)\, p\,(T_0)}{q(T_0)\, p\,(T_A)} \\ &\qquad + k \log \frac{p(T_A)}{p(T_0)} \\ S_B &= p(T_B)\, k \log \frac{q(T_0)\, p\,(T_B)}{q(T_B)\, p\,(T_0)} \\ &\qquad + k \log \frac{q(T_B)}{q(T_0)} \end{aligned}\right\}. \qquad (26)$$

Here q and p are functions of T given by equation (25). The arguments of the function are to be T_0, T_A, or T_B, whichever applies.

If we wish to draw a dependable conclusion from the energy content of the body K to the position co-ordinate of the pointer, we have to see to it that the body surely gets into the lower energy state when it gets into contact with T_A, and surely into the upper energy state when it gets into contact with T_B. In other words:

$$p(T_A) = 1, q(T_A) = 0; p(T_B) = 0, q(T_B) = 1. \qquad (27)$$

This of course cannot be achieved, but may be arbitrarily approximated by setting T_A near absolute zero and by making the statistical weighting g infinitely large when the threshold is crossed. T_0 is also changed, so that $p(T_0)$ and $q(T_0)$ become fixed. The equation (26) accordingly becomes:

$$S_A = -k \log p(T_0); \quad S_B = -k \log q(T_0) \qquad (28)$$

And if we form the expression $e^{-S_A/k} + e^{-S_B/k}$, we find:

$$e^{-S_A/k} + e^{-S_B/k} = 1. \qquad (29)$$

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$$e^{-S_A/k} + e^{-S_B/k} = 1. \qquad (29)$$

Hence the permissible limiting case could just be realized. The use of semipermeable walls according to Figure 1 allows full utilization of the measurement. Inequality (1) certainly cannot be sharpened.

As we have seen in this example, a simple nonliving device can achieve the same essential result as would be achieved by the intervention of intelligent beings. We have examined the "living functions" of a nonliving device and have seen that exactly the quantity of entropy is in it produced that is required by thermodynamics.

APPENDIX

In the case considered, when the frequency of the two states depends on the temperature according to the equations:

$$p(T) = \frac{1}{1 + ge^{-u/kT}}\,; q(T) = \frac{ge^{-u/kT}}{1 + ge^{-u/kT}} \qquad (30)$$

and the mean energy of the body is given by:

$$u(T) = uq(T) = \frac{uge^{-u/kT}}{1 + ge^{-u/kT}}. \qquad (31)$$

u

The following identity becomes valid:

$$\frac{1}{T}\frac{du}{dT} = \frac{d}{dT}\left\{\frac{u(T)}{T} + k \log\left(1 + e^{-u/kT}\right)\right\}. \qquad (32)$$

Therefore we can write the equation:

$$B_A = \frac{u(T_A) - (T_0)}{T_0} + \int_{T_A}^{T_0} \frac{1}{T}\frac{du}{dT}\, dT. \qquad (33)$$

Also this way:

$$S_A = \frac{u(T_A) - u(T_0)}{T_0} + \left\{\frac{u(T)}{T} + k \log(1 + ge^{-u/kT})\right\}_{T_A}^{T_0}, \qquad (34)$$

and by substituting the limit we obtain:

$$S_A = u(T_A)\frac{1}{T_0} - \frac{1}{T_A} + k \log \frac{1 + ge^{-u/kT_0}}{1 + ge^{-u/kT_A}}. \qquad (35)$$

If we express the latter equation according to (25):

$$1 + ge^{-u/kT} = \frac{1}{p(T)} \qquad (36)$$

For T_A and T_0, then we obtain:

$$S_A = u(T_A)\left(\frac{1}{T_0} - \frac{1}{T_A} + k \log \frac{p(T_A)}{p(T_0)}\right) \qquad (37)$$

And if we thus write according to (31):

$$u(T_A) = uq(T_A) \qquad (38)$$

then we obtain:

$$S_A = q(T_A)\left(\frac{u}{T_0} - \frac{u}{T_A}\right) + k \log \frac{p(T_A)}{p(T_0)}. \qquad (39)$$

We finally write according to (25):

$$\frac{u}{T} = -k \log \frac{q(T)}{gp(T)}. \qquad (40)$$

For T_A and T_0, then we obtain:

$$S_A = p(T_A)\, k \log \frac{p}{q}\frac{(T_0)}{(T_0)}\frac{q(T_A)}{p(T_A)} + k \log \frac{p(T_A)}{p(T_0)}. \qquad (41)$$

We obtain the corresponding equation for S_B, if we interchange the index A with B. Then we obtain:

$$S_B = q(T_B)\, k \log \frac{p(T_0)}{q(T_0)}\frac{q((T_B)}{p((T_B)} + k \log \frac{p(T_B)}{p(T_0)}. \qquad (42)$$

Formula (41) is identical with (26), given for S_A in the text.

We can also obtain the formula for S_B, if we write:

$$q(T_B) = p(T_B), \qquad (43)$$

expand and collect terms, then we get

$$S_B = p(T_B)\, k \log \frac{q(T_0)}{p(T_0)}\frac{p(T_B)}{q(T_B)} + k \log \frac{q(T_B)}{q(T_0)}. \qquad (44)$$

This is the formula given in the text for S_B.

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