365-16

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RESUME -- Lest We Forget

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8-31-44 - The investment and coal consumption of a separation plant is taken to be proportionate to the entropy change which is required to transform normal material containing a_o kg of 25 and b_o kg of 28 into an enriched mixture containing a_2 kg of 25 and b_2 kg of 28 and an impoverished mixture containing a_1 kg of 25 and b_1 kg of 28.

We describe by α the ratio of the amount of 25 entering the separation plant in natural uranium to the amount of 25 which comes out in the form of the enriched mixture and we write $\beta = \alpha - 1$. We describe by f the enrichment factor defined by $f \frac{a_0}{b_0} = \frac{a_2}{b_2}$

$$\cancel{a^2 \text{ or } b^2 = f \frac{a^o}{b^o}}$$

The entropy change (ΔS) required is

$$\Delta S = S_0 - (S_1 + S_2)$$

and we are interested in $\frac{\Delta S}{a_2}$ or the amount of entropy change required per kg 25 contained in the enriched mixture.

Since the ratios a/b are small for natural uranium and for impoverished uranium the expressions for S_o and S_1 are simplified to

$$S_i = a_i \left\{ 1 + \log \frac{b_i}{a_i} \right\}$$

Consequently we can write

(1) $$\frac{\Delta S}{a_2} = \log f - (\alpha - 1) \log \frac{\alpha - 1/f}{\alpha - 1} + \left\{ 1 - \left(\frac{b_2}{a_2} + 1 \right) \ln\left(1 + \frac{a_2}{b_2}\right) \right\}$$

The third term is independent of α and in addition it is small so that leaving it off gives an error of only about 2% for f = 10. and less for $f < 10$.

It is $\frac{a_1}{b_1} = \frac{\alpha - 1}{(\alpha - 1/f)} \frac{a_0}{b_0}$ or

(2) $$\alpha = \frac{\frac{a_0/b_0}{a_1/b_1} - \frac{1}{f}}{\frac{a_0/b_0}{a_1/b_1} - 1}$$

~~Feld checked it~~ Feld checked it Sept 24 '44 per short ton

Price for Unit ΔS.--For an actual separation plant for which α = 2 and f is 1.25 corresponding to .6% of 25 in the impoverished mixture and .9% of 25 in the enriched mixture we may have an 80% efficiency in converting from coal to steam and may use 660 tons of coal per kg of 25 contained in the enriched mixture. This is based on coal which has 7500 large calories per kg. Since equation (1) gives for f = 1.25 and α = 2, $\frac{\Delta S}{\Delta z}$ = 25 we would need per unit ΔS 660 x 25 = 16,500 tons (short) of coal, or at a price of \$2.00, \$33,500 per unit ΔS. This does not include amortization of plant and ~~maintenance~~ other operating cost.

~~Since $\alpha = 2$ amount of 25 entering plant is 2×25 kg per unit ΔS and amount of normal uranium $\frac{2 \times 25}{\frac{0.72}{100}} = 6700$ kg~~

P_1, the price for 1 kg of 25 in natural uranium is Price of a Ton of U taking \$7200 per Ton (long) of $\frac{7.2}{\ }$ U $P_1 = \$1000$ and $\frac{P_2}{P_1} = 32$

minimizing the price

$$\text{Price} = \alpha P_1 + \frac{\Delta S}{\Delta z} P_2$$ should be made minimum with respect to α for a given f. This gives (same as minimizing $\left\{(\alpha - 1)P_1 + \frac{\Delta S}{\Delta z} P_2\right\}$)

$$(3) \qquad \frac{P_1}{P_2} = \ln \frac{\alpha - 1/f}{\alpha - 1} - \frac{1 - 1/f}{\alpha - 1/f}$$

for $\left.\begin{array}{l}\alpha = 2 \\ f = 1.25\end{array}\right\} \quad \frac{P_2}{P_1} = 64$

These would be optimal if cost$/_{\Delta S}$ were twice as much as above cost for coal alone (or of cost in \#4 ~~per ton of coal~~)

(3)

3.) If this cost is less optimal $\alpha < 2$.
For high values of f ~~the~~ value of $\frac{\Delta n}{\Delta S}$ becomes small (the coal cost per kg 25 high) and optimum shifts to higher values of α. For instance at $\frac{P_2}{P_1} = 32$, $f = 10$ $\alpha_m = 4$. — → fig (1)
(fig 3)

Plot α_m and $\left(\alpha_m P_1 + \frac{\Delta S}{\Delta n} P_2\right)$ as a function of f for $\frac{P_2}{P_1} = 64$!
This will be needed to determine which is best α to use in power plant from point of view of economy. —

for $\alpha = 2$ and $f - 1 \ll 1$ ratio of $\frac{\text{U expend}}{\text{Separation expend}}$

$$\frac{\alpha P_1}{\frac{\Delta S}{\Delta n} P_2} = 1$$ or more generally for $f - 1 \ll 1$

$$= \frac{1}{\alpha - 1}$$ neglected any ε^3 terms

goes to zero for infinite α; this is reasonable for if U cost nothing we have $\alpha \to \infty$ but still a finite ΔS and finite cost of C; halfing up the U cost would then not appreciably change the C cost so the U cost must be very small. —
H U price $\to 0$
$\alpha \to 1$

Putting equation (2) into (3) and writing $\frac{a_0}{b_0} \Big/ \frac{a_1}{b_1} = F_1 \ (>1)$ we get $= \left(\frac{1}{f_1}\right)$

(4) $\frac{P_1}{P_2} = \ln F_1 + \frac{1-F_1}{F_1}$ i.e. for a given price ratio optimum means fixed concentration of impoverished mixture

"cost" of U25 in impoverished mixture

$$\frac{(\alpha-1)P_1}{\frac{\Delta S}{a_2} P_2} = \cancel{\frac{1}{\alpha}} = \cancel{\frac{F_1-1}{F_1-\frac{1}{F}}} \ \cancel{\left\{ \text{if } f-1 \ll 1, \ F-1 \ll 1 \right.}$$

(5) ~~value of~~ corresponding separation cost ~~[illegible]~~ $= \dfrac{\dfrac{1-F}{F_1-1}\ln F_1 + \dfrac{F-1}{F_1}}{\ln\dfrac{1}{F} - \dfrac{1-F}{F_1-1}\ln F_1}$

$F = \frac{1}{f} = \frac{1}{\varphi}$

Fig 2 { Plot this (as function of F) for two values of F_1 / corresponding to 32 and 64 ratios

while $\frac{\Delta S}{a_2}$ goes with ε^2 for a constant α, the situation is different for optimum Price i.e. fixed composition of the impoverished mixture; in this latter case as ε goes to zero α goes to 1. — (see Fig 3)