Conditions For A Chain Reaction

~~If~~ Since q denotes the fraction of fast neutrons emitted by uranium which are slowed down to the thermal region and are absorbed as thermal neutrons by uranium and ~~if~~ since μ denotes the number of fast neutrons produced on the average by uranium for one thermal neutron absorbed by uranium then obviously

(49) $$\mu q > 1$$

is the condition for the possibility of a chain reaction. If this condition is fulfilled then a divergent chain reaction can be maintained in a sufficiently large system from which only a small fraction of the neutrons emitted by the uranium within ~~the system~~ can escape across the boundary of the system without being absorbed within ~~the system~~.

Accordingly, the condition for the possibility of a chain reaction in a system composed of a lattice of uranium spheres embedded in carbon

(50) $$\mu q_{corr} > 1$$

and using equation No. 30 we find

$$\mu q_m \times 0.9 > 1 \quad \text{or} \quad q_m > 1.11/\mu$$

From which we find by using equation No. 27 as a sufficient condition for the possibility of a chain reaction

(51) $$\frac{4.44\mu}{(\mu - 1.1)^2} < \varepsilon$$

Using the value of $\mu = 2$ ~~as a presumably conservative value~~ we have as a sufficient condition

$$11.3 < \varepsilon$$

In order to see now whether a chain reaction is possible we have to calculate from our formulae the numerical value of ε. We shall do that in the following under the assumption that the energy liberated in the chain reaction will maintain the carbon at a temperature of about 900C.° and in order to be on the conservative side *(27) we shall assume that the temperature of the uranium spheres in which most of the energy is liberated is , in spite of efficient cooling, about the same.

Since we have at room temperature $\sigma_c(C) \leq 0.01$ we shall have at 900 C. a capture cross-section of carbon half of this value. The scattering cross-section of uranium for thermal neutrons we take to be $\sigma_s(U) \cong 9$. Finally, at room temperature we take $\frac{\sigma_a(U)}{\sigma_s(U)} = 1/2$ and correspondingly we take at 900 C. $\frac{\sigma_a(U)}{\sigma_s(U)} = 1/4$. For a density of graphite of 1.7 and a density of uranium of 15 we then obtain from No. 14 for $R = 8$ cm *(28)

$$\varepsilon = 14$$

This being larger than the value required by No. 51 ~~we conclude that in the circumstances~~ we can just about expect a divergent chain reaction to take place in the system ~~which we have investigated.~~ for $\sigma_c(C) = 0.01$

In reality the capture cross-section of carbon ~~may of course be~~ is perhaps much smaller than the upper limit which has so far been established and consequently there is hope that conditions will be much more favorable for a chain reaction than would seem from the values so far quoted.

The amount of carbon and uranium required to reach the point of divergence at which nuclear transmutation will proceed at a rate limited only by the necessity of avoiding over-heating is essentially determined by the value of

$$(\mu q - 1)$$

~~In order to see clearly how this value depends on the capture cross-section of carbon on the one hand the values of and~~

In the following we shall calculate how the value of this expression depends on the value of the carbon capture cross-section at room temperature.

We shall take the density of graphite to be 1.7; the density of uranium metal to be 15 and choose R = 8cm. *28

We then obtain for a capture cross-section of carbon at room temperature of 0.005 the following set of values at 900 C°: $A = 64\,cm$; $\frac{A^2}{B^2} = 90$; $\varphi = 0.666$ $\varepsilon = 27$; $q_m = 0.681$; $q_{corr} = 0.66$ and for $\mu = 2$

$$(\mu q - 1) = 0.32$$

~~If the capture cross-section of carbon at room temperature were 0.0033 then at 900 C. our equations would give the following set of values:~~

$\cancel{A = 75.5\,cm;\ \frac{A^2}{B^2} = 135;\ \varphi = 0.645;\ \varepsilon = 39;\ q_m = 0.728}$

$\cancel{q_{corr} = 0.71}$ ~~and for~~ $\cancel{\mu = 2}$

$$\cancel{(\mu q - 1) = 0.42}$$